DISCRETE STRUCTURE SECI1013

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GROUP/ASSIGNMENT :

GROUP 12/ASSIGNMENT 3

LECTURER'S NAME : DR NOR AZIZAH ALI

GROUP MEMBERS :

NAME	MATIRC NO.
HONG PEI GEOK	A20EC0044
KONG JIA ROU	A20EC1098
MAIZATUL AFRINA BINTI SAIFUL AZWAN	A20EC0204
RADIN DAFINA BINTI RADIN ZULKARNAIN	A20EC0135

Question 1

a) Let A={1, 2, 3, 4, 5, 6, 7, 8}, B={2, 5, 9}, and C={a, b}. Find each of the following:

i. $A-B$

$A-B = \{1, 3, 4, 6, 7, 8\}$

ii. $(A\cap B)\cup C$

$(A\cap B) = \{2, 5\}$

$C = \{a, b\}$

$(A\cap B)\cup C = \{2, 5, a, b\}$

iii. $A\cap B\cap C$

$A\cap B\cap C = \emptyset$

iv. $B\times C$

$B\times C = \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$

v. $P(C)$

$P(C) = \{\{\}, \{a\}, \{b\}, \{a, b\}\}$

b) By referring to the properties of set operations, show that:

$$(P\cap((P'\cup Q')) \cup (P\cap Q) = P$$

$(P\cap((P'\cup Q')) \cup (P\cap Q)$

$= (P\cap((P\cap Q)') \cup (P\cap Q)$ (De Morgan's Law)

$= P \cap U$ (Complement Law)

$= P$

c) Construct the truth table for, A = $(\neg p \vee q) \leftrightarrow (q \rightarrow p)$.

p	q	$(\neg p \vee q)$	$(q \rightarrow p)$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then $(x+2)^2$ is odd"

Let $x = 5$; $(x+2)^2 = 49$ (both odd)

Let $x = 11$; $(x+2)^2 = 169$ (both odd)

P(x) = x is an odd integer

Q(x) = $(x+2)^2$ is an odd integer

$$a = 2n + 1$$

$$(a+2)^2 = ((2n+1)+2))^2$$

$$(a+2)^2 = (2n+3)^2$$

$$(a+2)^2 = 4n^2 + 12n + 9$$

$$(a+2)^2 = 2(2n^2 + 6n + 4) + 1$$

$(a+2)^2 = 2m + 1 \Longrightarrow m = (2n^2 + 6n + 4)$ is an integer $(a + 2)^2 \Longrightarrow$ is an odd integer

e) Let $P(x,y)$ be the propositional function $x \geq y$. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\exists x \exists y\, P(x, y)$

There are some x positive integers and there are some y positive integers, $x \geq y$.

$\Longrightarrow$ TRUE

ii. $\forall x \forall y\, P(x, y)$

For every x positive integers and for every y positive integers, $x \geq y$.

$\Longrightarrow$FALSE

Question 2

1 1 a) 0

Suppose that the matrix of relation R on {1, 2, 3} is [0 1 1 0] relative to the ordering 1, 2, 3. 0 0

i. Find the domain and the range of R.

R = { (1 , 1) , (1 , 2) , (2 , 2) , (3 , 1) }

Domain = { 1 , 2 , 3 }

Range = { 1 , 2 }

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

Irreflexive : *R* is not an irreflexive relation because there are value 1 on its diagonal.

Antisymmetric : *R* is an antisymmetric relation because when i ≠ j, then $m_{ij} = 0$ or $m_{ji} = 0$.

When $m_{ij} = 1$ and $m_{ji} = 1$, then i = j

b) Let *S*={(*x*,*y*)| *x*+*y* ≥9} is a relation on *X*={2, 3, 4, 5}. Find:

i. The elements of the set *S*.

S = {(4, 5), (5, 4), (5, 5)}

ii. **Is *S* reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.**

Reflexive: S is not a reflexive relation because $(4, 4) \notin S$.

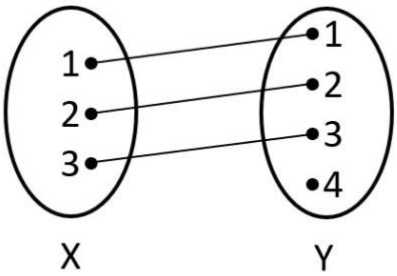
Symmetric: S is a symmetric relation.

Transitive: S is a transitive relation.

Equivalence: S is not an equivalence relation because S is not a reflexive relation.

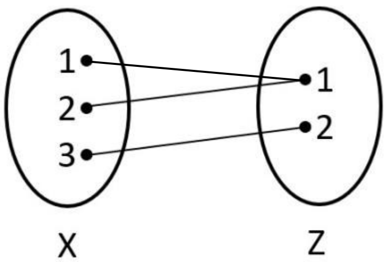
c) Let X={1, 2, 3}, Y={1, 2, 3, 4}, and Z={1, 2}.

i. Define a function f: X→Y that is one-to-one but not onto.



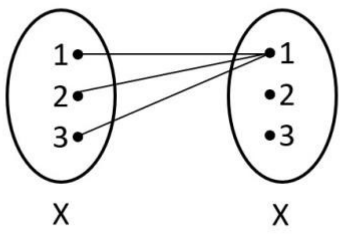
$$f(x) = x$$

ii. Define a function g: X→Z that is onto but not one-to-one.



$$g(1) = 1 \;, g(2) = 1 \,, g(3) = 2$$

iii. Define a function h: X→X that is neither one-to-one nor onto.



$h(x) = 1$

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

m(x) = 4x+3, n(x) = 2x−4

i. Find the inverse of m.

$y = 4x + 3$ $4x = 3 - y$

$x = \frac{3 - y}{4}$

$\therefore m^{-1}(x) = \frac{3 - x}{4}$

ii. Find the compositions of n o m.

$n \; o \; m = 2(4x + 3) - 4$ $= 6x + 6 - 4$

$= 6x + 2$

Question 3

a) Given the recursively defined sequence. $a_k = a_{k-1} + 2k$, for all integers $k \geq 2$, $a_1 = 1$

i. Find the first three terms.

$a_1 = 1$

$a_2 = 1 + 2\,(2)$ = 5

$a_3 = 5 + 2\,(3)$ =

11

ii. Write the recursive algorithm.

Input : k

Output : f (k) f

(k) { if (k = 1

)

return 1

return f (k – 1) + 2*k)

}

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k−1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for r_1, r_2, …. r_k.

When the algorithm is run with an input of size 1, then it executes 7 operations.

$$r_1 = 7$$

The number of operations with an input of size k is twice the number of operations with an input of k – 1.

$$r_k = 2r_{k-1}\ , when\ k > 1$$

c) Given the recursive algorithm:

Input: n

Output: S (n)

S(n) { if

(n=1)

return 5

return 5*S(n−1)

}

Trace S(4).

S (1) = 5

S (2) = 5*5

= 25 S

(3) = 5*25

$= 125$

$S(4) = 5 * 125 = 625$

Question 4

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

From 3 to B = 9 possibilities (first place)

From 5 to F = 11 possibilities (last place)

Therefore,

$$9 \times 16 \times 16 \times 11 = 25344$$

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0? (4 marks)

A _ _ _ _ | _ _ 0

Letters $= 26 \times 26 \times 26 \times 26$

$= 456976$

Digits $= 9 \times 10$

$= 90$

$\therefore$ Number of choices = 41127840

c) How many arrangements in a row of no more than three letters can be formed using the letters of word COMPUTER (with no repetitions allowed)?

To arrange one letter,

Since COMPUTER has 8 letters, thus 8 ways.

To arrange two letters,

First letter : 8 ways

Second letter : 7 ways

Use the multiplication rule $= 8 \times 7$

$= 56\ ways$

To arrange three letters,

First letter : 8 ways

Second letter : 7 ways

Third letter : 6 ways

Use the multiplication rule $= 8 \times 7 \times 6$

$= 336\ ways$

Therefore, the total arrangements are:

$= 8 + 56 + 336 =$

$400\ ways$

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

To choose women $C\ (7, 4)$

$$C\ (7,4) = \frac{7!}{4!\,(7-4)!}$$

$$= !\ \frac{7!}{4!\,(3)}$$

$$= \frac{7\ \times 6\ \times 5\ \times 4!}{4!\ \times 3!}$$

$= 35$

To choose men $C\ (6, 3)$

$$C\ (6,3) = \frac{6!}{3!\,(6-3)!}$$

$$= !\ \frac{6!}{3!\,(3)}$$

$$= \frac{6 \times 5\ \times 4\ \times 3!}{3!\ \times 3!}$$

$= 20$

Therefore, $35 \times 20 = 700$

e) How many distinguishable ways can the letters of the word PROBABILITY be arranged?

Since repetition is allowed,

For first letter : 11 ways

For second letter : 11 ways

For third letter : 11 ways

For fourth letter : 11 ways

For fifth letter : 11 ways

For sixth letter : 11 ways

For seventh letter : 11 ways

For eighth letter : 11 ways

For ninth letter : 11 ways

For tenth letter : 11 ways

For eleventh letter : 11 ways

Thus, $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \;\; = 2.8531 \times 10^{11}\, ways$

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

10 pastries are chosen from 6 different kinds of pastry.

n = 6 r

=10

$$C\,(n + r - 1, r) = \frac{(n + r - 1)!}{r!\,(n - 1)!}$$

$$C\,(6 + 10 - 1, 10) = \frac{15}{10!\,(5)!}$$

$$C\,(15, 10) = 3003$$

Hence, number of selection of pastries is 3003

Question 5

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

X = {(Ali, Daud), (Bahar, Daud), (Carlie, Daud), (Ali, Elyas), (Bahar, Elyas), (Carlie, Elyas)}

$$n m = \lceil \rceil \ k$$

$$= \lceil \frac{18}{6} \rceil$$

$$= 3$$

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

X = { 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 }

$|X| = 10$

20 – 10 = 10

Minimum integers one must pick to be sure of getting at least one that is odd = 10 + 1 = 11

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

X = { 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100 }

$|X| = 20$

100 – 20 = 80

Minimum integers one must pick to be sure of getting one that is divisible by 5 = 80 + 1 = 81