Assignment 4

SECI 1013 SECTION 01

DISCRETE STRUCTURE

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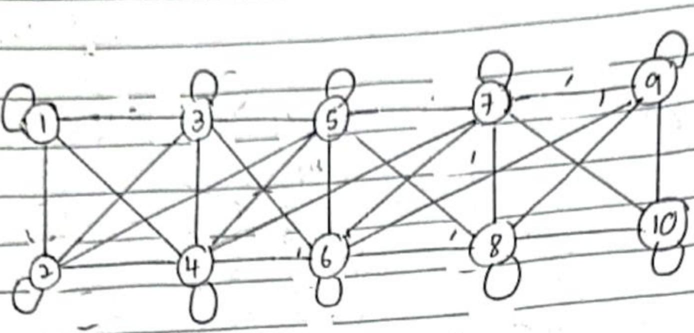
Semester 1 2020/2021

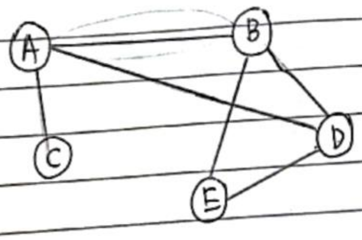
No.: ....

1. $V(G) = \{1, 2, 3, \ldots, 10\}$



$\therefore e(G) = 34$

2. a)



$$A_G = \begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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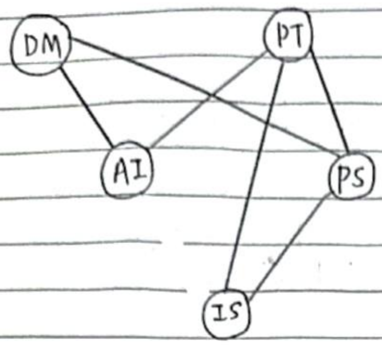
b



$$A_G = \begin{array}{cc} & \begin{array}{ccccc} \text{DM} & \text{PT} & \text{AI} & \text{PS} & \text{IS} \end{array} \\ \begin{array}{c} \text{DM} \\ \text{PT} \\ \text{AI} \\ \text{PS} \\ \text{IS} \end{array} & \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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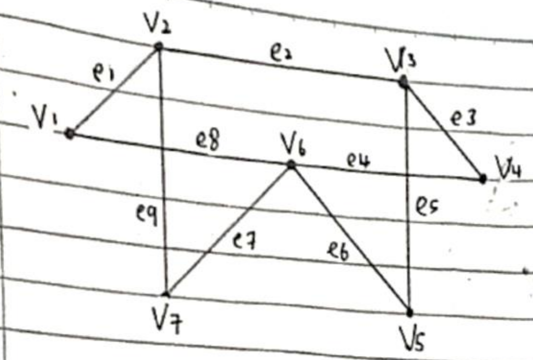
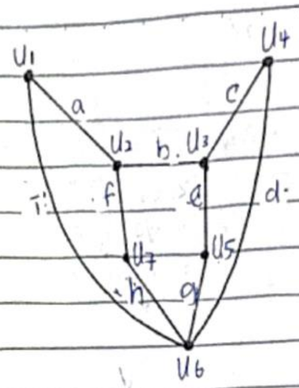
3.





$f(V_1) = U_1$
$f(V_2) = U_2$
$f(V_3) = U_3$
$f(V_4) = U_4$
$f(V_5) = U_5$
$f(V_6) = U_6$
$f(V_7) = U_7$

$E(e_1) = a$
$E(e_2) = b$
$E(e_3) = c$
$E(e_4) = d$
$E(e_5) = e$
$E(e_6) = g$
$E(e_7) = h$
$E(e_8) = i$
$E(e_9) = f$

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4. i)

$$A_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

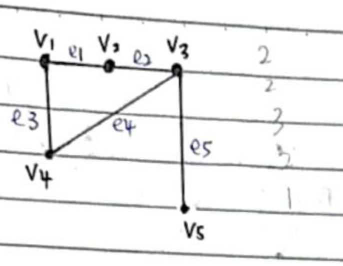
$$I_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

ii)

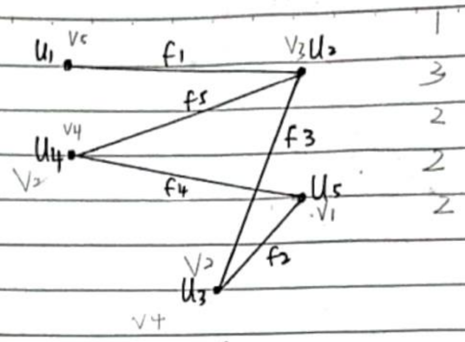
$$A_H = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

$$I_H = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

5.a)



G1



G2

Both graph have five vertices and five edges.
Both graph are connected and simple graph.
Both have one vertex with 1 degree, three vertex with 2 degree and one vertex with 3 degree.

$$A_{G1} = \begin{array}{c|ccccc} & V_1 & V_2 & V_3 & V_4 & V_5 \\ \hline V_1 & 0 & 1 & 0 & 1 & 0 \\ V_2 & 1 & 0 & 1 & 0 & 0 \\ V_3 & 0 & 1 & 0 & 1 & 1 \\ V_4 & 1 & 0 & 1 & 0 & 0 \\ V_5 & 0 & 0 & 1 & 0 & 0 \end{array}$$

$$A_{G2} = \begin{array}{c|ccccc} & U_5 & U_3 & U_2 & U_4 & U_1 \\ \hline U_5 & 0 & 1 & 0 & 1 & 0 \\ U_3 & 1 & 0 & 1 & 0 & 0 \\ U_2 & 0 & 1 & 0 & 1 & 1 \\ U_4 & 1 & 0 & 1 & 0 & 0 \\ U_1 & 0 & 0 & 1 & 0 & 0 \end{array}$$

Since A_{G_1} and A_{G_2} are the same.
G_1 and G_2 are isomorphic.

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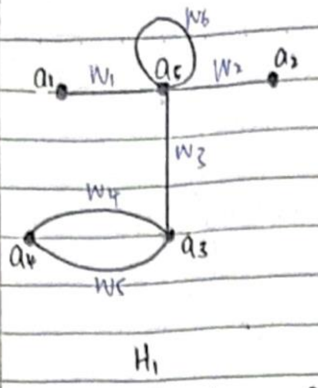
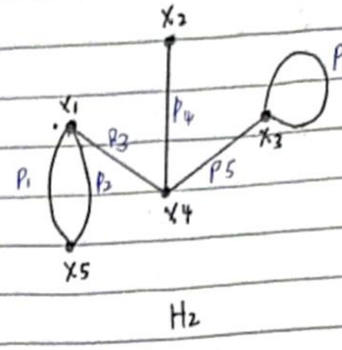
b.



1
5
1
3
2



2
3
3
1
3

Both graph have five vertices and six edges.
Both graph are connected graph.
Both graph have different degree for corresponding vertices.
Graph H_1 have 2 vertices with 1 degree, 1 vertex with 2 degree, 1 vertex with 3 degree and 1 vertex with 5 degree.
Graph H_2 have 1 vertex with 1 degree, 1 vertex with 2 degree and 3 vertices with 3 degree.
$f: H_1 \rightarrow H_2$ cannot be defined
$\therefore$ H_1 and H_2 are not isomorphic.

6a) It is a trail. This is because it is a walk from V_0 to V_1 that does not contain repeated edge. It contain repeated vertex, V_1.

b) It is a walk. This is because it contains a repeated edge, e_9 and repeated vertex, V_5.

c) It is a closed walk. This is because it is a walk that start and ends at the same vertex, V_2.

d) It is a circuit. This is because it is a closed walk that have at least one edge and does not contain repeated edges. It contains a repeated vertex, V_4.

e) It is a closed walk. This is because it is a walk that start and end at the same vertex, V_2. It contains repeated edges, e_4 and e_5 and repeated vertices V_3 and V_4 besides the first and the last vertices

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f. It is a path. This is because it is a trail from V_3 to V_2 that does not contain a repeated vertex.

7a) $v_1, e_1, v_2, e_2, v_3, e_5, v_4$

$v_1, e_1, v_2, e_3, v_3, e_5, v_4$

$v_1, e_1, v_2, e_4, v_3, e_5, v_4$

$\therefore$ number of path from v_1 to $v_4 = 3$

b) number of trails from v_1 to $v_4 = 3 + 3!$

$= 9$

c) number of walks from v_1 to v_4 is infinite. This is because a walk from one vertex v to a vertex w is a finite alternating sequence of adjacent vertices and edges of G. From vertex v_1 to v_4, there are infinite possible of walks.

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8. a) Graph in (a)

vertex	v_1	v_2	v_3	v_4	v_5
Degree	2	4	2	4	4

Graph in (a) has an Euler circuit since every vertex has even degree. One of the Euler circuit is $(v_1, e_1, v_2, e_2, v_5, e_7, v_4, e_6, v_5, e_3, v_2, e_4, v_3, e_5, v_4, e_8, v_1)$.

b) Graph in (b)

Vertex	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
Degree	2	5	2	2	2	4	2	3	3	3

Graph in (b) does not have an Euler circuit. This is because not all vertices have even degree. Vertices v_1, v_7, v_8 and v_9 have odd degree. $\deg(v_1) = 5$, $\deg(v_7) = 3$, $\deg(v_8) = 3$ and $\deg(v_9) = 3$.

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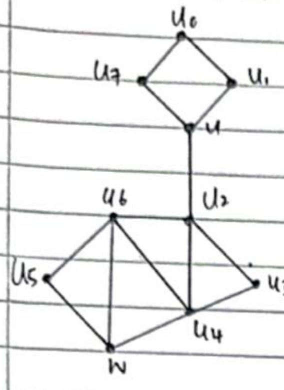
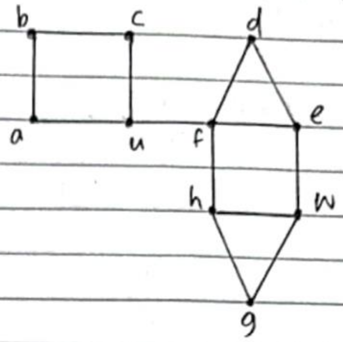


(a)



(b)

9) Graph a

Vertex	u_0	u_1	u	u_2	u_3	u_4	w	u_5	u_6	u_7
Degree	2	2	3	4	2	4	3	2	4	2

There is a euler path from u to w since it is connected and u and w are the only vertices having odd degree.
The euler path is $u, u_1, u_0, u_7, u, u_2, u_3, u_4, u_2, u_6, u_4, w, u_6, u_5, w$.

Graph b

Vertex	a	b	c	u	f	d	e	w	h	g
Degree	2	2	2	3	4	2	3	3	3	2

There is no euler path from u to w. This is because u and w are not the only vertices have odd degree. Vertices e and h also have odd degree. $\deg(e) = 3$, $\deg(h) = 3$.

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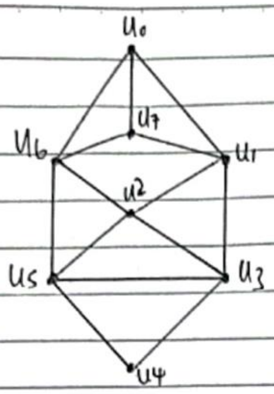
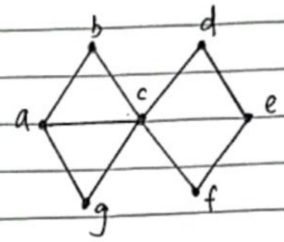
10.



(a)



(b)

Graph a has a Hamiltonian circuit.
$u_0 - u_6 - u_2 - u_5 - u_4 - u_3 - u_1 - u_7 - u_0$

Graph b does not have a Hamiltonian circuit.

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11. $m = 3$, $n = 100$

$$l = \frac{(m-1)n+1}{m}$$

$$= \frac{(3-1)100+1}{3}$$

$$= \frac{201}{3}$$

$$= 67$$

∴ A full 3-ary tree with 100 vertices will have 67 leaves.

12a) Root = a

b) Internal vertices = b, e, g, n, d, h, j

c) leaves = c, f, k, l, m, r, s, o, i, p, q

d) children of n = r, s

e) Parent of e = b

f) Siblings of k = l, m

g) proper ancestors of q = a, d, j

h) proper descendants of b = e, f, g, k, l, m, n, r, s

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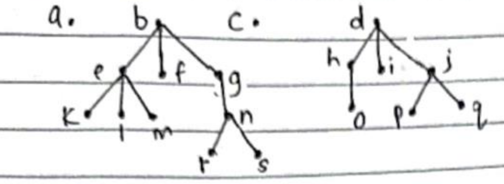
Preorder

13.



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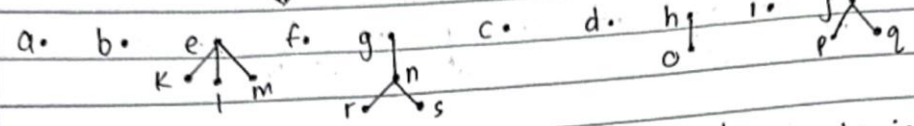

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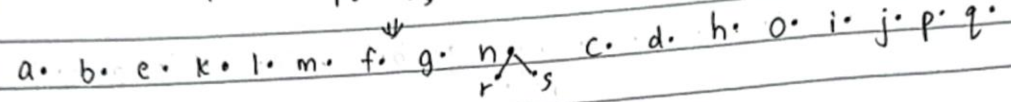
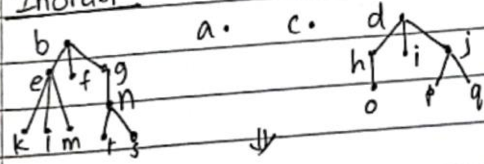


⇓

a. b. e. k. l. m. f. g. n. r. s. c. d. h. o. i. j. p. q.

∴ preorder = a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

Inorder



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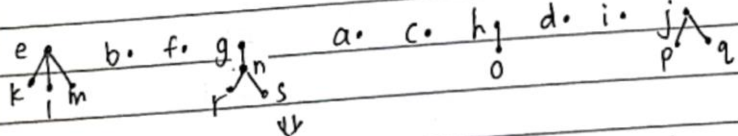

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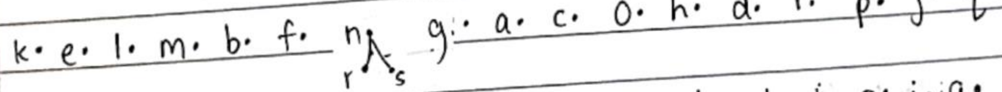
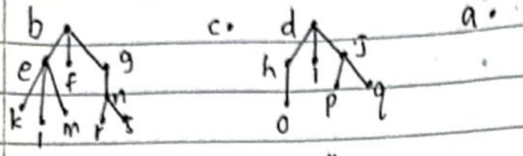


k. e. l. m. b. f. r. n. s. g. a. c. o. h. d. i. p. j. q.

Inorder = k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

Postorder



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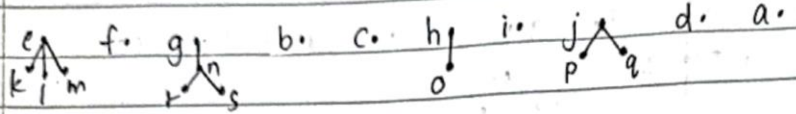

⇓

k. l. m. e. f. n g. b. c. o. h. i. p. q. j. d. a.
r s

⇓

k. l. m. e. f. r. s. n. g. b. c. o. h. i. p. q. j. d. a.

∴ Postorder = k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

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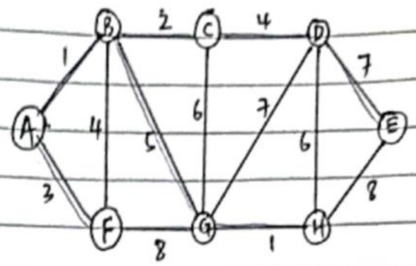
14\. AB = 1 ✓
GH = 1 ✓
BC = 2 ✓
AF = 3 ✓
BF = 4
CD = 4 ✓

BG = 5 ✓
CG = 6
DH = 6
DE = 7 ✓
DG = 7
FG = 8

EH = 8



∴ Minimum spanning tree = 1 + 1 + 2 + 3 + 4 + 5 + 7
= 23

15\.

i	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	{ }	{M, N, O, P, Q, R, S, T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N, O, P, Q, R, S, T}	0	4	∞	2	∞	5	∞	∞
2	{M, P}	{N, O, Q, R, S, T}	0	4	∞	2	6	5	5	∞
3	{M, P, N}	{O, Q, R, S, T}	0	4	10	2	6	5	5	∞
4	{M, P, N, R}	{O, Q, S, T}	0	4	7	2	6	5	5	6
5	{M, P, N, R, S}	{O, Q, T}	0	4	7	2	6	5	5	6
6	{M, P, N, R, S, T}	{O, Q}	0	4	7	2	6	5	5	6

∴ Shortest path from M to T is 6, M – R – T.