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QUESTION 1

$A = \{1,2,3,4,5,6,7,8\}$

(a) ~~$A = \{1,2,3,4,5,6,7,\}$~~ , $B = \{2,5,9\}$ $C = \{a,b\}$

i) ~~$A - B = \{1,3,4,6,7\}$~~ $A - B = \{1,3,4,6,7,8\}$

ii) $(A \cap B) \cup C$

$A \cap B = \{2,5\}$

$(A \cap B) \cup C = \{2,5,a,b\}$

iii) $A \cap B \cap C = \emptyset$

iv) $B \times C = \{(2,a),(5,a),(9,a),(2,b),(5,b),(9,b)\}$

v) $P(C) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

(b) $(P\cap((P'\cup Q)'))\cup(P\cap Q)=P$

LHS

$=(P\cap((P')'\cap Q'))\cup(P\cap Q)$ → De Morgan's laws

$=(P\cap(P\cap Q'))\cup(P\cap Q)$ → Double complement laws

$=((P\cap P)\cap Q')\cup(P\cap Q)$ → Associative laws

$=(P\cap Q')\cup(P\cap Q)$ → Idempotent laws

$=P\cap(Q'\cup Q)$ → Complement laws

$=P\cap U$ → Properties of universal set

$=P$

$\therefore (P\cap((P'\cup Q)'))\cup(P\cap Q)$

(c) $A=(\neg p\vee q)\leftrightarrow(q\to p)$

p	q	$\neg p$	$\neg p\vee q$	$q\to p$	$(\neg p\vee q)\leftrightarrow(q\to p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

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(d) "For all integer x, if x is odd, then $(x+2)^2$

Let $P(x)$: x is odd

$Q(x)$: $(x+2)^2$ is odd

$\forall x\,(P(x) \rightarrow Q(x))$

Domain discourse is set Z of all integer

Let k is an odd integer. Then,

$k = 2n+1$

$k+2 = 2n+1+2$

$k+2 = 2n+3$

$(k+2)^2 = (2n+3)^2$

$(k+2)^2 = 4n^2+12n+9$

$(k+2)^2 = 4n^2+12n+8+1$

$(k+2)^2 = 2(2n^2+6n+4)+1$

$(k+2)^2 = 2m+1$ where $m = 2n^2+6n+4$ is an integer

$(k+2)^2$ is odd

$\therefore$ For all integer x, if x is odd, then $(x+2)^2$ is odd

(e) i) $\exists x \exists x\ P(x, y)$

When $x=1$, $y=1$

$P(x,y) = 1 \geqslant 1$ (true)

$\therefore$ This statement is true because there is at least 1 x and 1 y to make proposition true

ii) $\cancel{\forall x \forall x}$ $\forall x \forall y\ P(x,y)$

When $x=1$, $y=2$,

$P(x,y) = 1 \ngeqslant 2$ (false)

$\therefore$ This statement is false because when $x=1$ and $y=2$, the proposition is false.

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Question 2

(a) (i) domain = $\{1, 2, 3\}$

range = $\{1, 2\}$

(ii) The relation is not irreflexive because its main diagonal is not all 1.

$R = \{(1,1), (1,2), (2,2), (3,1)\}$

$(1,2) \in R$ and $(2,1) \notin R$

$(3,3) \notin R$

$(3,1) \in R$ and $(1,3) \notin R$

Therefore, this relation is antisymmetric because for all $a, b \in R$, if $(a,b) \in R$ and $a \neq b$, then $(b,a) \notin R$.

(b) (i) $S = \{(4,5), (5,4), (5,5)\}$

(ii)

$$M_S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

It is not reflexive as its main diagonal is not all 1.

$m_{45} = m_{54}$

$m_{12} = m_{21}$ $\therefore$ It is symmetric.

$m_{34} = m_{43}$

$$M_S \otimes M_S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

It is not transitive because $M_S \otimes M_S \neq M_S$.

It is not an equivalence relation because it is not reflexive, not transitive although symmetric.

Therefore, S is symmetric only.

(c) (i) $f = \{(1,1), (2,2), (3,3)\}$

(ii) $g = \{(1,1), (2,2), (3,1)\}$

(iii) $h = \{(1,3), (2,3)\}$

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(d) (i) Let $y = m(x)$,

$y = m(x)$

$y = 4x + 3$

$y - 3 = 4x$

$x = \frac{y-3}{4}$

$\therefore m^{-1}(x) = \frac{x-3}{4}$

(ii) $n \circ m(x) = n(4x+3)$

$= 2(4x+3) - 4$

$= 8x + 6 - 4$

$= 8x + 2$

3) a) i) $a_1 = 1$

$a_2 = a_1 + 2(2)$
$= 1 + 4$
$= 5$

$a_3 = a_2 + 2(3)$
$= 5 + 6$
$= 11$

ii) Input : k integer positive
Output : a(k)

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a(k)
{  if (k=1)
       return 1
   return a(k-1) + 2k
}
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b) Input of size 1 : $r_1 = 7$

Input of size k : $r_k = 2k-1$, when $k \geq 2$

c) $S_1 = 5$

$S_2 = 5S_1$
$= 5(5)$
$= 25$

$S_3 = 5S_2$
$= 5(25)$
$= 125$

$S_4 = 5S_3$
$= 5(125)$
$= 625$

$\therefore S_4 = 625$

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4) a) No. of ways $= 9 \times 16 \times 16 \times 11$

$= 25344$

b) No. of ways $= 1 \times 26^3 \times 10^2 \times 1$

$= 1757600$

c) No. of ways $= 8 + (8 \times 7) + (8 \times 7 \times 6)$

$= 400$

d) No. of ways $= C(7,4) \times C(6,3)$

$= \dfrac{7!}{4!\,3!} \times \dfrac{6!}{3!\,3!}$

$= 700$

e) No. of ways $= \dfrac{11!}{2!\,2!}$

$= 9979200$

f) $C(10+6-1, 10) = C(15, 10)$

$= \dfrac{15!}{10!\,5!}$

$= 3003$

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5) a) $S = \{$(Ali, Daud), (Ali, Elyas), (Bahar, Daud), (Bahar Elyas),
(Carlle, Daud), (Carlle, Elyas)$\}$

$k = 6, \ n = 18$

$m = \left\lceil \frac{18}{6} \right\rceil = 3$

$\therefore$ At least 3 persons have the same first and last name.

b) $S = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$
10 odd numbers, 10 even numbers

If all 10 even numbers are picked, an odd number can be guaranteed on the next pick. So, $10 + 1 = 11$, there must be at least 11 integers to be picked to be sure of getting at least one that is odd.

c) n(divisible by 5) = 20 integers
n(not divisible by 5) = $100 - 20$
$= 80$ integers

If all 80 integers that are not divisible by 5 are picked, a divisible-by-5 integer is guaranteed on the next pick. So, $80 + 1 = 81$, there must be at least 81 integers to be picked in order to be sure of getting one that is divisible by 5.