ASSIGNMENT 1 DISCRETE STRUCTURE

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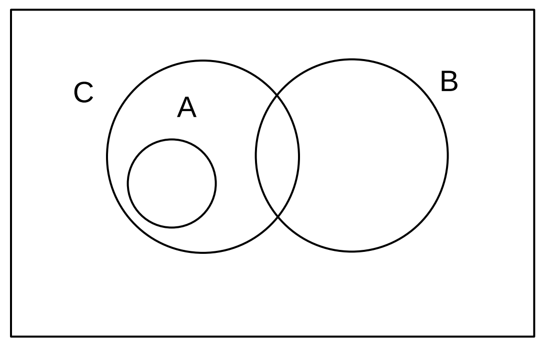
QUESTION 1

1. Let the universal set be the set R of all real numbers and let A={x∈R | $0 < x \le 2$}, B={x∈R | $1 \le x < 4$} and C={x∈R | $3 \le x < 9$}. Find each of the following:

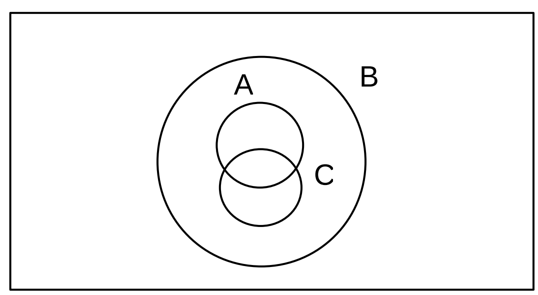
a) $A \cup C$ = { x∈R | $0 < x \le 2$ or $3 \le x < 9$ }
b) $(A \cup B)'$ = { x∈R | x<=0 or x>=4 }
c) $A' \cup B'$ = { x∈R | x<=0 or x>=4 }

QUESTION 2

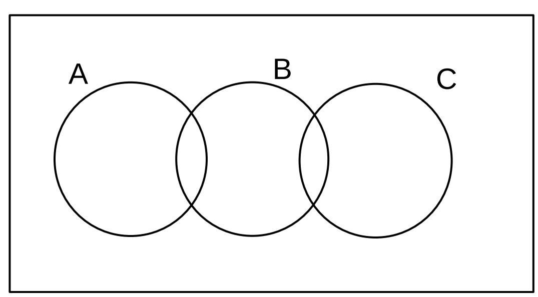
2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.
a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$



b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$



c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subset B$, $C \not\subset B$



QUESTION 3

3. Given two relations S and T from A to B,
$S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$

$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$
Let A={−1, 1, 2, 4} and B={1,2} and defined binary relations S and T from A to B as follows:
For all $(x,y) \in A \times B$, $x\ S\ y \leftrightarrow |x| = |y|$
For all $(x,y) \in A \times B$, $x\ T\ y \leftrightarrow x - y$ is even.
State explicitly which ordered pairs are in A×B, S, T, S ∩ T, and S ∪ T

A x B = { (-1 ,1) , (-1 , 2) , (1,1) , (1, 2) ,(2 ,1), (2 , 2) ,(4 ,1) ,(4,2) }

S = { (-1 , 1) , (1 , 1) , (2 , 2) }

T= { (-1 , 1) , (1 , 1) , (2 , 2), (4 , 2) }

S ∩ T= { (-1, 1) , (1, 1), (2 , 2) }

S ∪ T= { (-1 , 1) , (1 , 1) ,(2 , 2), (4 , 2) }

QUESTION 4

4. Show that $\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

LHS = $\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= \neg (\neg p \wedge (q \vee \neg q)) \vee (p \wedge q)$	Distribution Laws
$= (p \wedge (\neg q \vee q)) \vee (p \wedge q)$	Double negation
$= p \wedge (p \wedge q)$	Absorption Laws
$= p$	

QUESTION 5

5. R1={(x,y)| x+y ≤6}; R1 is from X to Y;
R2={(y,z)| y>z}; R2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5.
Find:
a) The matrix A1 of the relation R1 (relative to the given orderings)

R1 = { (1 , 1) , (1 , 2) , (1 , 3) , (1, 4) , (1, 5), (2, 1) , (2 , 2), (2 , 3) , (2 , 4) , (3 , 1) , (3 , 2) , (3 , 3), (4 , 1) , (4 , 2) , (5 , 1) }

A1 =	**1**	**2**	**3**	**4**	**5**
1	1	1	1	1	1
2	1	1	1	1	0
3	1	1	1	0	0
4	1	1	0	0	0
5	1	0	0	0	0

b) The matrix A2 of the relation R2 (relative to the given orderings)

R2 = { (2 , 1) ,(3 , 1) , (3 , 2) , (4 ,1) , (5 , 1) , (4 , 2) , (4 , 3) , (5 , 4) , (5 , 3) , (5 , 2) }

A2=	**1**	**2**	**3**	**4**	**5**
1	0	0	0	0	0
2	1	0	0	0	0
3	1	1	0	0	0
4	1	1	1	0	0
5	1	1	1	1	0

c) Is R1 reflexive, symmetric, transitive, and/or an equivalence relation?

R1 is not reflexive, not transitive but symmetric,

d) Is R2 reflexive, antisymmetric, transitive, and/or a partial order relation?

R2 is not reflexive, not transitive and antisymmetric

QUESTION 6

6. Suppose that the matrix of relation R1 on {1, 2, 3} is

[1 0 0]
[0 1 1]
[1 0 1]

relative to the ordering 1, 2, 3, and that the matrix of relation R2 on {1, 2, 3} is

[0 1 0]
[0 1 0]
[1 0 1]

relative to the ordering 1, 2, 3.

Find:

a) The matrix of relation R1 ∪ R2

R1 = { (1, 1) , (2 , 2) , (2 , 3) , (3 , 1) , (3 , 3) }

R2 = { (1, 2) , (2 , 2) , (3 , 1) , (3 , 3) }

R1 ∪ R2 = { (1, 1) , (1 , 2) , (2 , 2) , (2 , 3) , (3 ,1) , (3 , 3) }

R1 ∪ R2 =	**1**	**2**	**3**
1	1	1	0
2	0	1	1
3	1	0	1

b) The matrix of relation R1 ∩ R2

R1 = { (1, 1) , (2 , 2) , (2 , 3) , (3 , 1) , (3 , 3) }

R2 = { (1, 2) , (2, 2) , (3, 1) , (3, 3) }

R1 ∩ R2 = { (2 , 2) , (3 , 1) , (3 , 3) }

R1 ∩ R2 =	**1**	**2**	**3**
1	0	0	0
2	0	1	0
3	1	0	1

QUESTION 7

7. If f :R→ R and g:R→ R are both one-to-one, is f + g also one-to-one? Justify your answer.

Since f and g function are real numbers;

If f defined as f(x) = x , g(x) = -x , this are one to one function,
However (f+g)x = 0 for all ,

This shows that f+g is not one to one.

QUESTION 8

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase. Find a recurrence relation for $c_1, c_2, \ldots, c_n$.

$c_1 = 1$

$c_2 = 2$

The first one is if we climb one stair at a time, the general formula is:

$C_n = (C_{n-1}) + 2$, $n \geq 1$

The 2nd one is if we climb two stairs at a time, the general formula is:

$C_n = (C_{n-1}) + 1$, $n \geq 1$

If more than 3 steps;

$C_n = (C_{n-1}) + (C_{n-2})$, $n \geq 3$

QUESTION 9

9. The Tribonacci sequence (t_n) is defined by the equations, $t_0 = 0$, $t_1 = t_2 = 1$, $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$.

a) Find t_7.

$t_3 = 1 + 1 + 0 = 2$

$t_4 = 2 + 1 + 1 = 4$

$t_5 = 4 + 2 + 1 = 7$

$t_6 = 7 + 4 + 2 = 13$

$t_7 = 13 + 7 + 4 = 24$

b) Write a recursive algorithm to compute tn, $n \geq 3$.

Input : n

Output : t_n

t (n)

```
t ( n )
{

   if ( n = 1 or n = 2 )
        return 1

     return t ( n - 1 ) + t ( n - 2 ) + t ( n - 3 )
 }
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