DISCRETE STRUCTURE

SECI 1013 (04)

ASSIGNMENT 4

(Group Assignment)

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DUE DATE: 23 January 2020

1. Let G be a graph with $V(G)$={1,2,….,10}, such that two numbers ‘v’ and ‘w’ in $V(G)$ are adjacent if and only if | v – w| ≤3. Draw the graph G and determine the numbers of edges, $e(G)$.

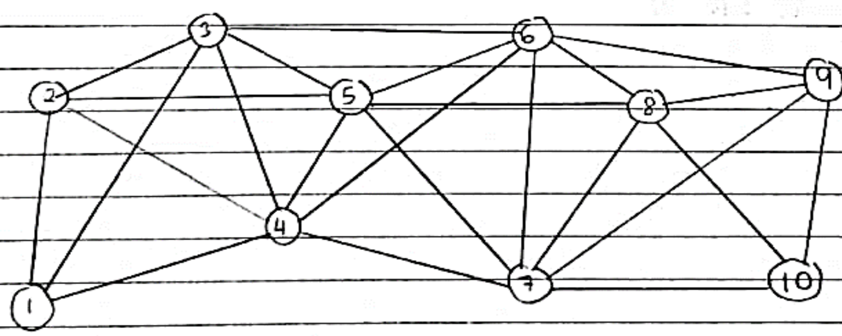
Question 1

$V(G) = \{1, 2, ..., 10\}$ v and w are adjacent

$|v - w| \leq 3$

Draw graph G, determine the number of edges. e(G)

	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	0	0	0	0	0	0
2	1	0	1	1	1	0	0	0	0	0
3	1	1	0	1	1	1	0	0	0	0
4	1	1	1	0	1	1	1	0	0	0
5	0	1	1	1	0	1	1	1	0	0
6	0	0	1	1	1	0	1	1	1	0
7	0	0	0	1	1	1	0	1	1	1
8	0	0	0	0	1	1	1	0	1	1
9	0	0	0	0	0	1	1	1	0	1
10	0	0	0	0	0	0	1	1	1	0



2. Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.

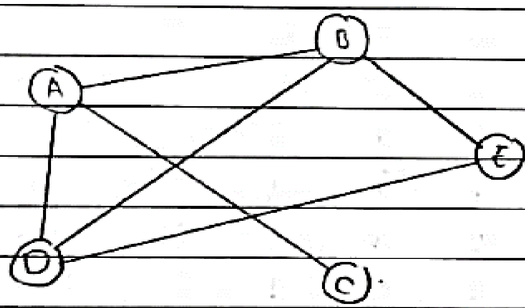
(a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.
(Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)

(b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot: -
i. DM and IS
ii. DM and PT
iii. AI and PS
iv. IS and A

Question 2

a) - Ahmad ⟷ Bakri are friends
↕ ↘
- David Chong
- David ⟷ Bakri ⟷ Ehsan

A = Ahmad
B = Bakri
C = Chong
D = David
E = Ehsan

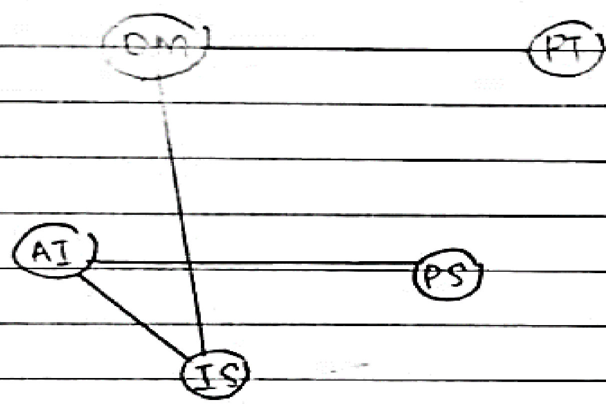


	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	0	0
D	1	1	0	0	1
E	0	1	0	1	0

b) 5 subjects ; DM, PT, AI, PS , IS

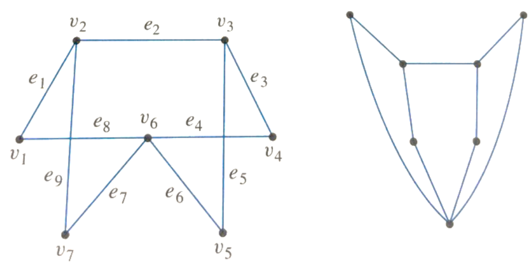
cannot be scheduled in the same time

- DM & IS
- DM & PT
- AI & PS
- IS & AI

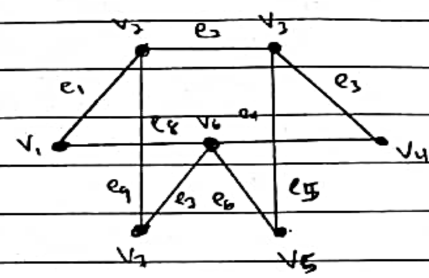


	DM	IS	PT	AI	PS
DM	0	1	1	0	0
IS	1	0	0	1	0
PT	0	0	0	0	0
AI	0	1	0	0	1
PS	1	0	0	0	0

3. Show that the two drawing represent the same graph by labelling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.



Question 3



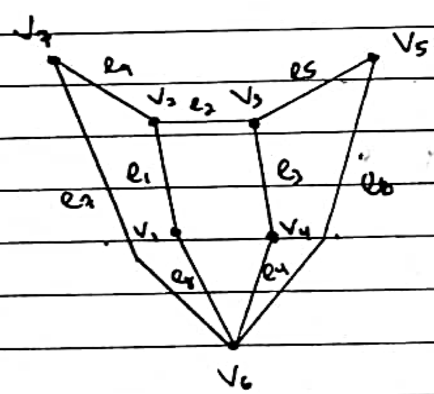
- 7 vertices & 9 edges
- all the vertices have 3 degree except V_6 have 4 degree
- all adjacent of V_6 :

deg V_1 = 2

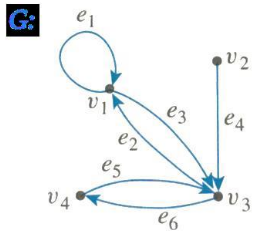
deg V_4 = 2

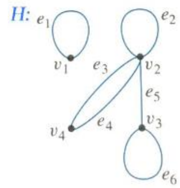
deg V_5 = 2

deg V_7 = 2

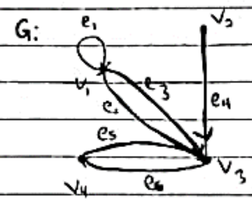


4. Find the adjacency and incidence matrices for the following graphs.





Question 4



Adjacency =

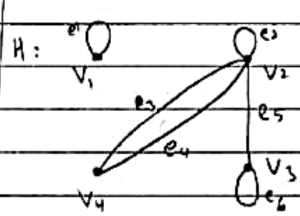
	V_1	V_2	V_3	V_4
V_1	1	0	1	0
V_2	0	0	1	0
V_3	1	0	0	1
V_4	0	0	1	0

$$G: \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Incidence =

	e_1	e_2	e_3	e_4	e_5	e_6
V_1	1	0	1	0	0	0
V_2	0	0	0	1	0	0
V_3	0	1	0	0	0	1
V_4	0	0	0	0	1	0

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



Adjacency =

	V_1	V_2	V_3	V_4
V_1	1	0	0	0
V_2	0	1	1	1
V_3	0	1	1	0
V_4	0	1	0	0

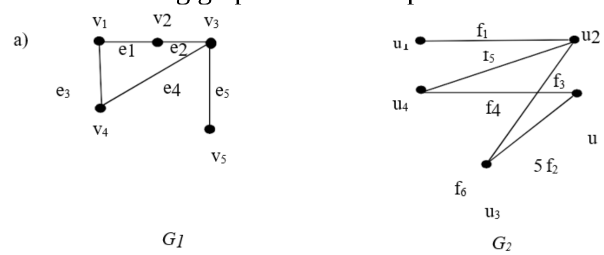
$$H: \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

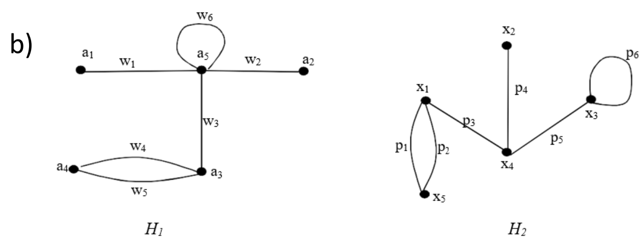
incidence :

	e_1	e_2	e_3	e_4	e_5	e_6
V_1	1	0	0	0	0	0
V_2	0	1	1	1	1	0
V_3	0	0	0	0	1	1
V_4	0	0	1	1	0	0

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

5. Determine whether the following graphs are isomorphic.





Question 5

a)

vertices	edges	degree
$G_1 = 5$	$G_1 = 5$	G_1 : has degree of 2
$G_2 = 5$	$G_2 = 5$	G_2 : has degree of 2, 3

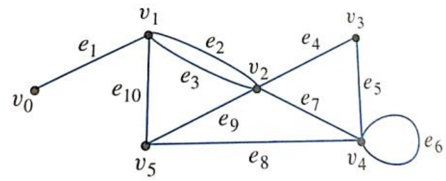
∴ These graph are not isomorphic

b)

vertices	edges	degree
$H_1 = 5$	$H_1 = 6$	H_1 : have degree of 1, 2
$H_2 = 5$	$H_2 = 6$	H_2 : have degree of 1, 2, 3

∴ These graph are not isomorphic

6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



a) v0e1v1e10v5e9v2e2v1
b) v4e7v2e9v5e10v1e3v2e9v5
c) v2
d) v5e9v2e4v3e5v4e6v4e8v5
e) v2e4v3e5v4e8v5e9v2e7v4e5v3e4v2
f) v3e5v4e8v5e10v1e3v2

6a) Trail, because it has no repeated edges and it has repeated vertices, v1.

b) Just a walk, because it has repeated edges and repeated vertices, e9, v5 and it is not closed.

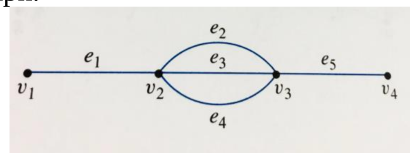
c) Closed walk, because it starts with vertex V2 and ends at vertex v2 also.

d) Circuit, because it has at least one edge and no repeated edges, it has repeated vertices, V4, so not simple circuit.

e) Closed walk, because it has repeated edges, e4 and e5 and repeated vertices, v2 and V3, it also starts and ends at same vertice, V2.

f) Trail and path, because it has no repeated edges and no repeated vertices.

7. Consider the following graph.



a) How many paths are there from $v1$ to $v4$?

b) How many trails are there from $v1$ to $v4$?

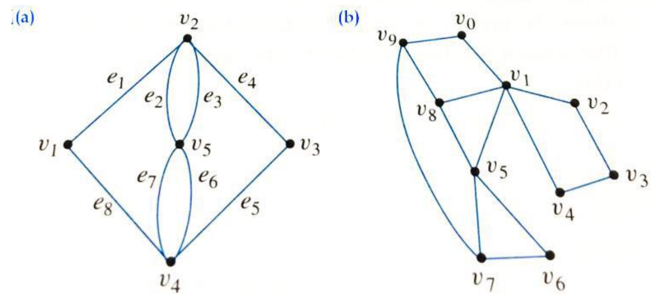
c) How many walks are there from $v1$ to $v4$?

7a) Three paths, $V_1 e_1 V_2 e_2 V_3 e_5 V_4$,
$V_1 e_1 V_2 e_3 V_3 e_5 V_4$,
$V_1 e_1 V_2 e_4 V_3 e_5 V_4$.

b) Nine trails, $V_1 e_1 V_2 e_2 V_3 e_5 V_4$,
$V_1 e_1 V_2 e_3 V_3 e_5 V_4$,
$V_1 e_1 V_2 e_4 V_3 e_5 V_4$,
$V_1 e_1 V_2 e_2 V_3 e_3 V_2 e_4 V_3 e_5 V_4$,
$V_1 e_1 V_2 e_2 V_3 e_4 V_2 e_3 V_3 e_5 V_4$,
$V_1 e_1 V_2 e_3 V_3 e_2 V_2 e_4 V_3 e_5 V_4$,
$V_1 e_1 V_2 e_3 V_3 e_4 V_2 e_2 V_3 e_5 V_4$,
$V_1 e_1 V_2 e_4 V_3 e_2 V_2 e_3 V_3 e_5 V_4$,
$V_1 e_1 V_2 e_4 V_3 e_3 V_2 e_2 V_3 e_5 V_4$.

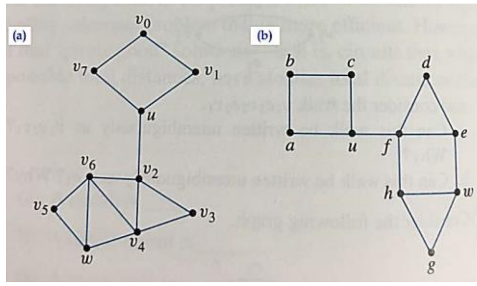
c) Infinity walk, because walk is finite sequence of adjacent vertices and edges, it has no restriction of repeated edges or vertices.

8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



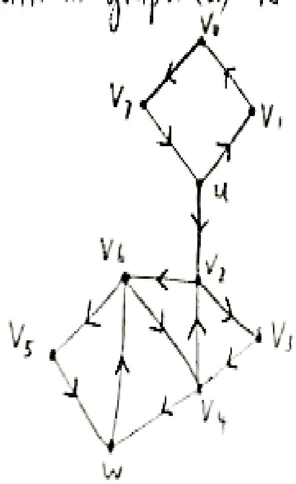
8) Graph in (a) has Euler circuit while graph in (b) has no Euler circuit.
It is because although graph in (b) has circuit that include all edges of graph,
all vertices in graph (b) are not all even degrees.
Graph in (a) has Euler circuit, $v_1 e_1 v_2 e_2 v_5 e_7 v_4 e_6 v_5 e_3 v_2 e_4 v_3 e_5 v_4 e_8 v_1$.

9. For each of graph in (a) – (b), determine whether there is an Euler path from u to w. If there is, find such a path.



9) Graph in (a) has Euler path, which is a path uses every edge on graph exactly one time.

Euler path in graph (a) is $u\ v_1\ v_0\ v_7\ u\ v_2\ v_3\ v_4\ v_2\ v_6\ v_4\ w\ v_6\ v_5\ w$.



Graph in (b) has no Euler path, because not zero or two vertices have odd degree. Vertex u, vertex e, vertex w, vertex h have odd degree, which is 3.

10. For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

10) There is no hamiltonian circuit in both graphs (a) and (b), because hamiltonian circuit is a simple circuit that includes every vertex that appears exactly once, except first and last vertices, and does not need to include all edges.

In both graphs, vertex u will appear more than once, because there are closed loops exist.

11. How many leaves does a full *3-ary* tree with 100 vertices have?

11. How many leaves $= \dfrac{(m-1)n+1}{m}$

$$l = \frac{(3-1) \times 100 + 1}{3} = 67 \text{ leaves.}$$

12. Find the following vertex/vertices in the rooted tree illustrated below.

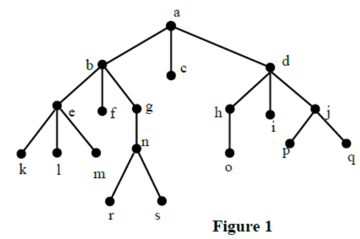


Figure 1

a) Root

b) Internal vertices

c) Leaves

d) Children of n

e) Parent of e

f) Siblings of k

g) Proper ancestors of *q*

h) Proper descendants of b

12. (a) a

(b) b, e, g, n, c, d, h, j

(c) k, l, m, f, r, s, o, i, p, q

(d) r, s

(e) b

(f) l, m

(g) j, d, a

(h) e, f, g, k, l, m, n, r, s

13. In which order are the vertices of ordered rooted tree in **Figure 1** is visited using *preorder, inorder* and *postorder*.

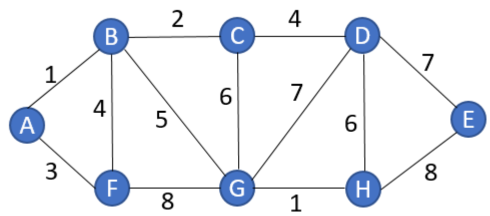
13.

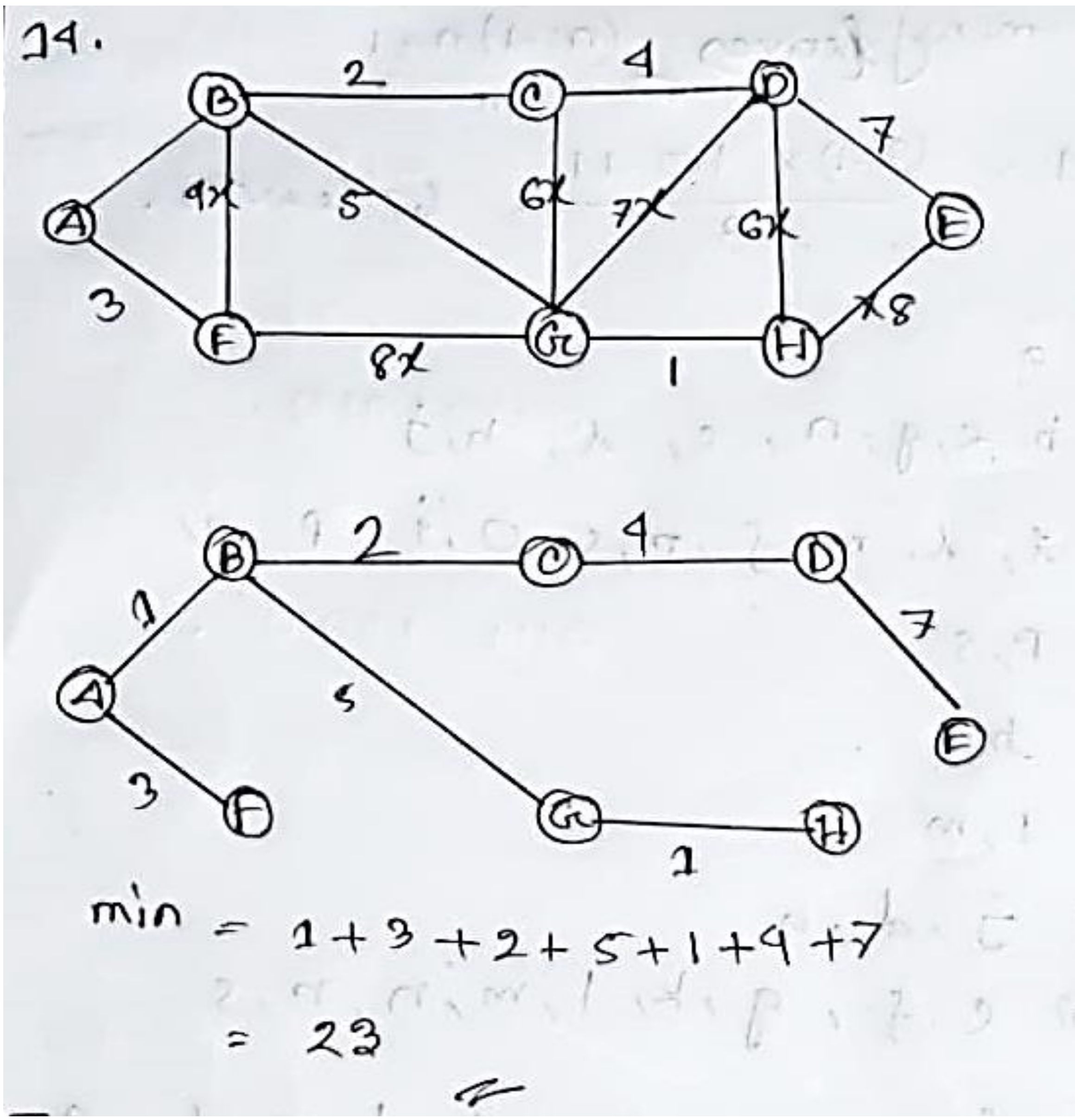
Preorder : a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

Post order : k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

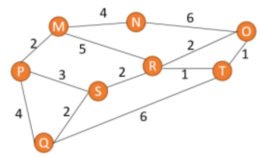
Inorder : k, e, l, m, b, f, g, r, n, s, a, c, o, h, d, i, p, j, q.

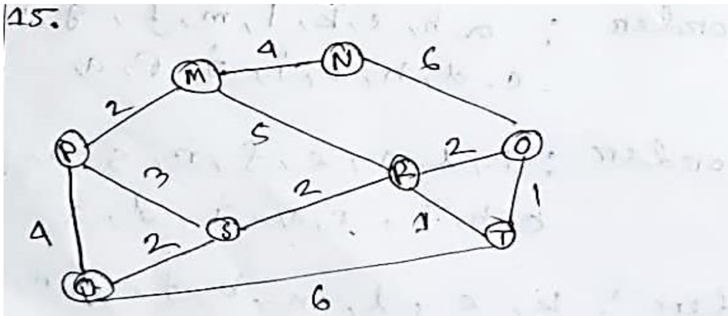
14. Find the minimum spanning tree for the following graph using Kruskal's algorithm.





15. Use Dijsktra's algorithm to find the shortest path from **M** to **T** for the following graph.





i	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	ϕ	{M, N, P, Q, R, S, T}	∞	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N, O, P, Q, R, S, T}	0	4	∞	2	∞	5	∞	∞
2	{M, P}	{N, O, Q, R, S, T}	0	4	∞	2	6	5	5	∞
3	{M, P, S}	{N, O, Q, R, T}	0	4	∞	2	6	5	5	12
4	{M, P, S, R}	{N, O, Q, T}	0	4	7	2	6	5	5	6
5	{M, P, S, R, T}	{N, O, Q}	0	4	7	2	6	5	5	6

∴ Minimum distance from M to T is 6

∴ The shortest path is M – R – T