



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

DISCRETE STRUCTURE SECI1013

20202021/1

GROUP/ASSIGNMENT :

GROUP 12/ASSIGNMENT 1

LECTURER'S NAME :

DR NOR AZIZAH ALI

GROUP MEMBERS :

NAME	MATIRC NO.
HONG PEI GEOK	A20EC0044
KONG JIA ROU	A20EC1098
MAIZATUL AFRINA BINTI SAIFUL AZWAN	A20EC0204
RADIN DAFINA BINTI RADIN ZULKARNAIN	A20EC0135

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Question 1

Let the universal set be the set $\mathbb{R}$ of all real numbers and let $A = \{x \in \mathbb{R} \mid 0 < x \leq 2\}$, $B = \{x \in \mathbb{R} \mid 1 \leq x < 4\}$ and $C = \{x \in \mathbb{R} \mid 3 \leq x < 9\}$. Find each of the following :

(a) $A \cup C$

$$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

(b) $(A \cup B)'$

$$(A \cup B)' = \{4, 5, 6, 7, 8\}$$

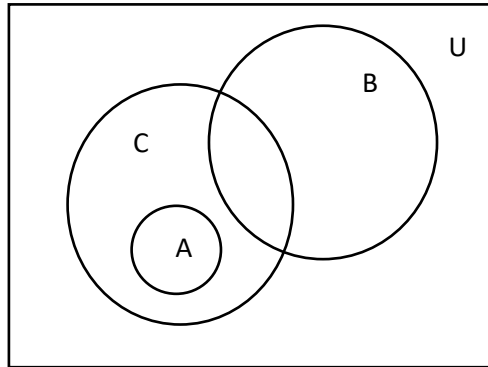
(c) $A' \cup B'$

$$A' \cup B' = \{3, 4, 5, 6, 7, 8\}$$

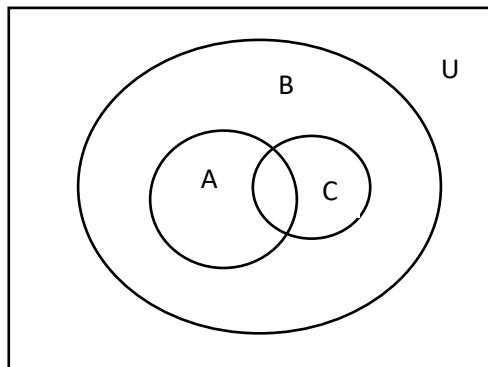
Question 2

Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

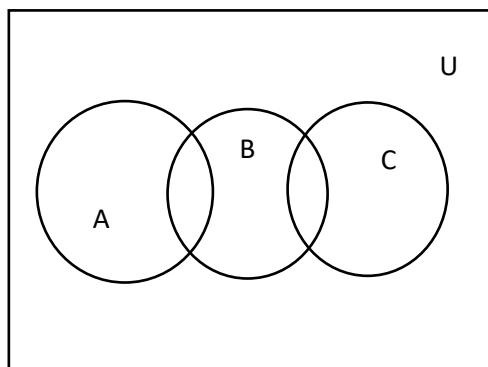
a) $A \cap B = \emptyset, A \subseteq C, C \cap B \neq \emptyset$



b) $A \subseteq B, C \subseteq B, A \cap C \neq \emptyset$



c) $A \cap B \neq \emptyset, B \cap C \neq \emptyset, A \cap C = \emptyset, A \not\subseteq B, C \not\subseteq B$



Question 3

Given two relations S and T from A to B ,

$$S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$$

$$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$$

Let $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$ and defined binary relations S and T from A to B as follows:

$$\text{For all } (x,y) \in A \times B, x S y \leftrightarrow |x| = |y|$$

$$\text{For all } (x,y) \in A \times B, x T y \leftrightarrow x - y \text{ is even}$$

State explicitly which ordered pairs are in $A \times B$, S , T , $S \cap T$, and $S \cup T$.

$$A = \{-1, 1, 2, 4\} \text{ and } B = \{1, 2\}$$

$$A \times B = \{(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)\}$$

$$S = \{(x, y) \in A \times B, x S y \leftrightarrow |x| = |y|\}$$

Since,

$$(-1, 1) \in A \times B$$

$$|-1| = |1|$$

$$(1, 1) \in A \times B$$

$$|1| = |1|$$

$$(2, 2) \in A \times B$$

$$|2| = |2|$$

$$\therefore S = \{(-1, 1), (1, 1), (2, 2)\}$$

$$T = \{ (x, y) \in A \times B, x T y \leftrightarrow x - y \text{ is even} \}$$

Thus,

$$(-1, 1); -1 - 1 = -2 \text{ is even}$$

$$(-1, 2); -1 - 2 = -3 \text{ is odd}$$

$$(1, 1); 1 - 1 = 0 \text{ is even}$$

$$(1, 2); 1 - 2 = -1 \text{ is odd}$$

$$(2, 1); 2 - 1 = 1 \text{ is odd}$$

$$(2, 2); 2 - 2 = 0 \text{ is even}$$

$$(4, 1); 4 - 1 = 3 \text{ is odd}$$

$$(4, 2); 4 - 2 = 2 \text{ is even}$$

$$\therefore T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$$

$$S \cap T = \{(-1, 1), (1, 1), (2, 2)\}$$

$$S \cup T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$$

Question 4

Show that $\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

$\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$	Distributive law
$= \neg (\neg p \wedge (q \vee \neg q)) \vee (p \wedge q)$	Complement law
$= \neg (\neg p \wedge U) \vee (p \wedge q)$	Properties of universal law
$= \neg (\neg p) \vee (p \wedge q)$	Double negation law
$= p \vee (p \wedge q)$	Absorption law
$= p$	

Question 5

$R_1 = \{(x, y) \mid x + y \leq 6\}$; R_1 is from X to Y; $R_2 = \{(y, z) \mid y > z\}$; R_2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5. Find:

a) The matrix A_1 of the relation R_1 (relative to the given orderings)

$$M_{A_1} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

b) The matrix A_2 of the relation R_2 (relative to the given orderings)

$$M_{A_2} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

c) Is R_1 reflexive, symmetric, transitive, and/or an equivalence relation?

Reflexive : R_1 is not a reflexive relation because there are value 0 and 1 on its diagonal and there are only some vertex that have loops.

Symmetric : R_1 is a symmetric relation because $m_{R_1} = m_{R_1}^T$ and $m_{ij} = m_{ji}$.

Transitive : R_1 is not a transitive relation.

Equivalence : R_1 is not an equivalence relation because R_1 is not a reflexive and a transitive relation.

d) Is R_2 reflexive, antisymmetric, transitive, and/or a partial order relation?

Reflexive : R_2 is not a reflexive relation because there is value 0 on its diagonal.

Antisymmetric : R_2 is an antisymmetric relation because $m_{ij} \neq m_{ji}$.

Transitive : R_2 is a transitive relation.

Partial order : R_2 is not a partial order relation because R_2 is not a reflexive relation

Question 6

Suppose that the matrix of relation R1 on {1, 2, 3} is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R2 on {1, 2, 3} is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation $R1 \cup R2$

$$M_{R_1} \cup M_{R_2} = M_{R_1} \vee M_{R_2} = \begin{array}{c} \begin{matrix} & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

b) The matrix of relation $R1 \cap R2$

$$M_{R_1} \cap M_{R_2} = M_{R_1} \wedge M_{R_2} = \begin{array}{c} \begin{matrix} & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

Question 7

If $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

A function is said to be one-to one if $f(x_1) = f(x_2)$ and $x_1 = x_2$

Assume that,

$$\begin{aligned}f(x) &= x \\g(x) &= -x\end{aligned}$$

For all real number for x ,

$$\begin{aligned}(f + g)(x) &= f(x) + g(x) \\&= (x) + (-x) \\&= 0\end{aligned}$$

For example,

$$\begin{aligned}(f + g)(5) &= f(5) + g(5) \\&= (5) + (-5) \\&= 0\end{aligned}$$

$$\begin{aligned}(f + g)(3) &= f(3) + g(3) \\&= (3) + (-3) \\&= 0\end{aligned}$$

$$\begin{aligned}(f + g)(5) &= (f + g)(3) = 0, \text{ but } 5 \neq 3 \\(f + g)(x_1) &= (f + g)(x_2) = 0 \text{ but } x_1 \neq x_2\end{aligned}$$

Therefore,

$\therefore f + g$ is not one-to-one.

Question 8

With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase.

Find a recurrence relation for $c_1, c_2, \dots, c_n$.

$$c_1 = 1,$$

$$c_2 = 2,$$

$$\therefore c_n = c_{n-1} + c_{n-2}, \text{ when } n \geq 3$$

Question 9

The Tribonacci sequence (t_n) is defined by the equations,

$$t_0 = 0, t_1 = t_2 = 1, t_n = t_{n-1} + t_{n-2} + t_{n-3} \text{ for all } n \geq 3.$$

a) Find t_7

$$\begin{aligned} t_0 &= 0 \\ t_1 &= 1 \\ t_2 &= 1 \\ t_3 &= 1 + 1 + 0 = 2 \\ t_4 &= 2 + 1 + 1 = 4 \\ t_5 &= 4 + 2 + 1 = 7 \\ t_6 &= 7 + 4 + 2 = 13 \\ t_7 &= 13 + 7 + 4 = 24 \end{aligned}$$

$$\therefore t_7 = 24$$

b) Write a recursive algorithm to compute t_n , $n \geq 3$.

Input: n

Output: $t(n)$

```
t(n) {  
    if (n = 0)  
        return 0  
    if (n = 1 or n = 2)  
        return 1  
    return t (n - 1) + t (n - 2) + t (n - 3)  
}
```