DISCRETE STRUCTURE SECI1013

20202021/1

GROUP/ASSIGNMENT :

GROUP 12/ASSIGNMENT 2

LECTURER'S NAME :

DR NOR AZIZAH ALI

GROUP MEMBERS :

NAME	MATIRC NO.
HONG PEI GEOK	A20EC0044
KONG JIA ROU	A20EC1098
MAIZATUL AFRINA BINTI SAIFUL AZWAN	A20EC0204
RADIN DAFINA BINTI RADIN ZULKARNAIN	A20EC0135

Question 1

Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7

a. How many numbers are there?

$6 \times 6 \times 6 = 216$

b. How many numbers are there if the digits are instinct?

$P(6,3) = 120$

c. How many numbers between 300 to 700 is only odd digits allow?

$4 \times 6 \times 3 = 72$

Question 2

Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table

a. Men insist to sit next to each other

$(6-1)! \times (5)! = (5)! \times (5)!$
$= 14400$ ways

b. The couple insisted to sit next to each other

$(9-1)! \times (2)! = (8)! \times (2)!$
$= 80640$ ways

c. Men and women sit in alternate seat

$(5-1)! \times (5)! = (4)! \times (5)!$
$= 2880$ ways

d. Before her friend left, Anita wants to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other?

Let Anita and husband = 1 person. Thus, total 11 persons.

$(11)! \times (2)! = 79833600$ ways.

Question 3

In a school sport day, five sprinters are competing in a 100-meter race. How many ways are there for the sprinter to finish

a. If no ties

P(5,5) = 120 ways

b. Two sprinters tie

$2 \times P(4,4) =$ 48 ways

c. Two group of two sprinters tie

$2 \times 2 \times P(3,3) =$ 24 ways

Question 4

A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose:

a. a dozen croissants?

The order does not matter, thus use combination. We have 6 types of croissants and need to choose 12 croissants from these types of croissants.

n = 6, r = 12

Repetition of croissants is allowed

$$C\,(n + r - 1, r) = \frac{(n + r - 1)!}{r!(n - 1)!}$$

$$C\,(6 + 12 - 1, 12) = \frac{(6 + 12 - 1)!}{12!(6 - 1)!}$$

$$C\,(17, 12) = \frac{17!}{12!(5)!}$$

$$= 6188 \text{ ways}$$

b. two dozen croissants with at least two of each kind?

The order does not matter, thus use combination.

We have 6 types of croissants and need to choose 24 croissants from these types of croissants and also, we required to choose at least two of each kind, which are 12 croissants in total. Then we still need to choose 24 – 12 = 12 croissants.

n = 6, r = 12

$$C\,(n + r - 1, r) = \frac{(n + r - 1)!}{r!(n - 1)!}$$

$$C\,(6 + 12 - 1, 12) = \frac{(6 + 12 - 1)!}{12!(6 - 1)!}$$

$$C\,(17, 12) = \frac{17!}{12!(5)!}$$

$$= 6188 \text{ ways}$$

c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

We have 6 types of croissants and need to choose 24 croissants from these types of croissants and also, we required to choose at least five chocolate croissants and at least three almond croissants, which are 8 croissants in total. Then we still need to choose 24 – 5 – 3 = 16 croissants.

n = 6, r = 16

$$C\,(n + r - 1, r) = \frac{(n + r - 1)!}{r!(n - 1)!}$$

$$C\,(6 + 16 - 1, 16) = \frac{(6 + 16 - 1)!}{16!(6 - 1)!}$$

$$C\,(21, 16) = \frac{21!}{16!(5)!}$$

= 20349 ways

Question 5

This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

2 wins from 4 games = $C(4,2)$

$$= \frac{4!}{2!(4-2)!}$$

= 6

1 win from 3 games = $C(3,1)$

$$= \frac{3!}{1!(3-1)!}$$

= 3

and

2 wins and 1 ties = 6 x C(3,1) x 2

= 36

1 win and 3 ties = 3 x C(4,3) x 8

= 96

Since we have two teams, thus,

= 2 x (36 + 96)

= 2 x 132

= 264 possible scoring scenarios

b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

Total possible scoring scenarios for 10 penalty kicks : 2^{10} = 1024

If the game is done in the first round : 264 scenarios (refer to a)

If the game is not done in the first round : 1024 – 264

= 760

Second round = 264

Therefore,

Number of different scoring scenarios = Game is not done in first round x second round

= 760 x 264

= 200640 scenarios.

c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Game is not done in first round = 760 scenarios

Game is not done in second round = 760 scenarios

Sudden death-shoot out = 10 scenarios

Thus,

Number of scoring scenarios = 760 x 760 x 10

= 5776000 scenarios

Question 6

A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

$4^{10} = 1048576$

$2(1048576) + 1 \quad = 2097153$

$\therefore$ 2097153 students

Question 7

In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Let A = Students who passed in history

Let B = Students who passed in Mathematics

P (A) = 0.75

P (B) = 0.65

$P(A \cap B) = 0.5$

$P(A \cap B)' = 0.5$

Since the probability of students failed in both subjects is 0.5,

Thus, 35 candidates equal to probability of 0.5.

And we know the total probability will be 1, so the number of candidates sit for the exam:

Number of candidates sit for the exam = $\frac{35}{0.5}$

= 70

Question 8

An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Total number of possible outcomes = 780-299

= 481 outcomes

3 digits of number 1 = not possible (number range from 300 to 780)

2 digits of number 1 = 311,411,511,611,711

= 5 possible integers

1 digit of number 1 =

301,310,312,313,314,315,316,317,318,319,321,331,341,351,361,371, 381, 391
(18 integers)

401, 410,412,413,414,415,416,417,418,419,421,431,441,451,461,471, 481, 491
(18 integers)

501, 510,512,513,514,515,516,517,518,519,521,531,541,551,561,571, 581, 591
(18 integers)

601, 610,612,613,614,615,616,617,618,619,621,631,641,651,661,671, 681, 691
(18 integers)

701, 710,712,713,714,715,716,717,718,719,721,731,741,751,761,771
(16 integers)

Therefore, (4 x 18 integers) + 16 integers = 88 possible integers

Total successful outcomes = 0 + 5 + 88

= 93

∴ The probability that the number is chosen will have 1 as at least one digit = $\frac{93}{481}$ = 0.1933

Question 9

Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same colour are not distinguishable, and the parking lots are chosen at random.

a. In how many ways can the cars be parked in the parking lots?

There are 10 objects (parking lots) with three different types which blue cars, yellow cars and empty lots.

$$n = 10, k = 3$$

$$P(n) = \frac{n!}{(n_1!\, n_2! \ldots n_k!)}$$

$$P(10) = \frac{10!}{(2!\, 4!\, 4!)}$$

$$= 3150 \text{ ways}$$

b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

Since there have 10 parking lots and parked by 6 cars, then it has remaining 4 empty lots. According to the question, these four empty lots are next to each other, so we consider four empty lots as one empty lot.

$$n = 7, k = 3$$

$$P(n) = \frac{n!}{(n_1!\, n_2! \ldots n_k!)}$$

$$P(7) = \frac{7!}{(2!\, 4!\, 1!)}$$

$$= 105 \text{ ways}$$

Question 10

A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively

P(E): The coach use email to give a message to a trainee

P(L): The coach use letter to give a message to a trainee

P(H): Th coach use handphone to give a message to a trainee

P(R): The trainee receives the message

P(E) = 0.4; P(L) = 0.1; P(H) = 0.5

P(R|E) = 0.6; P(R|L) = 0.8; P(R|H) = 1

a. **Find the probability the trainee receives the message**

$$P(R) = P(R|E).P(E) + P(R|L).P(L) + P(R|H).P(H)$$

$$= (0.6 \text{ x } 0.4) + (0.8 \text{ x } 0.1) + (1 \text{ x } 0.5)$$

$$= 0.82$$

b. **Given that the trainee receives the messages, find the conditional probabilities that he receives it via email**

$$P(E|R) = \frac{P(R|E).P(E)}{P(R|E).P(E) + P(R|L).P(L) + P(R|H).P(H)}$$

$$= \frac{0.6\ x\ 0.4}{(0.6\ x\ 0.4) + 0.8\ x\ 0.1) + (1\ x\ 0.5)}$$

$$= \frac{0.24}{0.82}$$

$$= 0.2927$$

Question 11

In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

P(C): Cars
P(L): Light truck
P(A): Fatal accident
P(A'): Not a fatal accident

P(L) = 0.4
P(C) = 0.6

$$P(A|C) = \frac{20}{100000}$$

$$= 0.0002$$

$$P(A|L) = \frac{25}{100000}$$

$$= 0.00025$$

$$P(L|A) = \frac{P(A|L).P(L)}{P(A|L).P(L) + P(A|C).P(C)}$$

$$= \frac{0.00025\ x\ o.4}{(0.00025\ x\ 0.4) + (0.0002\ x\ 0.6)}$$

$$= 0.4545$$

Question 12

There are 9 letters having different colours (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

Total letters = 9
Total boxes = 4

Possible ways without restrictions:

$= 4^9$
= 262144 ways

Disallowed ways:

$= 4 \times 3^9$
= 78732 ways

If all letters in one box:

$= 4 \times 1^9$
= 4 ways

9 letters into 4 boxes:

= 262144 – 78732 + 4
= 183416 ways