Siew Yu Xuan A20EC0146 , Muhammad Sabiq Ahsan A20EC0322
Lai Ting Ying A19EC3033 , Muhammad Harith Irfan A20EC0089

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Assignment 1

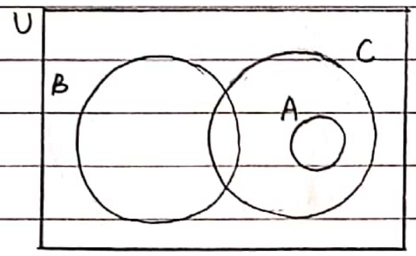
1) $A = \{1, 2\}$, $B = \{1, 2, 3\}$, $C = \{3, 4, 5, 6, 7, 8\}$

a) $A \cup C = \{1, 2\} \cup \{3, 4, 5, 6, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

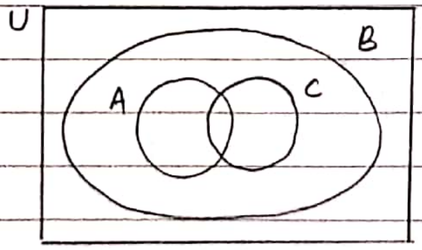
b) $(A \cup B)' = (\{1, 2\} \cup \{1, 2, 3\})'$
$= \{1, 2, 3\}'$
$= \{x \mid x$ is a real number, $x < 1$ and $x > 3\}$

c) $A' \cup B' = (A \cap B)'$
$= (\{1, 2\} \cap \{1, 2, 3\})'$
$= \{1, 2\}'$
$= \{x \mid x$ is a real number, $x < 1$ and $x > 2\}$

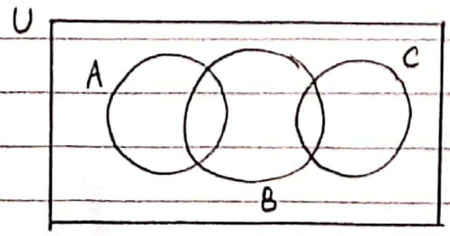
2) a) $A \cap B = \phi$, $A \subseteq C$, $C \cap B \neq \phi$



b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \phi$



c) $A \cap B \neq \phi$, $B \cap C \neq \phi$, $A \cap C = \phi$, $A \notin B$, $C \notin B$



NO: DATE:

3) $A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$

$S = \{(-1,1), (1,1), (2,2)\}$

$T = \{(-1,1), (4,2)\}$

$S \cap T = \{(-1,1), (1,1), (2,2)\} \cap \{(-1,1), (4,2)\}$
$= \{(-1,1)\}$

$S \cup T = \{(-1,1), (1,1), (2,2)\} \cup \{(-1,1), (4,2)\}$
$= \{(-1,1), (1,1), (2,2), (4,2)\}$

4) $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= \neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q) \vee (p \wedge q)$	De Morgan's Laws
$= (\neg\neg p \vee (\neg q)) \wedge (\neg\neg p \vee \neg\neg q) \vee (p \wedge q)$	De Morgan's Laws
$= (p \vee (\neg q)) \wedge (p \vee q) \vee (p \wedge q)$	Double Negative Laws
$= p \vee (\neg q \wedge q) \vee (p \wedge q)$	Distributive Laws
$= p \vee (p \wedge q) \vee \phi$	Commutative, Complement Laws
$= p$ # (shown)	Absorption Laws

NO: DATE:

5) $R_1 = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)\}$

$R_2 = \{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(5,4)\}$

a)

$$M_{R_1} = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

b)

$$M_{R_2} = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

c)

$$M_{R_1} = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Since $(a,b) \in R$ and $(b,a) \in R$ it is symmetric.

Since the main diagonal elements are 0 and 1, it is not reflexive.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Since $M_{R_1} \otimes M_{R_1} \neq M_{R_1}$, it is not transitive.

$\therefore R_1$ is symmetric relation.

NO: .... DATE: ....

5) d)

$$M_{R_2} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 \end{array}$$

Since the main diagonal elements are 0, it is irreflexive

Since $\forall x, y \in A$, $(x,y) \in R \wedge x \neq y \rightarrow (y,x) \notin R$, it is antisymmetric.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Since $M_{R_2} \otimes M_{R_2} \neq M_{R_2}$, it is not transitive.

$\therefore R_2$ is an antisymmetric relation.

NO: .... DATE: ....

6) $R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$

$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

a) $R_1 \cup R_2 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\} \cup \{(1,2), (2,2), (3,1), (3,3)\}$
$= \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

b) $R_1 \cap R_2 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\} \cap \{(1,2), (2,2), (3,1), (3,3)\}$
$= \{(2,2), (3,1), (3,3)\}$

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

7) Yes, $f+g$ is also one-to-one

$(f+g)(R) = f(R) + g(R)$
$= R + R$
$= 2R$

$(f+g)(R_1) = (f+g)(R_2)$
$2R_1 = 2R_2$
$R_1 = R_2$

8) $C_1 = 1$

$C_2 = 2$

$C_3 = 3$

$C_4 = 5$

$C_n = C_{n-2} + C_{n-1}$, where $n \geqslant 3$

9) a) $t_n = t_{n-1} + t_{n-2} + t_{n-3}$

$t_7 = t_{7-1} + t_{7-2} + t_{7-3}$

$= t_6 + t_5 + t_4$

$= 9 + 5 + 3$

$= 17$ #

$t_4 = t_3 + t_2 + t_1$

$= 1 + 1 + 1$

$= 3$

$t_5 = t_4 + t_3 + t_2$

$= 3 + 1 + 1$

$= 5$

$t_6 = t_5 + t_4 + t_3$

$= 5 + 3 + 1$

$= 9$

b)
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t(n)
{ if(0<n<4)
      return 1

   return t(n-1)+t(n-2)+t(n-3)
}
```