SECI1013-03 DISCRETE STRUCTURE

SEMESTER 1, 2020/2021

ASSIGNMENT 1

GROUP MEMBERS:

| NO. | NAME | MATRIC NO. |
|---|---|---|
| 1. | ADAM WAFII BIN AZUAR | A20EC0003 |
| 2. | MUHAMMAD DINIE HAZIM BIN AZALI | A20EC0084 |
| 3. | AFIF HAZMIE ARSYAD BIN AGUS | A20EC0176 |

SUBMITTED TO:

DR. NOR AZIZAH ALI

**DISCRETE STRUCTURE (SECI 1013)**

**2020/2021 – SEMESTER 1**

# ASSIGNMENT# 1

1. Let the universal set be the set **R** of all real numbers and let $A=\{x\in\mathbf{R} \mid 0 < x \leq 2\}$, $B=\{x\in\mathbf{R} \mid 1 \leq x < 4\}$ and $C=\{x\in\mathbf{R} \mid 3 \leq x < 9\}$. Find each of the following:

a) $A \cup C$

b) $(A \cup B)'$

c) $A' \cup B'$

**Solution:**

A = {1, 2}  B = {1, 2, 3}  C = {3, 4, 5, 6, 7, 8}

a) A ∪ C = {1, 2, 3, 4, 5, 6, 7, 8}

b) (A ∪ B)' = {4, 5, 6, 7, 8}

c) A' ∪ B' = {3, 4, 5, 6, 7, 8}

2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

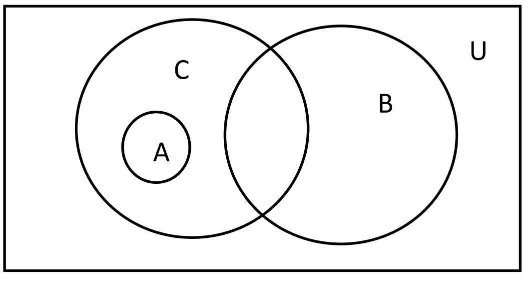
a) A ∩ B = ∅, A ⊆ C, C ∩ B ≠ ∅

b) A ⊆ B, C ⊆ B, A ∩ C ≠ ∅

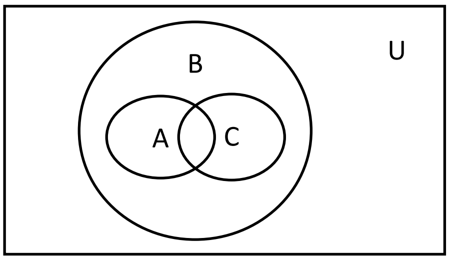
c) A ∩ B ≠ ∅, B ∩ C ≠ ∅, A ∩ C = ∅, A ⊄ B, C ⊄ B

**Solution:**

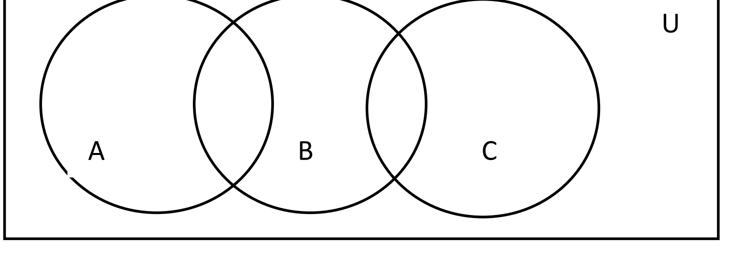
a)




b)




c)



3. Given two relations $S$ and $T$ from $A$ to $B$,

$S \cap T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$

$S \cup T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$

Let $A=\{-1, 1, 2, 4\}$ and $B=\{1,2\}$ and defined binary relations $S$ and $T$ from $A$ to $B$ as follows:

For all $(x,y) \in A\times B$, $x\ S\ y \leftrightarrow |x| = |y|$

For all $(x,y) \in A\times B$, $x\ T\ y \leftrightarrow x - y$ is even

State explicitly which ordered pairs are in $A\times B$, $S$, $T$, $S \cap T$, and $S \cup T$.

**Solution:**

A = {-1, 1, 2, 4}     B = {1, 2}

A × B = {(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)} #

S = $\{(x,y) \in A\times B \mid x\ S\ y \leftrightarrow |x| = |y|\}$

S = {(-1, 1), (1, 1), (2, 2)} #

T = $\{(x,y) \in A\times B \mid x\ T\ y \leftrightarrow x - y$ is even$\}$

T = {(-1, 1), (1, 1), (2, 2), (4, 2)} #

$S \cap T$ = {(-1, 1), (1, 1), (2, 2)} #

$S \cup T$ = {(-1, 1), (1, 1), (2, 2), (4, 2)} #

4. Show that $\neg((\neg p\wedge q) \vee (\neg p\wedge\neg q)) \vee (p\wedge q) \equiv p$. State carefully which of the laws are used at each stage.

**Solution:**

$p \equiv \neg((\neg p\wedge q) \vee (\neg p\wedge\neg q)) \vee (p\wedge q)$

$\equiv (p\wedge\neg q) \vee (p\wedge q) \vee (p\wedge q)$ → Double negation law

$\equiv (p\wedge\neg q) \vee (p\wedge q)$ → Idempotent laws

$\equiv p \wedge (\neg q\vee q)$ → Distributive laws

$\equiv p \wedge (\mathbf{T})$ → Negation laws

$\equiv p$ → Identity laws

5. $R1=\{(x,y)|\ x+y \leq 6\}$; $R1$ is from $X$ to $Y$; $R2=\{(y,z)|\ y>z\}$; $R2$ is from $Y$ to $Z$; ordering of $X$, $Y$, and $Z$: 1, 2, 3, 4, 5.

Find:

a) The matrix $A1$ of the relation $R1$ (relative to the given orderings)

b) The matrix $A2$ of the relation $R2$ (relative to the given orderings)

c) Is $R1$ reflexive, symmetric, transitive, and/or an equivalence relation?

d) Is $R2$ reflexive, antisymmetric, transitive, and/or a partial order relation?

**Solution:**

R1 = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)}

R2 = {(2, 1), (3, 1), (4, 1), (5, 1), (3, 2), (4, 2), (5, 2), (4, 3), (5, 3), (5, 4)}

a) $A1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

b) $A2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

c) Symmetric because $A1 = A1^T$.

d) Antisymmetric because there is one directed relation and one way.

6. Suppose that the matrix of relation R1 on {1, 2, 3} is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R2 on {1, 2, 3} is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation R1∪ R2

b) The matrix of relation R1∩ R2

**Solution:**

a) $R_1$ = {(1, 1), (2, 2), (2, 3), (3,1), (3, 3)}

$R_2$ = {(1, 2), (2, 2), (3, 1), (3, 3)}

R1∪ R2= {(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 3)}

R1∪ R2= $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b) R1∩ R2= {(2, 2), (3, 1), (3, 3)}

R1∩ R2 = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

7. If $f$ :**R**→ **R** and $g$:**R**→ **R** are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

**Solution:**

Let $f(x) = x + 4,$ and $g(x) = -x + 5$ , (one to one equations)

$(f + g)(x) = ( x + 4) + (-x + 5)$

$= x + 4 - x + 5$

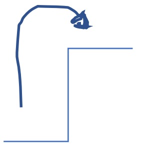
$= 9$, is a constant function

Therefore: $(f + g)$ is not one to one function

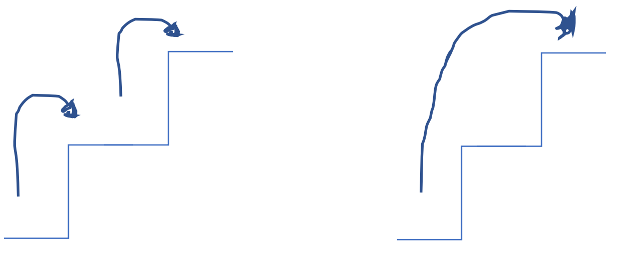
8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of $n$ stairs, let $c_n$ be the number of different ways to climb the staircase. Find a recurrence relation for $c_1, c_2, \ldots, c_n$.

**Solution:**

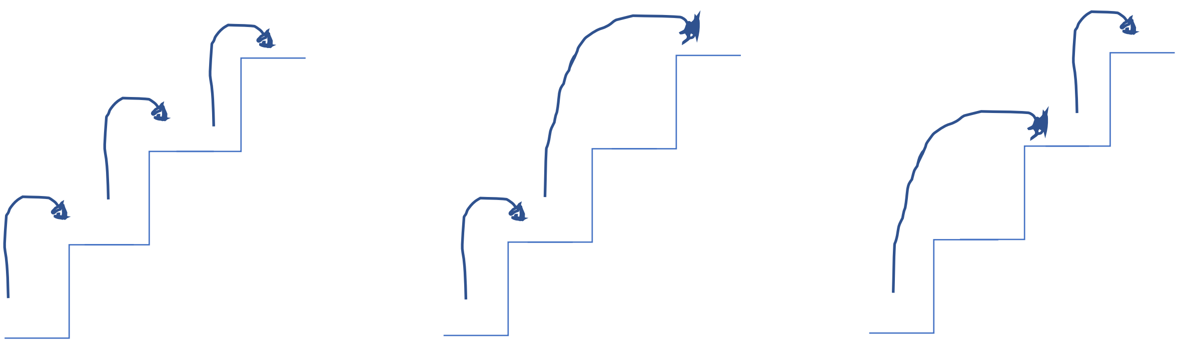
$C_1$ = One way



$C_2$ = 2 way



$C_3$ = 3 way



Thus, the recurrence relation is:

$C_n = C_{(n-1)} + C_{(n-2)}$ , $n \geq 3$ , $while\ C_1 = 1, C_2 = 2.$

9. The Tribonacci sequence ($tn$) is defined by the equations,

$t_0 = 0$, $t_1 = t_2 = 1$, $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$.

a) Find $t_7$.

b) Write a recursive algorithm to compute $t_n$, $n \geq 3$.

**Solution:**

a) $t_0 = 0$, $t_1 = t_2 = 1$, $t_n = t_{n-1} + t_{n-2} + t_{n-3}$

$T_7 = t_{7-1} + t_{7-2} + t_{7-3}$

$T_7 = t_6 + t_5 + t_4$

$T_7 = 13 + 7 + 4$

$= 24$

b) Input: n

Output: f(n)

```
f(n)
{       if (n=1 or n=2)
        return 1
        return f(n-1) + f(n-2) + f(n-3)
}
```