# ASSIGNMENT 4

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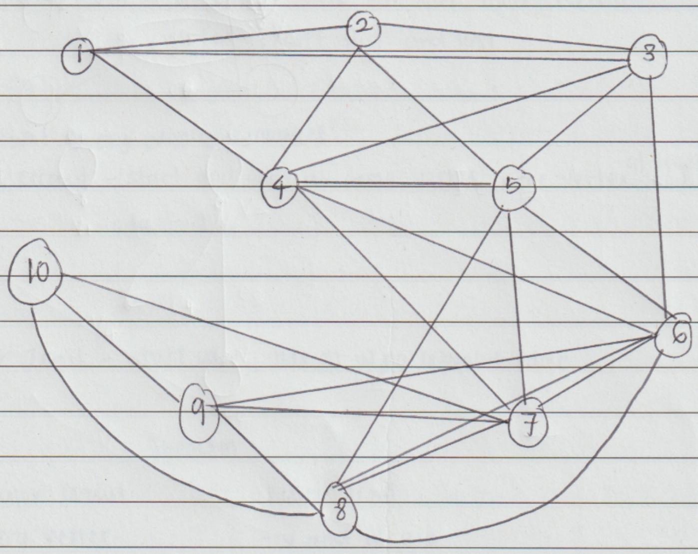
ASSIGNMENT 4.

1. Let G be a graph with $V(G) = \{1, 2, \ldots 10\}$ such two numbers $v, w$ in $V(G)$ are adjacent if and only if $|v-w| \leq 3$. Draw graph G and the number of edges.

$V(G) = \{1,2,3,4,5,6,7,8,9,10\}$

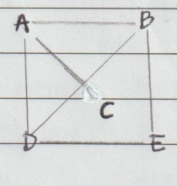
| $\lvert v-w\rvert \leq 3$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 8 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 9 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |




edges = 24

2. Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.

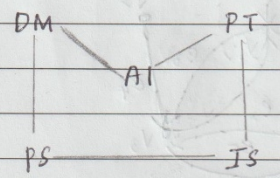
a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.




$$A_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \left[\begin{array}{ccccc} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{array}\right] \end{array}$$

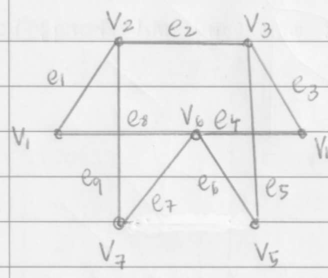
b) There are 5 subjects to be scheduled in the exam week. Discrete Mathematics (DM), Programming Technique (PT). Artificial intelligence (AI), Probability statistics (PS) and information system (IS). The following subjects cannot be scheduled in the same time slot :-

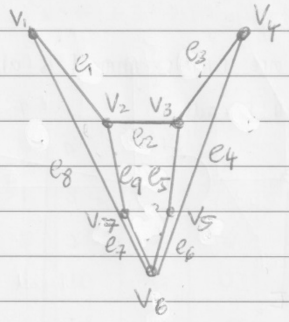
i. DM and IS
ii. DM and PT
iii. AI and PS
iv. IS and AI




$$A_H = \begin{array}{c} \\ DM \\ PT \\ AI \\ PS \\ IS \end{array} \begin{array}{c} \begin{array}{ccccc} DM & PT & AI & PS & IS \end{array} \\ \left[\begin{array}{ccccc} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{array}\right] \end{array}$$

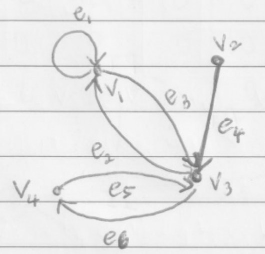
3. Show that the two drawing represent the same graph by labeling the vertices and edges of the right-hand drawing corresponding to the left-hand drawing.




4. Find the adjacency matrix and incidence matrices for the following graphs.

G:

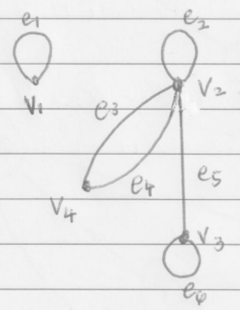



G will be undirected graph.

$$A_G = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 1 & 0 & 1 & 0 \\ V_2 & 0 & 0 & 1 & 0 \\ V_3 & 1 & 0 & 0 & 1 \\ V_4 & 0 & 0 & 1 & 0 \end{array}$$

$$I_G = \begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline V_1 & 2 & 1 & 1 & 0 & 0 & 0 \\ V_2 & 0 & 0 & 0 & 1 & 0 & 0 \\ V_3 & 0 & 1 & 1 & 1 & 1 & 1 \\ V_4 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}$$

H:

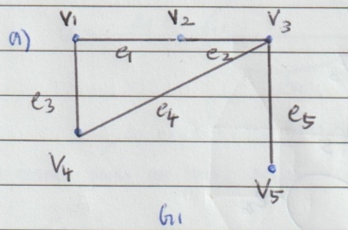



$$A_H = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 1 & 0 & 0 & 0 \\ V_2 & 0 & 1 & 1 & 2 \\ V_3 & 0 & 1 & 1 & 0 \\ V_4 & 0 & 2 & 0 & 0 \end{array}$$

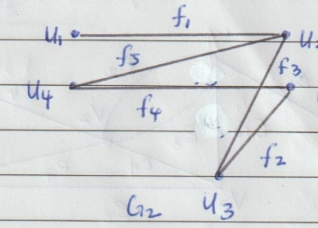
$$I_H = \begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline V_1 & 2 & 0 & 0 & 0 & 0 & 0 \\ V_2 & 0 & 2 & 1 & 1 & 1 & 0 \\ V_3 & 0 & 0 & 0 & 0 & 1 & 2 \\ V_4 & 0 & 0 & 1 & 1 & 0 & 0 \end{array}$$

5- Determine wether the following graphs are isomorphic.

a)




$G_1$




$G_2$

number vertices 5 : 5

number edges 5 : 5

deg ($G_1$) deg ($G_2$)

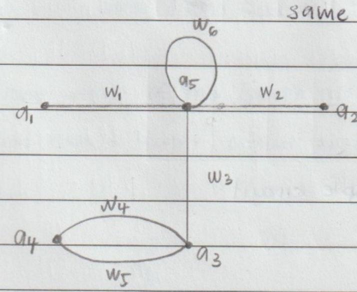
$G_1$ and $G_2$ is

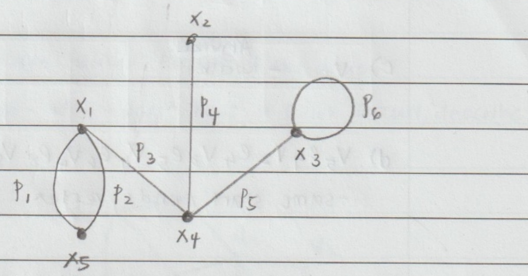
This graph is isomorphic

Degrees vertices 2,2,3,2,1 || 1,3,2,2,2

same

b)




number vertices : 5

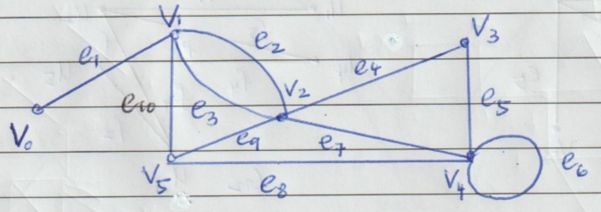
number edges 6 : 6

degree

1, 1, 2, 3, 5 || 1, 2, 3, 3, 3

So $H_1$ and $H_2$ is not isomorphic

6- In the graph below, determine wether the following walks are trails, paths, closed walks, circuit / cycles, simple circuits or just walks.




a) $V_0 e_1 V_1 e_{10} V_5 e_9 V_2 e_2 V_1$ - trail

- repeated vertex, u to w

b) $V_4 e_7 V_2 e_9 V_5 e_{10} V_1 e_3 V_2 e_9 V_5$ - just walks

- repeated vertex, v to w

c) $V_2$ - walk

d) $V_5 e_9 V_2 e_4 V_3 e_5 V_4 e_6 V_4 e_8 V_5$ - circuit

- same start / ends vertex

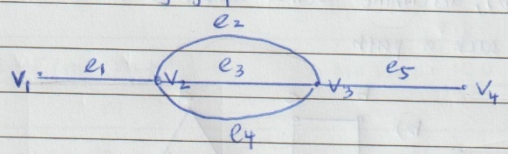
e) $V_2 e_4 V_3 e_5 V_4 e_8 V_5 e_9 V_2 e_7 V_4 e_5 V_3 e_4 V_2$ - closed walk

- repeated vertex, edges

f) $V_3 e_5 V_4 e_8 V_5 e_{10} V_1 e_3 V_2$ - Path

- no repeat vertex, edges

7 consider the following graph.




no repeat edge, vertex

a) How many path are there from $v_1$ to $v_4$?

3

b) How many trails, are there from $v_1$ to $v_4$?

repeat vertex

$3 \times 3 = 9$

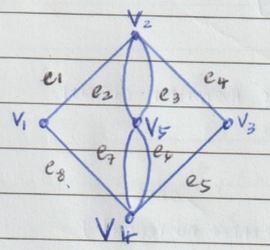
c) How many walks from $v_1$ to $v_4$?

repeated vertex, edges

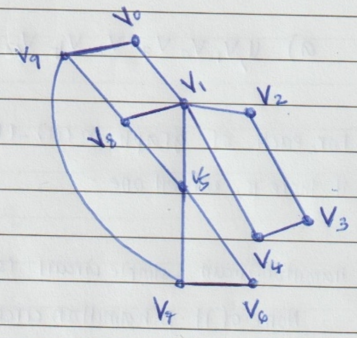
walks from $v_1$ to $v_4$ is infinite

8. Determine which of the graph in (a) - (b) have Euler circuit. If the graph does not have a Euler circuit explain why not. If it does have a Euler circuit, describe one.

a)




b)




(even)

Euler circuit - start / ends same vertex, no repeated edges.

a)

| Vertex | 1 | 2 | 3 | 4 | 5 |
| --- | --- | --- | --- | --- | --- |
| Degree | 2 | 4 | 2 | 4 | 4 |

(a) is a Euler circuit - pass through all edges only once, same starts and ends vertex

(b) is not Euler circuit - pass through edges with repetition

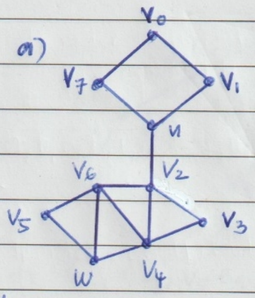
b)

| Vertex | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Degree | 2 | 5 | 2 | 2 | 2 | 4 | 2 | 3 | 3 | 2 |

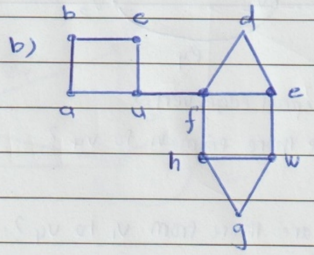
odd degree

9. For each of graph in (a) – (b), determine wether there is an Euler path from v to w. if there is, find such a path.

a)




b)




(atmost two vertices odd degree)

onnected uses every edges

Euler path – odd degree.

(known as known Euler Trail) (no repeated edge, start v ends w)

(a)

| Vertex | $V_0$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_6$ | $V_7$ | u | w |
|---|---|---|---|---|---|---|---|---|---|---|
| Degree | 2 | 2 | 4 | 2 | 4 | 2 | 4 | 2 | 3 | 3 |

(b)

| Vertex | a | b | c | d | e | f | g | h | u | w |
|---|---|---|---|---|---|---|---|---|---|---|
| Degree | 2 | 2 | 2 | 2 | 3 | 4 | 2 | 3 | 3 | 3 |

≠ atmost 2 vertices odd degree.

a) $u\ V_1\ V_0\ V_7\ V_2\ V_4\ V_6\ W\ V_4\ V_3\ V_2\ V_6\ V_5\ W$

10. For each of graph in (a) – (b) determine wether is Hamilton circuit if there is, exhibit one.

Hamilton circuit – simple circuit (start / end same vertex, vertices no repeat once)

None of it is hamilton circuit.

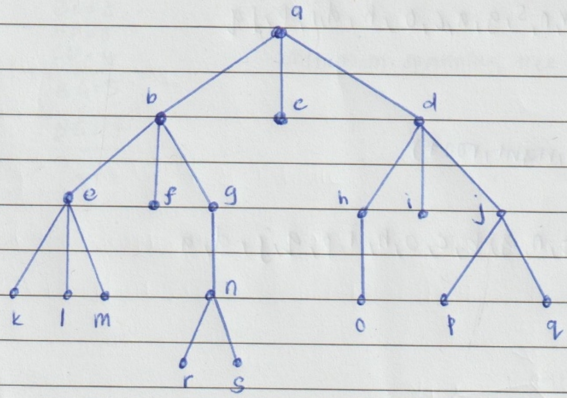
11. How many leaves does a full 3 - ary tree with 100 vertices have?

$$l = \frac{[(m-1)n+1]}{m} \qquad m=3,\ n=100$$

$$= \frac{(3-1)100+1}{3}$$

$$= 67$$

12. Find the following vertex / vertices in the rooted tree illustrated below.




a) Root  a,

b) Internal vertices  a, b, e, g, n, d, h, j

c) Leaves  f, k, l, m, c, r, s, o, i, p, q

d) Children of n  r, s

e) Parent of e  b

f) Siblings of k  l, m

g) Proper ancestors of q  a, d, j

h) Proper descendants of b - e, k, l, m, f, g, n, r, s

Typo

13. In which order are the vertices of ordered rooted tree in figure 1 is visited using preorder, inorder and postorder.

Preorder (root, left, right)

a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

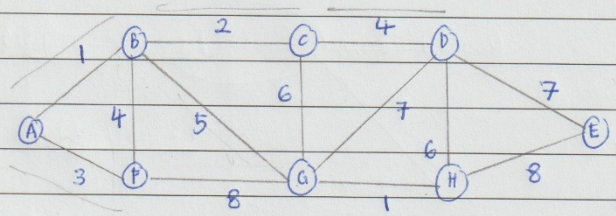
inorder (left, root, right)

k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

Postorder (left, right, root)

k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

14. Find the minimum spanning tree for the following graph using Kruskal's Algorithm.




AB = 1

GH = 1

BC = 2

AF = 3

BF = 4

CD = 4

BG = 5

CG = 6

DH = 6

DG = 7

DE = 7

FG = 8

HE = 8

select shortest edge, do not create cycle!

start, ends same vertex

repeated no edge

AB = 1
GH = 1
BC = 2
AF = 3
CD = 4
BG = 5
DE = 7

23

Minimum spanning tree : 23

15.

| Iteration | S | N | L(M) | L(N) | L(P) | L(Q) | L(S) | L(R) | L(O) | L(T) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | { } | {M, N, P, Q, S, R, O, T} | 0 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| 1 | {M} | {N, P, Q, S, R, O, T} | 0 | 4 | 2 | ∞ | ∞ | 5 | ∞ | ∞ |
| 2 | {M, N} | {P, Q, S, R, O, T} | 0 | 4 | 2 | ∞ | ∞ | 5 | 10 | ∞ |
| 3 | {M, N, P} | {Q, S, R, O, T} | 0 | 4 | 2 | 6 | 5 | 5 | 10 | ∞ |
| 4 | {M, N, P, R} | {S, Q, O, T} | 0 | 4 | 2 | 6 | 5 | 5 | 7 | 6 |
| 5 | {M, N, P, R, S} | {Q, O, T} | 0 | 4 | 2 | 6 | 5 | 5 | 7 | 6 |
| 6 | {M, N, P, R, S, Q} | {O, T} | 0 | 4 | 2 | 6 | 5 | 5 | 7 | 6 |
| 7 | {M, N, P, R, S, Q, T} | {O} | 0 | 4 | 2 | 6 | 5 | 5 | 7 | 6 |
| | | | | | | | | | | |

Shortest Path : M → R → T

Shortest Length : 6