UNIVERSITI TEKNOLOGI MALAYSIA

TOPIC:

ASSIGNMENT 3 DISCRETE STRUCTURE

SECTION:

SECTION 03 (2020/2021)

GROUP:

GROUP 10

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QUESTION 1

a) Let A={1, 2, 3, 4, 5, 6, 7, 8}, B={2, 5, 9}, and C={a, b}. Find each of the following:

i. $A–B$ = {1, 3, 4, 6, 7, 8}
ii. $(A\cap B)\cup C$

$(A\cap B)$ = {2, 5}

$(A\cap B)\cup C$ = {2, 5, a, b}

iii. $A\cap B\cap C$ = {}
iv. $B\times C$ = {(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)}
v. $P(C)$ = {ϕ, {a}, {b}, {a, b}}

b)

(P ∩ ((P’ U Q)’)) U (P ∩ Q)

=(P ∩ (P” ∩ Q’)) U (P ∩ Q)	De Morgan’s Law
= (P ∩ (P ∩ Q’)) ∪ (P ∩ Q)	Double Negation Law
= ((P ∩ P) ∩ Q’) ∪ (P ∩ Q)	Associative Law
= (P ∩ Q’) ∪ (P ∩ Q)	Idempotent Law
= P ∩ (Q’ ∪ Q)	Distributive Law
= P ∩ U	Complement Law
= P (shown)	Properties of Universal set

c) Construct the truth table for, A = $(\neg p \vee q) \leftrightarrow (q \rightarrow p)$

p	q	$\neg p$	$(\neg p \vee q)$	$(q \rightarrow p)$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Proof the following statement using direct proof “For all integer x, if x is odd, then (x+2)^2 is odd”

P(x) = x is odd

Q(x) = (x+2) ^ 2 is odd

Then we can say, $\forall\ (\boldsymbol{P}\ (\boldsymbol{x}) \rightarrow \boldsymbol{Q}(\boldsymbol{x}))$ with domain of discourse is set of all integers.

Let y is an odd integer;

y = 2n + 1

(y + 2)^2 = (2n + 1 + 2)^2

(y + 2)^2 = (2n + 3) ^ 2

(y + 2)^2 = $4n$ ^2 + $12n$ + 9

(y + 2)^2 = 2 (2n^2 + 6n + 4) + 1

(y + 2)^2 = 2m + 1, where m is 2n^2 + 6n + 4.

So, when x is odd then (x+2)^2 is odd.

e) Let P(x,y) be the propositional function x ≥ y. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\exists x\ \exists y\ P(x, y)$

Let $x = 3$; y = 7,
P(x ≥ y) = 3 ≥ 7 (FALSE)

Let $x = 7$; y = 3,
P(x ≥ y) = 7 ≥ 3 (TRUE)

∴ $\exists x\ \exists y$(x ≥ y) is TRUE

ii. $\forall x\ \forall y\ P(x, y)$

Let $x = 3$; y = 7,
P(x ≥ y) = 3 ≥ 7 (FALSE)

∴ $\forall x\ \forall y$(x ≥ y) is FALSE

QUESTION 2

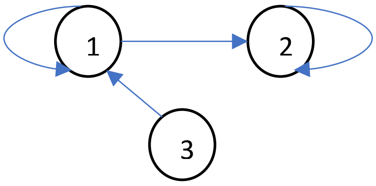
a) Suppose that the matrix of relation R on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

relative to the ordering 1, 2, 3.

i. Find the domain and the range of R.

Domain of R = {1, 2, 3}
Range of R = {1, 2}

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.



Digraph of R

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Diagonal of R

Irreflexive: No. Because :-

1. Diagonal contains non-zero.
2. Have loop at vertex 1 and vertex 2.

Antisymmetric: Yes. Because :-

1. No dual way in digraph
2. (1, 2) $\in$ R, but (2, 1) $\notin$ R or (3, 1) $\in$ R, but (1, 3) $\notin$ R

b) Let S={(x,y)| x+y ≥9}is a relation on X={2, 3, 4, 5}. Find:

i. The elements of the set S.

ii. Is S reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer

Answer

i. S={ (4,5) , (5,4) , (5,5) }

ii. -S is not reflexive, because the diagonal value in the matrix does not contain all 1's

-S is symmetric because the transpose of matrix S is equal to the original of matrix S

$M\_S = M\_S\text{^}T$

-S is not transitive because $M\_S \times M\_S \neq M\_S$

-S is not equivalence relation because it has only symmetric properties while not

Reflexive and not transitive

c) Let X={1, 2, 3}, Y={1, 2, 3, 4}, and Z={1, 2}. (6 marks)

i. Define a function f: X→Y that is one-to-one but not onto.

ii. Define a function g: X→Z that is onto but not one-to-one.

iii. Define a function h: X→X that is neither one-to-one nor onto.

- **i) { (1,1) , (2,3), (3,4) } ; because every element have different codomain and not all codomain have domain.**
- **ii) { (1,2) , (2,1) , (3,1) } ; because all codomain have its domain and domain have same codomain.**
- **iii) { (1,3) , (3,3) , (2,1) } ; because domain have same codomain and not all codomain have its domain.**

d)Let m and n be functions from the positive integers to the positive integers defined by the equations: m(x) = 4x+3, n(x) = 2x□4 (6 marks)

i. Find the inverse of m.

ii. Find the compositions of n o m.

i)

$m(x)= 4x+3$ let $y =x$

$m^{-1}(y)= 4x+3$

$y=4x+3$

$x = \frac{y-3}{4}$

$m^{-1}(x) = \frac{y-3}{4}$

ii)

$n \circ m = 2(4x +3) -4$

$= 8x + 6 - 4$

$= 8x + 2$

QUESTION 3

a) Given the recursively defined sequence.

$a_k = a_{k-1} + 2k$, for all integers $k \geq 2$, $a_1 = 1$

i)Find the first three terms

$a_1 = 1$

$a_2 = 1 + 2\ (2)$

$= 5$

$a_3 = 5 + 2(3)$

$= 11$

ii. Write the recursive algorithm.

```
Input : k

Output : f (k)

f (k) {

        if ( k = 1 )

                return 1

        return f ( k – 1 ) + 2*k )

        }
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k–1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let rk = the number of executes with an input size k. Find a recurrence relation for r1, r2, …. rk.

Answer
K= size of input
rk=number of executes, with k size input.

when k=1, r1=7
when k=2, r2=doubles=2 x r1

recurrence relation
rk= 2(r(k-1)) , where k is >= 1

c) Given the recursive algorithm:

Input: n

Output: S (n)

```
S(n) {
if (n=1)
   return 5
return 5*S(n - 1)
}
```

Trace S(4).

S (1) = 5

Return 5

S(2) = 25

Return 5 * S(1)

S (3) = 125

Return 5 * S (2)

S (4) = 625

Return 5 * S (3)

So, S(4) = 625

QUESTION 4

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long? (4 marks)\

From 3 to B = 9 possibilities (First digit)

From 5 to F = 11 possibilities (last digit)

Total possibilities for 4 digits= 9 X 16 X 16 X 11 = 25344

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

Since the plate have four letters and three digits ;

A --- --- --- --- --- 0

1 x 25 x 25 x 25 9 x 10 x 1

25 because there are 25 letters and 9 because plates cannot start with 0.

= 25 x 25 x 25 x 9 x 10

= 1406250 license plates.

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?

Case 1: A row with 1 letter

$\overline{8}$

Case 2: A row with 2 letters

$\overline{8} \times \overline{7} = 56$

Case 3: A row with 3 letters

$\overline{8} \times \overline{7} \times \overline{6} = 336$

Total: 8 + 56 + 336 = 400 arrangements

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

= 7C4 x 6C3

= 700 groups

e) How many distinguishable ways can the letters of the word PROBABILITY be arranged?

Total letters = 11

Total ways to arranged= $\frac{11!}{(1!1!1!2!1!2!1!1!1!)}$

= 9979200

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

Answer
Number of pastries to select =r=20
number of types of pastries= n= 6

$$\binom{20+6-1}{20} = \binom{25}{20} = \frac{25!}{20!(25-20)!} = \frac{25!}{20!(5!)} = 53130$$

QUESTION 5

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

Pigeon (n) : 18 persons

Pigeonholes (k) : 3 x 2 = 6 (possible combinations of first and last name)

The generalized pigeonhole principle is applied,

$\left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{18}{6} \right\rceil = \lceil 3 \rceil = 3$ people have the same first and last names

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

Answer
Number of odd numbers= 10
Number of even numbers= 10

Therefore, we must pick 11 integers. This is because it may happen that the first 10 integers picked turn out to be even, so the last integer picked must and can be sure to be odd.

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

Number that is divisible by 5 from 1- 100 = 19 (pigeonhole)

Number that are not divisible by 5 = 100-19

= 81

So, the least number you must pick to get one that is divisible by 5 is = 81 + 1

= 82 numbers (pigeon)