SECI 1013 SECTION 01

DISCRETE STRUCTURE

LECTURER:

Dr Razana Alwee

SUBMITTED BY:

FELICIA CHIN HUI FEN (A20EC0037)

IMAN EHSAN BIN HASSAN (A20EC0048)

NAVINTHRA RAO A/L VENKATAKUMAR (A20EC0104)

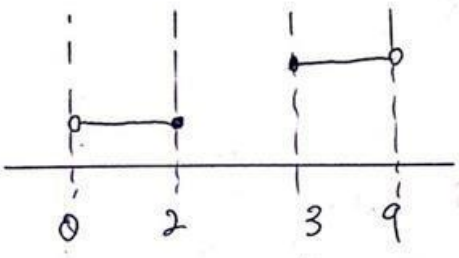
Semester 1 2020/2021

1) a) $A = \{x \in R \mid 0 < x \leq 2\}$

$B = \{x \in R \mid 1 \leq x < 4\}$

$C = \{x \in R \mid 3 \leq x < 9\}$

$A \cup C = \{x \in R \mid 0 < x \leq 2 \text{ or } 3 \leq x < 9\}$



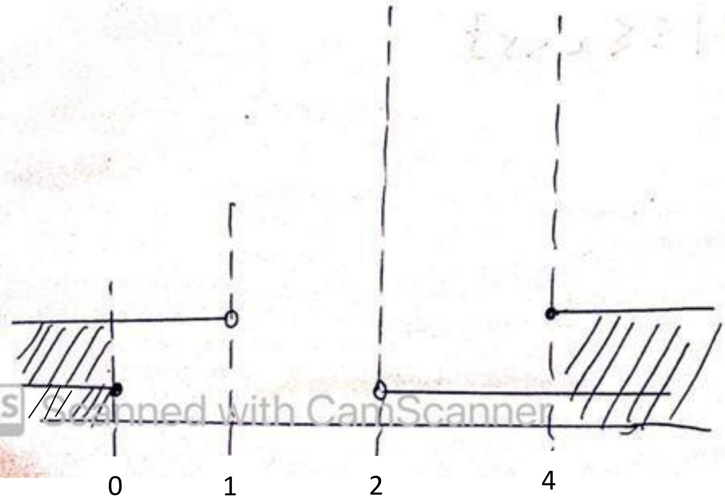
b) $A' = \{x \in R \mid x \leq 0 \text{ or } x > 2\}$

$B' = \{x \in R \mid x < 1 \text{ or } x \geq 4\}$

$(A \cup B)' = A' \cap B'$ De Morgan's Law

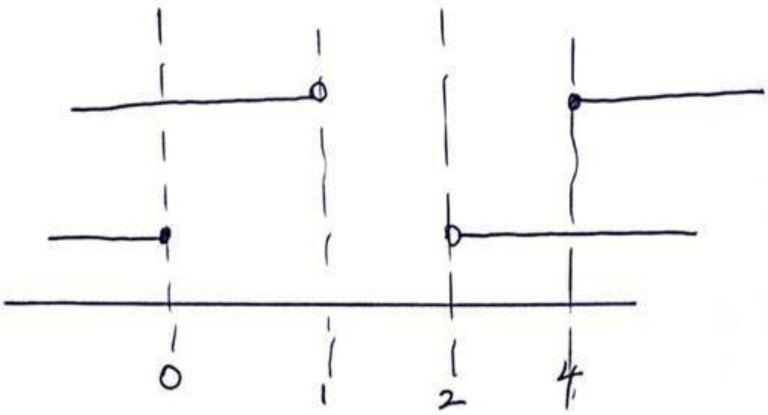
$= \{x \in R \mid (x \leq 0 \text{ or } x > 2) \text{ and } (x < 1 \text{ or } x \geq 4)\}$

$= \{x \in R \mid x \leq 0 \text{ or } x \geq 4\}$

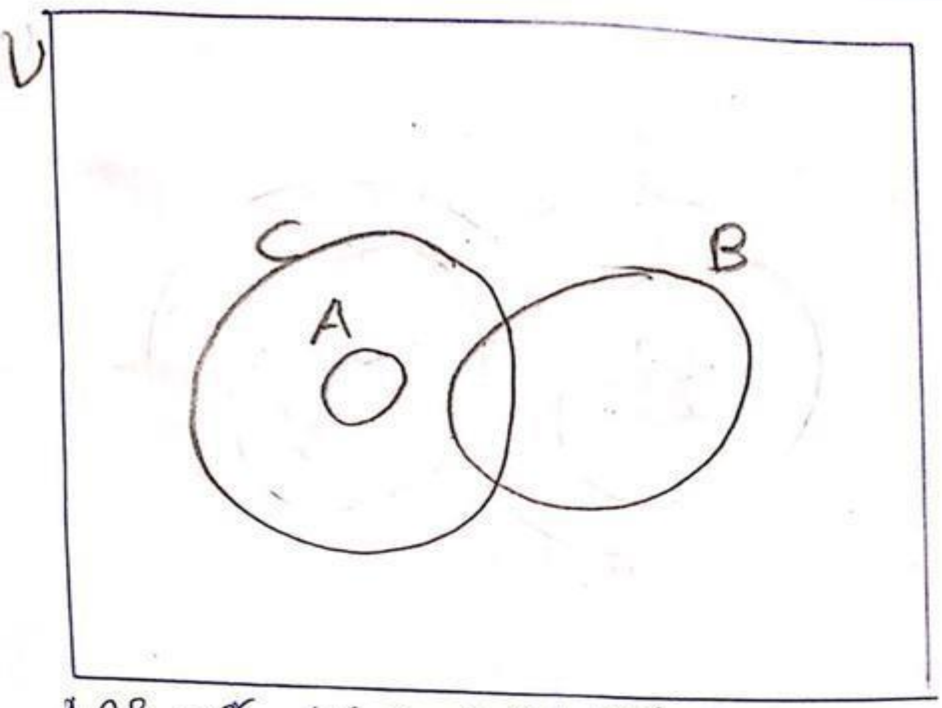


c) $A' \cup B' = \{x \in \mathbb{R} \mid (x \leq 0 \text{ or } x > 2) \text{ or } (x < 1 \text{ or } x \geq 4)\}$

$= \{x \in \mathbb{R} \mid x < 1 \text{ or } x > 2\}$

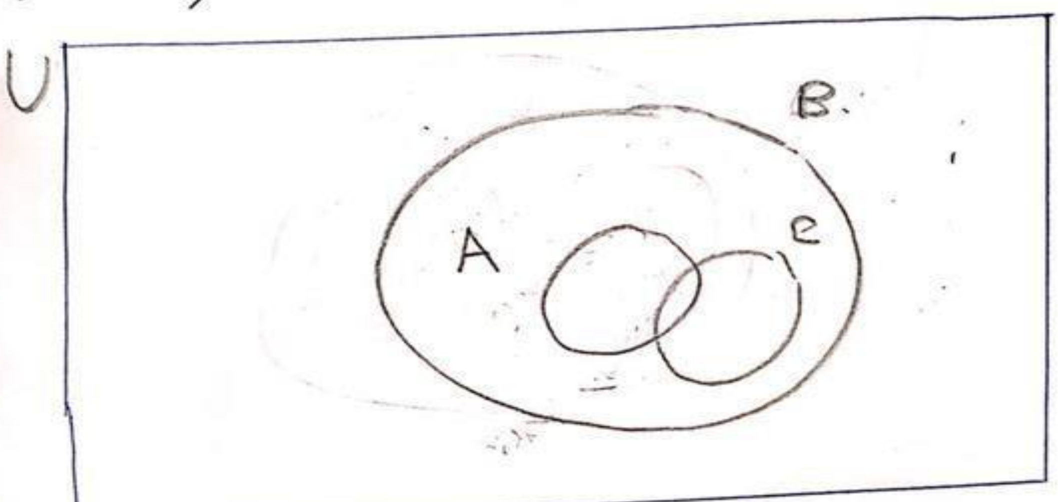


2) a)

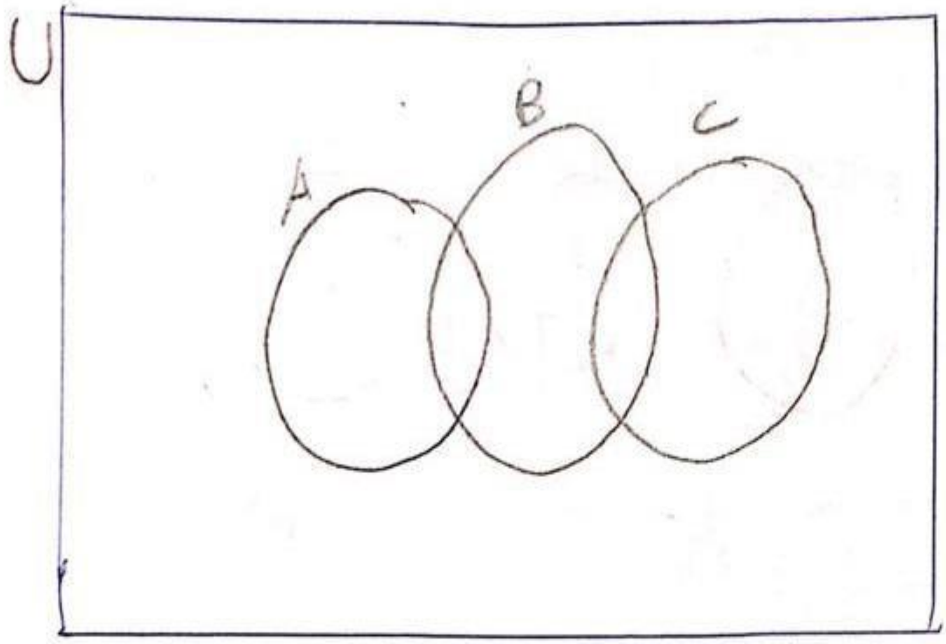


$A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$

b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$



(c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subset B$, $C \not\subset B$



3) $A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$

$S = \{(-1,1), (1,1), (2,2)\}$

$T = \{(-1,1), (1,1), (2,2), (4,2)\}$

$S \cap T = \{(-1,1), (1,1), (2,2)\}$

$S \cup T = \{(-1,1), (1,1), (2,2), (4,2)\}$

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4. $\neg[(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \vee (p \wedge q)$

$\equiv [\neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q)] \vee (p \wedge q)$ De Morgan's Law

$\equiv [(\neg(\neg p) \vee \neg q) \wedge (\neg(\neg p) \vee \neg(\neg q))] \vee (p \wedge q)$ De Morgan's Law

$\equiv [(p \vee \neg q) \wedge (p \vee q)] \vee (p \wedge q)$ Double negation Law

$\equiv [p \vee (\neg q \wedge q)] \vee (p \wedge q)$ Distribution Law

$\equiv (p \vee F) \vee (p \wedge q)$ $\neg q \wedge q \equiv$ False

$\equiv p \vee (p \wedge q)$ Indentity Law for False

$\equiv p$ — shown Absorption Law

5.a) $R_1 = \{(x,y) \mid x+y \leq 6\}$

$X, Y = 1, 2, 3, 4, 5$

$R_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$

$$M_{A1} = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

b) $R_2 = \{(y,z) \mid y > z\}$

$Y, Z = 1, 2, 3, 4, 5$

$R_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$

$$M_{A2} = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

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c) Is R_1 reflexive?

R_1 is not reflexive: because the main diagonal of matrix relation is 1's and 0's and not all 1's.

$$M_{A1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Is R_1 symmetric?

$$M_{A1}^{T} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

R_1 is symmetric because $M_{A1} = M_{R_1}^{T}$

Is R_1 transitive?

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

R_1 is not transitive since $M_{A1} \otimes M_{A1} \neq M_{A1}$

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Is R_1 an equivalence relation?

R_1 is not an equivalence relation since it is symmetric but not reflexive and transitive.

d) Is R_2 reflexive?

R_2 is not reflexive because the main diagonal of the matrix of relation is all 0's and not all 1's.

$$M_{A2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Is R_2 antisymmetric

R_2 is anticymmetric because,

$(2,1) \in R_2$ and $(1,2) \notin R_2$

$(3,1) \in R_2$ and $(1,3) \notin R_2$

$(3,2) \in R_2$ and $(2,3) \notin R_2$

$(4,1) \in R_2$ and $(1,4) \notin R_2$

$(4,3) \in R_2$ and $(3,4) \notin R_2$

$(5,1) \in R_2$ and $(1,5) \notin R_2$

$(5,2) \in R_2$ and $(2,5) \notin R_2$

$(5,3) \in R_2$ and $(3,5) \notin R_2$

$(5,4) \in R_2$ and $(4,5) \notin R_2$

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6.

$$M_{R1} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 \end{array}$$

$\therefore R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$

$$M_{R2} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}$$

$\therefore R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

a) $R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

$$\therefore M_{R_1 \cup R_2} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 \end{array}$$

b) $R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$

$$\therefore M_{R_1 \cap R_2} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}$$

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7. If $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ are both one-to-one, $f+g$ is not one-to-one.

Let $f(x) = x$, $\forall x \in R$

$f(x)$ is one-to-one since

$$f(a) = f(b)$$
$$a = b$$

Let $g(x) = -x$, $\forall x \in R$

$g(x)$ is one-to-one since

$$g(a) = g(b)$$
$$-a = -b$$
$$a = b$$

$$(f+g)(x) = f(x) + g(x)$$
$$= x + (-x)$$
$$= 0$$

$\therefore (f+g)(x) = 0$, $\forall x \in R$

Let $x = 1$,

$(f+g)(1) = 0$

Let $x = 2$,

$(f+g)(2) = 0$

$\therefore (f+g)(1) = (f+g)(2)$

$1 \neq 2$

$\therefore f+g$ is not one-to-one.

Subject :

8. For n staircase, $n \geq 1$

C_n : number of different ways to climb staircase

$C_1 = 1$

$C_2 = 2$

$C_3 = 2 + 1 = 3 \Rightarrow C_2 + C_1.$

$C_4 = 3 + 2 = 5 \Rightarrow C_3 + C_2$

$C_5 = 5 + 3 = 8 \Rightarrow C_4 + C_3$

$\vdots$

$C_n = C_{n-1} + C_{n-2}$, $C_1 = 1$, $C_2 = 2$, $n > 2$

9. $t_0=0$, $t_1=1$, $t_2=1$, $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$

a) $t_3 = t_2 + t_1 + t_0$
$= 1+1+0$
$= 2$

$t_5 = t_4 + t_3 + t_2$
$= 4+2+1$
$= 7$

$t_7 = t_6 + t_5 + t_4$
$= 13+7+4$
$= 24$

$t_4 = t_3 + t_2 + t_1$
$= 2+1+1$
$= 4$

$t_6 = t_5 + t_4 + t_3$
$= 7+4+2$
$= 13$

$\therefore t_7 = 24$

b)
```
t(n)
{ if (n=0)
      return 0
  else if (n=1 or n=2)
      return 1
  else
      return t(n-1)+ t(n-2)+ t(n-3)
}
```