SECI1013-03 DISCRETE STRUCTURE

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ASSIGNMENT 2

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1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7
   a. How many numbers are there?
   b. How many numbers are there if the digits are instinct?
   c. How many numbers between 300 to 700 is only odd digits allow?

   Solution:

   a. Form 3-digit number from 2, 3, 4, 5, 6 or 7;
   T_1 = 6 ways
   T_2 = 6 ways $\qquad$ 6×6×6 = 216 ways
   T_3 = 6 ways

   b. If the digits are instinct;
   T_1 = 6 ways
   T_2 = 5 ways $\qquad$ 6×5×4 = 120 ways
   T_3 = 4 ways

   c. Odd numbers between 300 to 700;
   First number only can be 3, 4, 5, or 6 = T_1 = 4 ways
   Second number can be any of the six digits = T_2 = 6 ways
   Third number can be 3, 5, or 7 because only odd digits allow = T_3 = 3 ways
   4×6×3 = 72 ways

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table
   a. Men insist to sit next to each other
   b. The couple insisted to sit next to each other
   c. Men and women sit in alternate seat
   d. Before her friend left, Anita want to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

   Solution:

   a. 5 men and 5 women. All men want to sit next to each other;
   1 group of men + 5 women = (6 – 1)! = 120 ways
   5 men can be arranged 5 ways = 5! = 120 ways
   120×120 = 14 400 ways

   b. 1 couple and 8 people. Couple want to sit next to each other;
   1 couple + 8 people = (9 – 1)! = 40 320 ways
   Couple can be arranged in 2 ways = 2! = 2
   40 320×2 = 80 640 ways

   c. Men sit in alternate seats = (5 – 1)! = 24 ways
   5 women can be arranged between 5 gaps = 5! = 120 ways
   24×120 = 2 880 ways

   d. 5 men, 5 women, Anita and her husband = 12 people
   Anita and her husband consider as 1 group = 11 people
   Ways to arrange them in line = 11! = 39 916 800 ways
   Anita and her husband can be arranged in 2 ways = 2! = 2
   39 916 800×2 = 79 833 600 ways

3. In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish
   a. If no ties
   b. Two sprinters tie
   c. Two group of two sprinters tie

Solution:

a. No ties;

$$5! = 120 \text{ ways}$$

b. Two sprinters tie;

Position = 4! = 24 ways

From 5 sprinters, which two sprinters will be tie = 5C2 = 10 ways

$$24 \times 10 = 240 \text{ ways}$$

c. Two group of two sprinters tie;

Position = 3! = 6 ways

From 5 sprinters, which two sprinters will be tie = 5C2 = 10 ways

From 3 sprinters, which two sprinters will be tie = 3C2 = 3 ways

$$6 \times 10 \times 3 = 180 \text{ ways}$$

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose
   a. a dozen croissants?
   b. two dozen croissants with at least two of each kind?
   c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

Solution:

a. n = 6

r = 12 (a dozen)

Repetition is allowed: $C(17, 12) = \frac{17!}{12!(17-12)!}$

$= \frac{17!}{12! \times 5!}$

= 6,188 ways

b. n = 6

r = 12 (we need to choose two dozen and select 2 of each kind, so we assume that we take 12 croissants first then we take another with the same way as the first 12)

Repetition is allowed: $C(17, 12) = \frac{17!}{12!(17-12)!}$

$= \frac{17!}{12! \times 5!}$

= 6,188 ways

c. n = 6

r = 16 (two dozen = 24, but we need to select 5 chocolate and 3 almonds, so 24 – 5 – 3 = 16)

Repetition is allowed: $C(21, 16) = \frac{21!}{16!(21-16)!}$

$= \frac{21!}{16! \times 5!}$

= 20,349 ways

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.
   a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?
   b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?
   c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Solution:

a. The round ends if it is impossible for a team to equal the number of goals scored by the other team.
   If there are 2 wins for one team with one additional tie or win for that team, or
   1 win for one of the teams with 3 additional ties or wins for that team.

   Wins = one team score and the other not, Ties = both team score or both team not score
   2 wins among 4 kicks: C(4, 2) = $\frac{4!}{2!(4-2)!}$ = 6
   3 wins among 4 kicks: C(4, 3) = $\frac{4!}{3!(4-3)!}$ = 4
   1 win among 3 kicks: C(3, 1) = $\frac{3!}{1!(3-1)!}$ = 3

   Possible scenarios;
   2 wins and 1 ties/wins: C(4, 2)×C(3, 1)×2 = 6×3×2 = 36
   1 win and 3 ties/wins: C(3, 1)×C(4, 3)×2^3 = 3×4×8 = 96
   Number of scenarios = 2(36 + 96) = 264 scenarios

b. 10 penalty kicks are played: 2^{10} = 1024
   264 of the 1024 scenarios result the game being settles in the first round:1024–264=760
   Settle in second round: 264
   Number of scenarios = 760×264 = 200,640 scenarios

c. First round: 760 scenarios
   Second round: 760 scenarios
   Sudden death: 2 + 2 + 2 + 2 + 2 = 10 scenarios
   Number of scenarios = 760×760×10 = 5,776,000 scenarios

6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical?
   (Assume that no answers are left blank.)

Solution:

Possible number of distinct answer sheets = 4^{10} = 1,048,576
At least three answer sheets must be identical = 2(1,048,576) + 1 = 2,097,153 students

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Solution:

Passed in history = 75% Failed in history = 100% - 75% = 25%

Passed in mathematics = 65% Failed in mathematics = 100% - 65% = 35%

Passed in both = 50%

Total passed students = Passed in history + passed in mathematics – passed both

= 75% + 65% - 50%

= 90%

If 90% passed then 10% will be failed. Let total students be x;

$$\frac{10}{100} \times x = 35$$

$$x = \frac{35 \times 100}{10}$$

$$x = 350\ students$$

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Solution:

Total possible outcomes = 780 – 299 = 481

1 in 3 digits = 0

1 in 2 digits = 5 (311, 411, 511, 611, 711)

1 in 1 digit = 88 (301, 310, 312, 313, 314, 315, 316, 317, 318, 319, 321, 331, 341, 351, 361, 371, 381, 391, ….., 771)

Total successful outcomes = 0 + 5 + 88 = 93

Probability = $\frac{93}{481}$ = 0.1933

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.

   a. In how many ways can the cars be parked in the parking lots?
   b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

Solution:

a. Parking lots = 10

Cars = 6

6 cars in 10 parking lots can be arranged in 10C6 ways = 210 ways

These 6 cars can be arranged in 6! ways;

210×6! =151,200 ways

b. X _ X _ X _ X _ X _

This is not possible because we can park only 5 cars instead of 6.

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively

a. Find the probability the trainee receives the message
b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

Solution:

a. A = trainee receive the message, E = email, L = letter, H = handphone

P(E) = 0.4, P(L) = 0.1, P(H) = 0.5

P(A | E) = 0.6, P(A | L) = 0.8, P(A | H) = 1.0

P(A) = P(E) P(A | E) + P(L) P(A | L) + P(H) P(A | H)

= (0.4)(0.6) + (0.1)(0.8) + (0.5)(1.0)

= 0.24 + 0.08 + 0.5

= 0.82

b. $P(E \mid A) = \frac{P\ (A \mid E)\ P(E)}{P(A \mid E)\ P(E)+P(A \mid L)\ P(L)+P(A \mid H)\ P(H)}$

$= \frac{(0.6)(0.4)}{(0.6)(0.4) + (0.8)(0.1) + (1.0)(0.5)}$

$= \frac{0.24}{0.82}$

= 0.2927

11. In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

Solution:

Let: A = Cars

A' = Light truck

B = Fatal accident

B' = Not fatal accident

Given: P (B | A) = 20/100,000

P (B | A') = 25/100,000

P (A') = 0.4

P (A) = 1 – 0.4 = 0.6

$P(A' \mid B) = \frac{P\ (B \mid A')P\ (A')}{P\ (B \mid A')P\ (A')+P\ (B \mid A)P\ (A)}$

$= \frac{(0.00025)(0.4)}{(0.00025)(0.4)+(0.00020)(0.6)}$

= 0.4545

12. There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

Solution:

Letters = 9

Boxes = 4

$4^9 = 262,144$