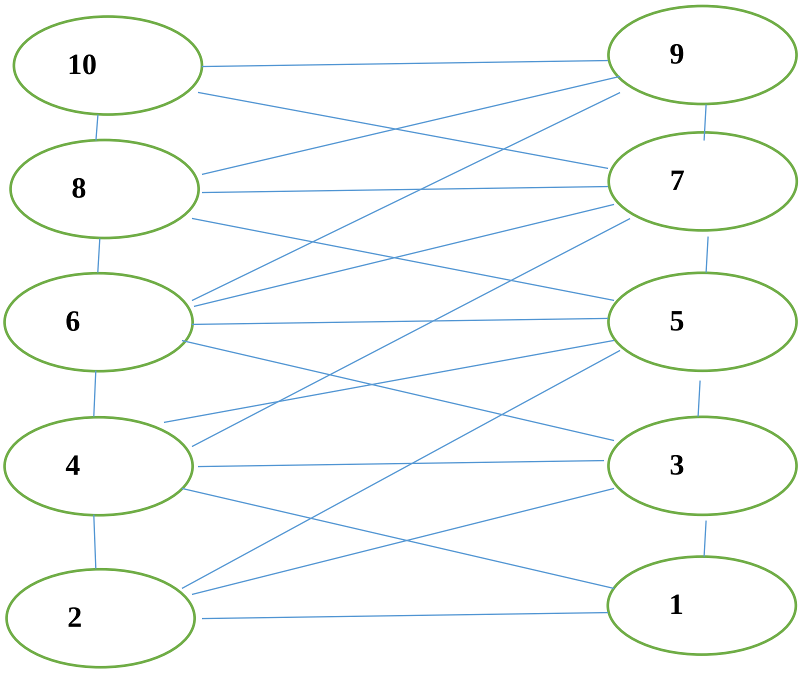
ASSIGNMENT 4

COURSE NAME: DISCRETE STRUCTURE
COURSE CODE: SECI 1013
SECTION: 03
LECTURER'S NAME: Dr. Nor Azizah Ali
GROUP NUMBER: 7

GROUP MEMBERS:

Name	Matric No.
LUQMAN ARIFF BIN NOOR AZHAR	A20EC0202
TERENCE A/L LOORTHANATHAN	A20EC0165
MADINA SURAYA BINTI ZHARIN	A20EC0203

1. Let G be a graph with $(G) = \{1, 2, ..., 10\}$, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if $| v – w| \leq 3$. Draw the graph G and determine the numbers of edges, $e(G)$.

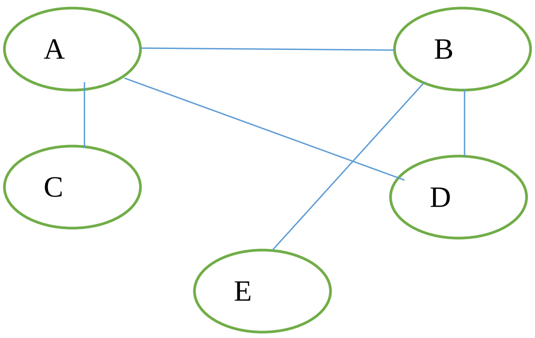


E(G) = 24

2. Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.

(a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.

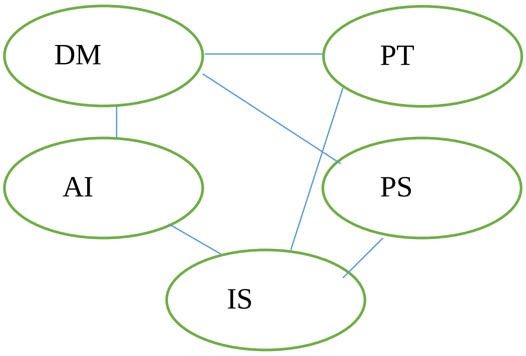
(Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)



	A	**B**	**C**	**D**	**E**
A	**0**	**1**	**1**	**1**	**0**
B	**1**	**0**	**0**	**1**	**1**
C	**1**	**0**	**0**	**0**	**0**
D	**1**	**1**	**0**	**0**	**1**
E	**0**	**1**	**0**	**1**	**0**

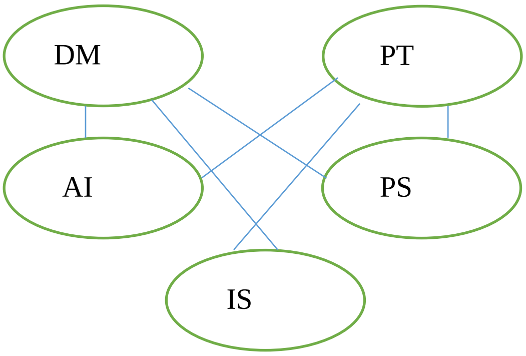
(b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot: -

i. DM and IS



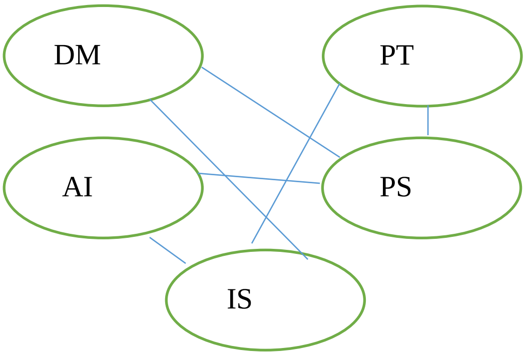
	DM	PT	AI	PS	IS
DM	**0**	**1**	**1**	**1**	**0**
PT	**1**	**0**	**0**	**0**	**1**
AI	**1**	**0**	**0**	**0**	**1**
PS	**1**	**1**	**0**	**0**	**1**
IS	**0**	**1**	**1**	**1**	**0**

ii. DM and PT



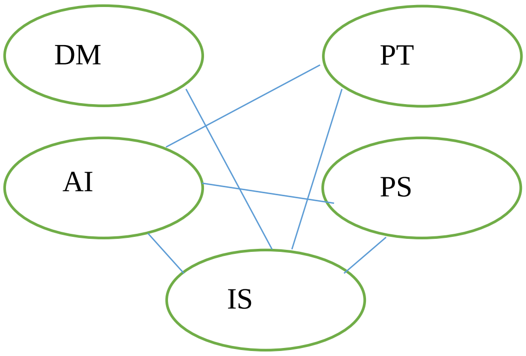
	DM	PT	AI	PS	IS
DM	**0**	**0**	**1**	**1**	**1**
PT	**1**	**0**	**1**	**1**	**1**
AI	**1**	**1**	**0**	**0**	**0**
PS	**1**	**1**	**0**	**0**	**0**
IS	**1**	**1**	**0**	**0**	**0**

iii. AI and PS



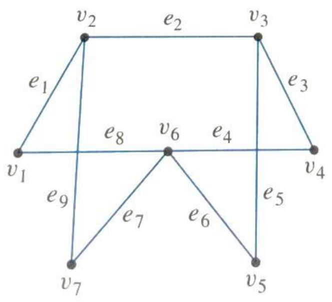
	DM	PT	AI	PS	IS
DM	**0**	**0**	**1**	**1**	**0**
PT	**0**	**0**	**1**	**1**	**0**
AI	**1**	**1**	**0**	**0**	**1**
PS	**1**	**1**	**0**	**0**	**1**
IS	**0**	**0**	**1**	**1**	**0**

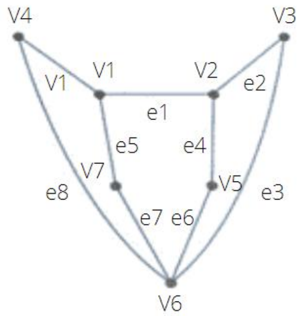
iv. IS and AI



	DM	PT	AI	PS	IS
DM	**0**	**0**	**1**	**0**	**1**
PT	**0**	**0**	**1**	**0**	**1**
AI	**1**	**1**	**0**	**1**	**0**
PS	**0**	**0**	**1**	**0**	**1**
IS	**1**	**1**	**0**	**1**	**0**

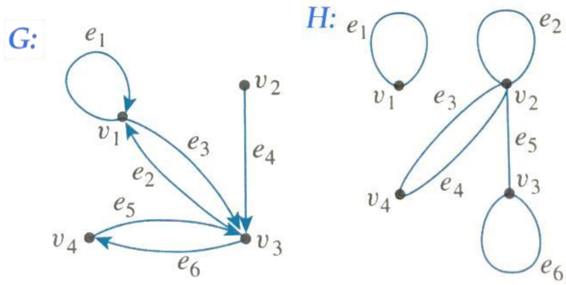
3. Show that the two drawing represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.





- **same number of vertices, edges**
- **same degree for corresponding vertices**
- **same number of connected components**
  These two graphs are isomorphic

4. Find the adjacency and incidence matrices for the following graphs.



Adjacency

G	V1	V2	V3	V4
V1	1	0	2	0
V2	0	0	1	0
V3	2	0	0	2
V4	0	0	2	0

H	V1	V2	V3	V4
V1	1	0	0	0
V2	0	1	1	2
V3	0	1	1	0
V4	0	2	0	0

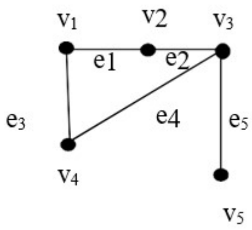
Incidence

G	E1	E2	E3	E4	E5	E6
V1	2	1	1	0	0	0
V2	0	0	0	1	0	0
V3	0	1	1	0	1	1
V4	0	0	0	0	1	1

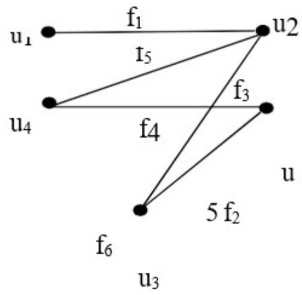
H	E1	E2	E3	E4	E5	E6
V1	2	0	0	0	0	0
V2	0	2	1	1	1	0
V3	0	0	0	0	1	2
V4	0	0	1	1	0	0

5. Determine whether the following graphs are isomorphic.

a)



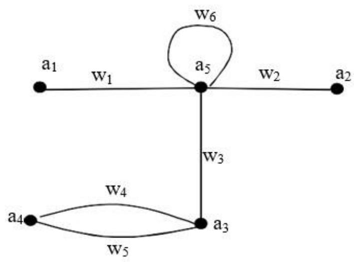
G1



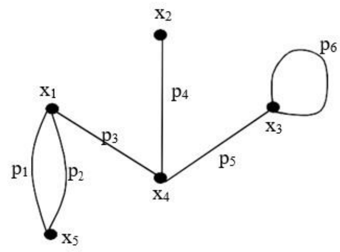
G2

Different degree for corresponding vertices, not isomorphic

b)



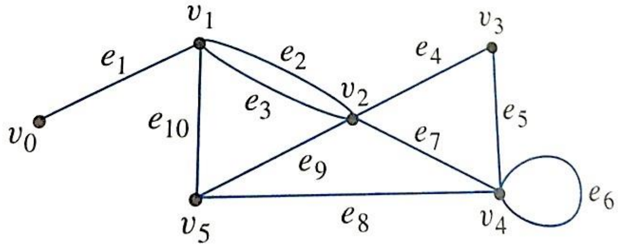
H1



H2

Different degree for corresponding vertices, not isomorphic

6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



a) $v_0 e_1 v_1 e_{10} v_5 e_9 v_2 e_2 v_1$

There is a vertex repeated but no edge is repeated, and it is not closed.
Therefore, it is a trail.

b) $v_4 e_7 v_2 e_9 v_5 e_{10} v_1 e_3 v_2 e_9 v_5$

Both vertex and edges are repeated but it is not closed.
Therefore, it is a walk.

c) v_2

Only one vertex is there so it is a walk.

d) $v_5 e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4 e_8 v_5$

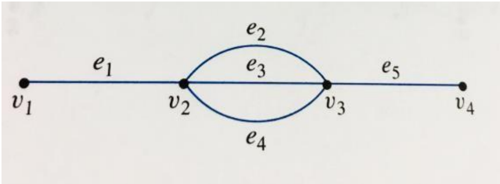
There is a vertex is repeated but edges are not, and it is closed.
Therefore, it is a closed trail that is a circuit.

e) $v_2 e_4 v_3 e_5 v_4 e_8 v_5 e_9 v_2 e_7 v_4 e_5 v_3 e_4 v_2$

Both edges and vertices are repeated, and it is closed.
Therefore, it is a closed walk.

There are no vertices that are repeated, nor the edges and it is not closed.
Therefore, it is just a path.

7. Consider the following graph.



(a) How many paths are there from v_1 to v_4?

(V1, e1, V2, e2, V3, e5, V4), (V1, e1, V2, e3, V3, e5, V4), (V1, e1, V2, e4, V3, e5, V4)

Therefore, there are 3 paths. No vertices and no edge are repeated.

(b) How many trails are there from v_1 to v_4?

(V1, e1, V2, e2, V3, e5, V4), (V1, e1, V2, e3, V3, e5, V4), (V1, e1, V2, e4, V3, e5, V4)

Therefore, there are 3 trails. No repeated edge.

(c) How many walks are there from v_1 to v_4?

Paths that are also walks:

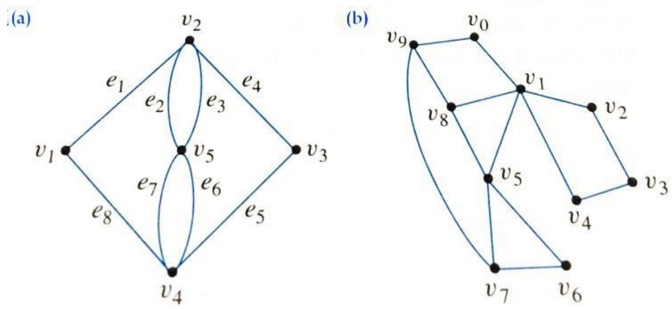
(V1, e1, V2, e2, V3, e5, V4), (V1, e1, V2, e3, V3, e5, V4), (V1, e1, V2, e4, V3, e5, V4)

Walk other than Path:

(V1, e1, V2, e2, V3, e4, V2, e3, V3 …)

From above there are 3 walks which are paths, excluding that there are infinite possible walks. Given graph has loops between V2 and V3 with non-directed edges which enables the possibility of edges and vertices infinite times.

8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



a) Graph (a) does have a euler circuit

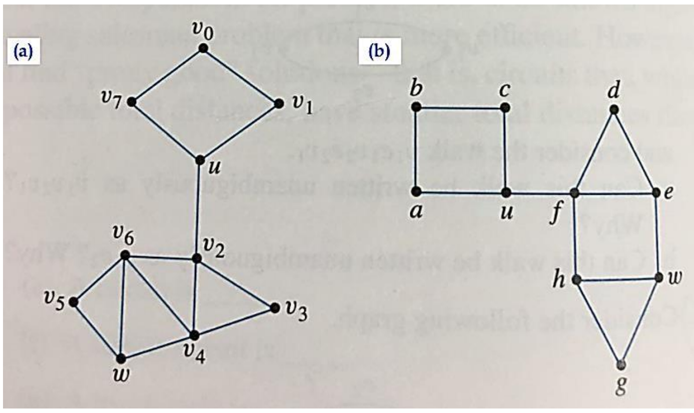
: $(V_1, e_1, V_2, e_2, V_5, e_3, V_2, e_4, V_3, e_5, V_4, e_6, V_5, e_7, V_4, e_8$
$V_1)$

b) According to Euler theorem, if graph has a euler circuit every vertex has even degree.

Vertex	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9
Degree	2	4	2	2	2	4	2	3	3	3

Therefore, observe that Vertex V_7, V_8, V_9 has 3 degree which is an odd amount of degree.
So (b) does not have a euler circuit.

9. For each of graph in (a) – (b), determine whether there is a Euler path from u to w. If there is, find such a path.



a) According to Euler theorem, u and w to have a euler trail must be connected and be the only vertices having odd degree.

Since u and w is connected and u and w only have odd degree, therefore (a) has a euler trail.

$(u, V_7, V_0, V_1, u, V_2, V_6, V_5, w, V_6, V_4, V_2, V_3, V_4, w)$

b) Based on Euler theorem, u and w to have a euler trail must be connected and be the only vertices having odd degree.

Vertices	u	a	b	c	d	e	f	g	h	w
Degree	3	2	2	2	2	3	4	2	3	3

Since vertices e, h has an odd number of degree (b) does not have a euler trail.

10. For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

Hamilton circuit does not exist for the graph in (a) – (b).

11. How many leaves does a full *3-ary* tree with 100 vertices have?

$$\boldsymbol{L = \frac{[(m-1)n+1]}{m}}$$

$$\boldsymbol{L = \frac{[(3-1)100+1}{3} = 67}$$

12. Find the following vertex/vertices in the rooted tree illustrated below.

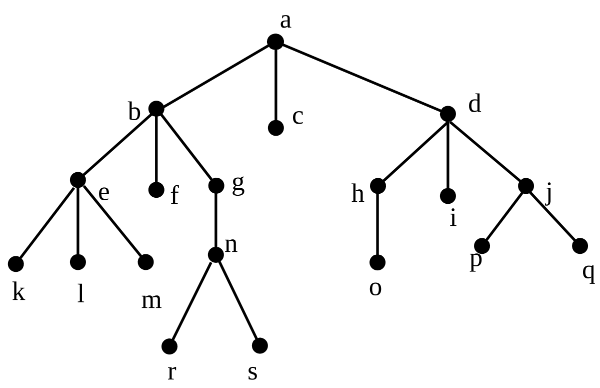


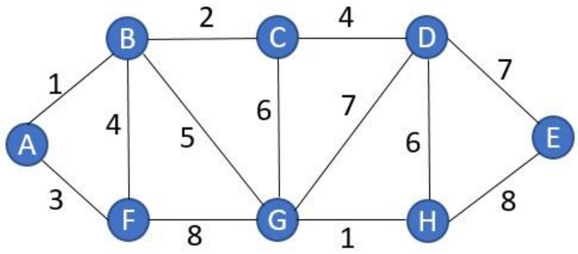
Figure 1

a) Root = **a**
b) Internal vertices = **none**
c) Leaves = **(c, f, i, k, l, m, r, s, o, p, q)**
d) Children of n = **(r, s)**
e) Parent of e = **(b)**
f) Siblings of k = **(l, m)**
g) Proper ancestors of q = **(a, d, j)**
h) Proper descendants of b = **(e, f, g, k, l, m, n, r, s)**

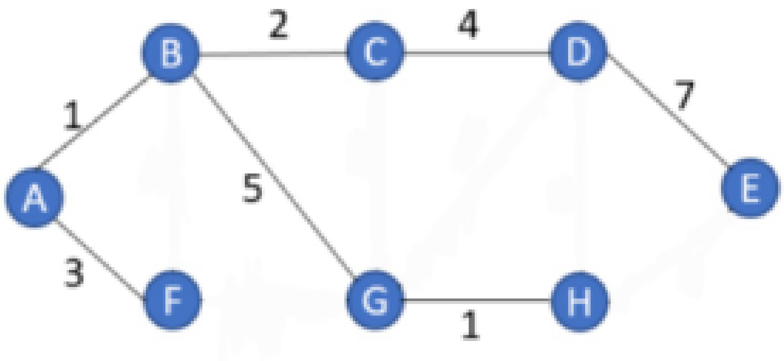
13. In which order are the vertices of ordered rooted tree in **Figure 1** is visited using *preorder, inorder* and *postorder*.

preorder = a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q
inorder = k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q
postorder = k, l, m, e, f, r, s, n, g, b, c, o, h, I, p, q, j, d, a

14. Find the minimum spanning tree for the following graph using Kruskal's algorithm.

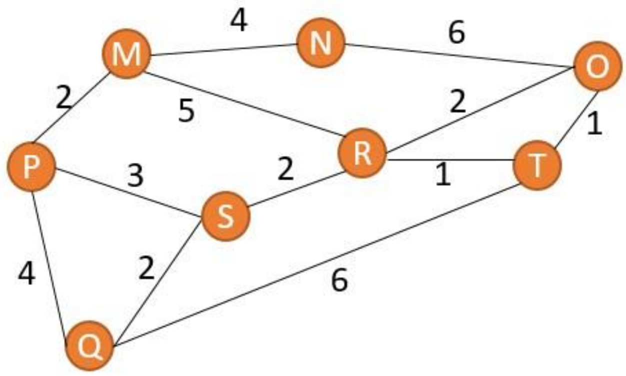


GH	**AB**	~~**BC**~~	**AF**	**BF**	**CD**	**BG**	~~**CG**~~	~~**DH**~~	~~**DG**~~	**DE**	~~**FG**~~	~~**HE**~~
1	**1**	~~**2**~~	**3**	**4**	**4**	**5**	~~**6**~~	~~**6**~~	~~**7**~~	**7**	~~**8**~~	~~**8**~~



Total weight = 1 + 1 + 2 + 3 + 4 + 5 + 7 = 23

15. Use Dijsktra's algorithm to find the shortest path from **M** to **T** for the following graph.



Iteration	*S*	*N*	*L(M)*	*L(N)*	*L(O)*	*L(P)*	*L(Q)*	*L(R)*	*L(S)*	*L(T)*
0	**{ }**	**{M, N, O, P, Q R, S, T}**	**0**	∞	∞	∞	∞	∞	∞	∞
1	**{M}**	**{N, O, P, Q, R, S, T}**	**0**	**4**	∞	**2**	∞	**5**	∞	∞
2	**{M, P}**	**{N, O, Q, R, S, T}**	**0**	**4**	∞	**2**	**6**	**5**	**5**	∞
3	**{M, P, N}**	**{O, Q, R, S, T**	**0**	**4**	**10**	**2**	**6**	**5**	**5**	∞
4	**{M, P, N, R}**	**{O, Q, S, T}**	**0**	**4**	**7**	**2**	**6**	**5**	**5**	**6**
5	**{M, P, N, R, S}**	**{O, Q, T}**	**0**	**4**	**7**	**2**	**6**	**5**	**5**	**6**
6	**{M, P, N, R, S, Q**	**{O, T}**	**0**	**4**	**7**	**2**	**6**	**5**	**5**	**6**
7	**{M, P, N, R, S, Q, T}**	**{O}**	**0**	**4**	**7**	**2**	**6**	**5**	**5**	**6**

Shortest path: $M \rightarrow R \rightarrow T$
Shortest length = 6