ASSIGNMENT 3

SUBJECT:
DISCRETE STRUCTURE (SECI1013-03)

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QUESTION 1 [25 marks]

a) Let A={1, 2, 3, 4, 5, 6, 7, 8}, B={2, 5, 9}, and C={a, b}. Find each of the following:

i. A−B = {1,3,4,6,7,8}}
ii. (A∩B) ∪C = {2,5,a,b}
iii. A∩B∩C = ∅
iv. B×C = {(2,a),(2,b),(5,a),(5,b),(9,a),(9,b)}
v. P(C) = { ∅, {a}, {b}, {a, b}}

b) By referring to the properties of set operations, show that: (4 marks)

(P ∩ ((P' ∪ Q)')) ∪ (P ∩ Q)
= (P ∩ (P'' ∩ Q')) ∪ (P ∩ Q) De Morgan's Law
= (P ∩ (P ∩ Q')) ∪ (P ∩ Q) Double Negation Law
= ((P ∩ P) ∩ Q') ∪ (P ∩ Q) Associative Law
= (P ∩ Q') ∪ (P ∩ Q) Idempotent Law
= P ∩ (Q' ∪ Q) Distributive Law
= P ∩ U Complement Law
= P (shown) Properties of Universal set

c) Construct the truth table for, **A** = $(\neg p \vee q) \leftrightarrow (q \rightarrow p)$.

p	q	$\neg p$	$(\neg p \vee q)$	$(q \rightarrow p)$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then $(x+2)^2$ is odd" (4 marks)

P(x) = x is an odd integer
Q(x) = $(x + 2)^2$ is an odd integer
Symbolically,
$\forall x\,(P(x) \rightarrow Q(x))$ with the domain of discourse is the set of all integer

$a = 2k + 1$
$a + 2 = 2k + 3$
$(a + 2)^2 = (2k + 3)^2$
$(a + 2)^2 = 4k^2 + 12k + 9$
$(a + 2)^2 = (4k^2 + 12k + 8) + 1$
$(a + 2)^2 = 4(k^2 + 3k + 2) + 1$ Let $k^2 + 3k + 2 = b$ where b is an integer
$(a + 2)^2 = 4b + 1$

$\therefore (a + 2)^2$ is an odd integer
Therefore, for all integer of x, if x is odd, $(x + 2)^2$ is odd.

e) Let $P(x,y)$ be the propositional function $x \geq y$. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE. (4 marks)

i. $\exists x\, \exists y\, P(x, y)$
Let x = 5, y = 3
$x \geq y$, $5 \geq 3$ (true)
Therefore this statement is TRUE.

ii. $\forall x\, \forall y\, P(x, y)$
Let x = 6, y = 9
$6 \geq 9$ (false)
x = 6, y = 9 is the counterexample of this statement, therefore this statement is FALSE.

QUESTION2 [25 marks]

a) Suppose that the matrix of relation R on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ relative to the ordering 1, 2, 3. (7 marks)

i. Find the domain and the range of R.

R = {(1,1), (1,2), (2,2), (3,1)}

Domain = {1, 2, 3}
Range = {1, 2}

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

The relation is not irreflexive because $\exists x \in A$ where (x, x) $\in R$ main diagonal contains 0 and 1

The relation is antisymmetric because (1,2)∈R but (2,1)∉R and (1,1)∈R

b) Let S= {(x,y)| x+y ≥9}is a relation on X={2, 3, 4, 5}. Find: (6 marks)

i. The elements of the set S.

S = {(4,5), (5,4), (5,5)}

ii. Is S reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.

$$S = \begin{matrix} & \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \end{matrix}$$

S is not reflexive, not symmetric and transitive. Thus, it is not equivalence relation.

- It is not reflexive because $\forall x, \epsilon\ S, (x, x)\ not\ belong\ to$ R. Only (1, 1) belong to R.
- It is symmetric because $\forall x\ , y\ \epsilon\ S\ , (x, y) \epsilon\ R\ , (y, x) \epsilon\ R$
- It is not transitive because $M_R \times M_R \neq M_R$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

c) Let X = {1, 2, 3}, Y={1, 2, 3, 4}, and Z={1, 2}. (6 marks)

i. Define a function f: $X \to Y$ that is one-to-one but not onto.

f(x) = {(1,1),(2,2),(3,3)}

ii. Define a function g: $X \to Z$ that is onto but not one-to-one.

g(x)={(1,1)(,2,2),(3,2)}

iii. Define a function h: $X \to X$ that is neither one-to-one nor onto.

h(x)={(1,1),(2,2),(3,2)}

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

$m(x) = 4x+3$, $n(x) = 2x - 4$ (6 marks)

i. Find the inverse of m.

Let m(x) =y, therefore $m^{-1}(y)$=x

$$m(x) = 4x+3$$

$$y = 4x+3$$

$$4x = y - 3$$

$$x = \frac{y - 3}{4}$$

$$m^{-1}(x) = \frac{x - 3}{4}$$

ii. Find the compositions of $n \cdot m$.

$$n \cdot m = n(m(x))$$

=2(4x+3)-4

=8x+6-4

=8x+2

QUESTION3 [25 marks]

a) Given the recursively defined sequence.

$$a = a_{k-1} + 2k, \text{ for all integers } k \geq 2, \qquad a_1 = 1$$

i) Find the first three terms. (2 marks)

$a_{1=}$ 1

$a_{2=}$ a_1+ 2(2) = 1 + 4 = 5

$a_{3=}$ a_2+ 2(3) = 5 + 6 = 11

$a_{1=}$ 1 , $a_{2=}$ 5, $a_{3=}$ 11

ii) Write the recursive algorithm. (5 marks)

```
Input : k
Output : f (k)
a(k) {
if ( k = 1 )
return 1
return a( k – 1 ) + 2*k )
}
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size *k* as it is run with an input of size *k*□1 (where *k* is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size *k*. Find a recurrence relation for r_1, r_2, …. r_k. (4 marks)

k = input size
r_k= the number of executes, based on k

When:

k = 1, $r_1 = 7$
k = 2, $r_2 = 2$ x r_1

The input size k is times two of the number input k – 1, thus recurrence relation:
$r_k = 2r_{k-1}$, when k>1

c) Given the recursive algorithm:

Input: *n*

Output: *S* (*n*)

```
S(n) {
        if (n=1)
        return 5
         return 5*S(n-1)
        }
```

Trace *S*(4). (4 marks)

S(1) = 5
S(2) = 5 x S(1) = 5 x 5 = 25
S(3) = 5 x S(2) = 5 x 25 = 125
S(4) = 5 x S(3) = 5 x 125 = 625

QUESTION 4 [25 marks]

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

(4 marks)

3 – B	_	_	5 - F
9	16	16	11

3 - B have 9 subscript
5 - F have 11 subscript

Total hex no. = 9 x 16 x 16 x 11
= 25344

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

(4 marks)

Letter have 26 subscript and number have 10 subscript. First plate number don't have 0.
A _ _ _ // _ _ 0

1 x 26 x 26 x 26 x 9 x 10 x 1 = 1581840 ways

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?

(5 marks)

Total number of letters: 8
No repeated letters

Case 1 (one word) = ${}^8P_1 = 8$
Case 2 (two words) = ${}^8P_2 = 56$
Case 3 (three words) = ${}^8P_3 = 336$

Total arrangement = case 1+ case 2 + case 3
= 8 + 56 + 336
= 400 ways

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

(4 marks)

From 7 women choose 4 women (7, 4)

$= \frac{7!}{4!(7-4)!}$

$= \frac{7!}{4!(3)!}$

=35

From 6 men choose 3 men (6, 3)

$= \frac{6!}{3!(6-3)!}$

$= \frac{6!}{3!(3)!}$

=20

Groups of seven that can be chosen = 35 x 20 = 700

e) How many distinguishable ways can the letters of the word *PROBABILITY* be arranged?

(4 marks)

There are 11 characters with 1P, 1R, 1O, 2B, 1A, 2I, 1L, 1T, 1Y

P(11) = $\frac{11!}{2!\ x\ 2!}$

= 9979200

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

(4 marks)

10 selection, r = 10

C(r + n - 1, r) = $\frac{(r+n-1)!}{r!(n-1)!}$

C(10 + 6 - 1, 10) = $\frac{(10+6-1)!}{10!(6-1)!}$

C(15, 10) = $\frac{15!}{10!(5)!}$

= 3003

QUESTION 5 [10 marks]

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

(4 marks)

X = {(Ali, Daud), (Bahar, Daud), (Carlie, Daud),

(Ali, Elyas), (Bahar, Elyas), (Carlie, Elyas)}

Total people with same first name and same last name

= 3 x 2

= 6

$m = \left\lceil \frac{18}{6} \right\rceil$

= 3 (at least 3 has same first and last names)

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

(3 marks)

Total Number, A = 20
Odd Number, O = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19} =10
Even Number , E = { 2 , 4 , 6 , 8 , 10 , 12 , 14 , 16 , 18 , 20 } = 10
20 – 10 = 10
Therefore, 11 integers should be picked.
To ensure at least 1 odd integer, it must be 10 + 1 = 11.

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

(3 marks)

Integers that is divisible by 5: (20 integers)

{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100}

integers that is not divisible by 5: 100 – 20 = 80

total integers to pick: 80 + 1 = 81 (at least one integer is divisible by 5)