SECI 1013

DISCRETE STRUCTURE

SEMESTER 1 2020/2021

TUTORIAL 2:

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No. :

Date :

TUTORIAL 2.

1 a. $\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ } = 6^3$

$6 \quad 6 \quad 6 = 216$

b. $\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ } = 6 \times 5 \times 4$

$6 \quad 5 \quad 4 = 120$

c. odd digits = 3, 5, 7

$\underline{3}\ \underline{\ \ }\ \underline{\ \ } = 1 \times 3 \times 3$ $\quad$ $\underline{5}\ \underline{\ \ }\ \underline{\ \ } = 1 \times 3 \times 3$

$1 \quad 3 \quad 3 = 9$ $\quad$ $1 \quad 3 \quad 3 = 9$

total numbers = 9 + 9

= 18

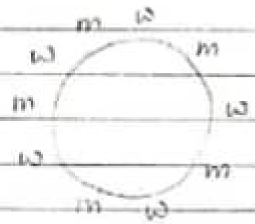
2. men = 5 women = 5

a. $(6-1)! \times 5! = 14400$

b. $(9-1)! \times 2! = 80640$

c. 5 women sits alternatively = $(5-1)!$

= 24 ways



The 5 mens is sat in the gaps = $5!$

= 120 ways

total num of ways = 24×120

= 2880

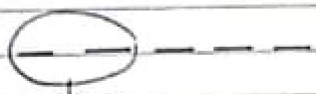
d. $\underbrace{\underline{A}\ \underline{H}}_{2! \times 11}\ \underbrace{\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }}_{10!}$

$2! \times 11! = 79833600$ ways

No. :

Date :

3 a. $5! = 120$

b. 

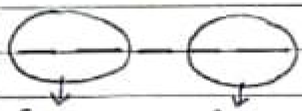
tie : ${}^5C_2$

$= C(5,2) \times 4!$

$= \dfrac{5!}{2!\,(5-2)!} \times 4!$

$= 10 \times 24$

$= 240$

c. 

${}^5C_2$ ${}^3C_2$

$= C(5,2) \times C(3,2) \times 3!$

$= \dfrac{5!}{2!\,(5-2)!} \times \dfrac{3!}{2!\,(3-2)!} \times 3!$

$= 10 \times 3 \times 6$

$= 180$

4 a. $n = 6 \quad r = 12$ $\quad C(6+12-1,\ 12) = \dfrac{(6+12-1)!}{12!\,(6-1)!}$

$= 6188$

b. croissants with 2 of each kind : $2 \times 6 = 12$

remaining croissants $= 24 - 12 = 12$

$n = 6 \quad r = 12$

repetitions is allowed :

$C((6+12-1),\ 12) = \dfrac{(6+12-1)!}{12!\,(6-1)!}$

$= \dfrac{17!}{12!\ 5!}$

$= 6188$

No. :

Date :

c. num of ways to choose ;

5 chocolate croissants = 5

3 almond croissants = 3

remaining croissants = 24 − 5 − 3 = 16

$n = 6 \quad r = 16$

since repetitions are allowed ;

$$C(6+16-1, 16) = \frac{(6+16-1)!}{16!\,(6-1)!}$$

$$= \frac{21!}{16!\,5!}$$

$$= 20349$$

NO:.............................. DATE:..............................

5. a) 3 options = Team A wins, or Team B wins, or Both Team ties

For the round ends once it is impossible for a team to equal to the number of goals scored by other team.

2 options = 2 wins for one of the teams, with 1 additional ties or wins for that team

or

= 1 win for one of the teams, with 3 additional ties or wins for that team

2 wins among 4 games : $C(4,2) = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = 6$

1 win among 3 games : $C(3,1) = \frac{3!}{1!(3-1)!} = \frac{3!}{1!2!} = 3$

Next, the other games need to have result in either a win or a tie

2 options :

2 wins and 1 ties / wins : $C(4,2) \cdot C(3,1) \cdot 2 = 36$

1 wins and 1 ties / wins : $C(3,1) \cdot C(4,3) \cdot 2^3 = 96$

total ways $= 2 \times (36 + 96)$

$= 2 \times 132$

$= 264$ ways

b) Without restrictions $= 2^{10}$

$= 1024$

ways settled in first round $= 264$

ways not settled in first round $= 1024 - 264$

$= 760$

If there is a second round of 10 penalty kicks and settled in this round, it means the first round of 10 penalty kicks is not settled.

So, first round $= 760$

Second round $= 264$

Total $= 760 \times 264$

$= 200640$ ways

NO:..............................

DATE:..............................

c) In sudden-death shootout, there is 3 options

1st Option: Team A win, Team B win, Both tie

2nd Option: Team A or Team B win, the game stops

3rd Option: There is a tie, there is another shootout

The game ends during this sudden-death shootout in 2 ways which is if Team A wins or Team B wins

If there is a sudden-death shootout, it means the first round or second round of the 10 penalty kicks are not settled, but the game was settled within the next 10 round of penalty kicks (sudden-death shootout)

First round = 760

Second round = 760

Sudden-death shootout $= 2+2+2+2+2$

$= 10$

Total $= 760 \times 760 \times 10$

$= 5776000$ ways #

NO:

DATE:

6)

$\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_$

$4\ \ 4\ \ 4\ \ 4\ \ 4\ \ 4\ \ 4\ \ 4\ \ 4\ \ 4 = 4^{10}$

$\therefore$ Possible answer sheets $= 4^{10}$

$= 1048576$

pigeonholes, k = answer sheets = 1048576

pigeons, n = number of students

$\left\lceil \frac{n}{1048576} \right\rceil = 3$

$n = 1048576 \times 2 + 1$

$= 2097153$

$\therefore$ The minimum number of students needed in the professor's class in order to guarantee that at least 3 answer sheets must be identical is the smallest integer n such that $\left\lceil \frac{n}{1048576} \right\rceil = 3$.

The smallest $n = 1048576 \times 2 + 1$

$= 2097153$.

If there is only 2097152 students, then it is possible for there is exactly 2 answer sheets are identical.

Thus, the minimum number of students that must be in the professor's class in order to guarantee that at least 3 answer sheets must be identical is 2097153.

NO:.................................   DATE:.................................

7) Let M = event of students passing Mathematics
Let H = event of students passing History

$P(H) = \frac{75}{100}$ $\qquad$ $P(M) = \frac{65}{100}$

$= 0.75$ $\qquad$ $= 0.65$

$P(H \cup M) = \frac{50}{100}$ $\qquad$ $|P(H \cup M)'| = 35$ students

$= 0.50$

$P(H \cup M) = P(H) + P(M) - P(H \cap M)$

$= 0.75 + 0.65 - 0.50$

$= 0.9$

$P(H \cup M)' = P(H' \cap M')$ De Morgan's Law

$= 1 - P(H \cup M)$

$= 1 - 0.9$

$= 0.1$

$P(H \cup M)' = 0.1$

$\frac{35}{\text{total students}} = 0.1$

$\text{total students} = \frac{35}{0.1}$

$= 350$ students #

NO:................ DATE:................

8) 3 _ _ = $1 \times 10 \times 10$ = 100

4 _ _ = $1 \times 10 \times 10$ = 100

5 _ _ = $1 \times 10 \times 10$ = 100

6 _ _ = $1 \times 10 \times 10$ = 100

7 _ _ = $1 \times 8 \times 10$ = 80

7 8 0 = $1 \times 1 \times 1$ = 1

Let S = Sample Space

= 100 + 100 + 100 + 100 + 80 + 1

= 481

3 1 _ = $1 \times 1 \times 10$ = 10

4 1 _ = $1 \times 1 \times 10$ = 10

5 1 _ = $1 \times 1 \times 10$ = 10

6 1 _ = $1 \times 1 \times 10$ = 10

7 1 _ = $1 \times 1 \times 10$ = 10

total = 10 + 10 + 10 + 10 + 10

= 50

3 _ 1 = $1 \times 9 \times 1$ = 9

4 _ 1 = $1 \times 9 \times 1$ = 9

5 _ 1 = $1 \times 9 \times 1$ = 9

6 _ 1 = $1 \times 9 \times 1$ = 9

7 _ 1 = $1 \times 7 \times 1$ = 7

total = 9 + 9 + 9 + 9 + 7

= 43

Let A = event of number is chosen will have 1 as at least one digit

= 50 + 43

= 93

$$P(A) = \frac{A}{S} = \frac{93}{481} = 0.193$$ *

No:

Date: ....

9\. a) $C(10,6) = \frac{10!}{6!4!}$

$= 210$

$\frac{6!}{4!2!} = 15$

$C(10,6) \times \frac{6!}{4!2!} = 210 \times 15$

$= 3150$ ways #

b) $C(7,6) \times \frac{6!}{4!2!} \times \frac{4!}{4!}$

$= \frac{7!}{6!1!} \times \frac{6!}{4!2!} \times \frac{4!}{4!}$

$= 7 \times 15 \times 1$

$= 105$ ways #

Let E denotes empty lots are next to one another.

$P(E) = \frac{\text{ways that empty lots are next to one another}}{\text{ways that cars be parked in the parking lots}}$

$= \frac{105}{3150}$

$P(E) = \frac{1}{30}$

$= 0.0333$ #

10\. Let E denotes "coach use email."

L denotes "coach use letter."

H denotes "coach use handphone."

M denotes "trainee receive message."

$P(E) = 0.4, P(L) = 0.1, P(H) = 0.5$

$P(M|E) = 0.6, P(M|L) = 0.8, P(M|H) = 1$

a) $P(M|E) = \frac{P(M \cap E)}{P(E)}$

$P(M \cap E) = P(E) \times P(M|E)$

$= 0.4 \times 0.6$

$P(M \cap E) = 0.24$

$P(M \cap L) = P(L) \times P(M|L)$

$= 0.1 \times 0.8$

$P(M \cap L) = 0.08$

$P(M \cap H) = P(H) \times P(M|H)$

$= 0.5 \times 1$

$= 0.5$

$P(M) = P(M \cap E) + P(M \cap L) + P(M \cap H)$

$= 0.24 + 0.08 + 0.5$

$= 0.82$ #

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No:

Date: ....

b) $P(E|M) = \dfrac{P(M|E)P(E)}{P(M|E)P(E) + P(M|L)P(L) + P(M|H)P(H)}$

$= \dfrac{0.24}{0.24 + 0.08 + 0.5}$

$= \dfrac{0.24}{0.82}$

$= \dfrac{12}{41}$

$= 0.2927$ #

11. Let L denotes light truck
C denotes cars
F denotes fatality

$P(L) = 0.4$ $P(C) = 0.6$

$P(F|C) = \dfrac{20}{100000} = 0.0002$

$P(F|L) = \dfrac{25}{100000} = 0.00025$

$P(F) = P(F|C)P(C) + P(F|L)P(L)$

$= 0.0002(0.6) + 0.00025(0.4)$

$= 0.00012 + 0.0001$

$= 0.00022$

$P(L|F) = \dfrac{P(F|L)P(L)}{P(F)}$

$= \dfrac{0.00025(0.4)}{0.00022}$

$= 0.4545$ #

No:

Date: ....

12. let X = total ways that a letter can be distributed into 4 boxes.
T = ways that tetrahedron box has no letters.
P = ways that polyhedron box has no letters.
C = ways that cube box has no letters.
D = ways that dodecahedron box has no letters.

Ways that at least 1 letter in each box
$= |X| - |T \cup P \cup C \cup D|$
$= 4^9 - (4 \times 3^9) - (4 \times 2^9) + (4 \times 1^9) - (4 \times 0^9)$
$= 262144 - 78732 - 2048 + 4 - 0$
$= 181368$ #

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