



UTM

UNIVERSITI TEKNOLOGI MALAYSIA

TOPIC:
ASSIGNMENT 3 DISCRETE STRUCTURE

SECTION:
SECTION 03 (2020/2021)

GROUP:
GROUP 10

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1. Let G be a graph with $V(G) = \{1, 2, \dots, 10\}$, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if $|v - w| \leq 3$. Draw the graph G and determine the numbers of edges, $e(G)$.

Vertex = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

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v \ w	1	2	3	4	5	6	7	8	9	10
1		1	2	3	4	5	6	7	8	9
2	1		1	2	3	4	5	6	7	8
3	2	1		1	2	3	4	5	6	7
4	3	2	1		1	2	3	4	5	6
5	4	3	2	1		1	2	3	4	5
6	5	4	3	2	1		1	2	3	4
7	6	5	4	3	2	1		1	2	3
8	7	6	5	4	3	2	1		1	2
9	8	7	6	5	4	3	2	1		1
10	9	8	7	6	5	4	3	2	1	

= obey $|v - w| \leq 3$ = invalid. v and w cannot be the same number

2a) Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.

Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri

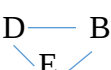
and Ehsan all friends.

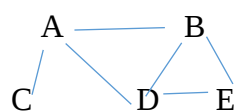
(Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)

A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan

Ahmad and Bakri are friends = A-B

Ahmad is also friends with David and Chong= A 

David, Bakri and Ehsan all friends.= D 

Therefore, 

adjacency matrix

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	0	0
D	1	1	0	0	1
E	0	1	0	1	0

2b)

There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and

Information System (IS). The following subjects cannot be scheduled in the same time

slot:-

i. DM and IS

ii. DM and PT

i and ii) = DM — AI
PS

iii. AI and PS

iv. IS and AI

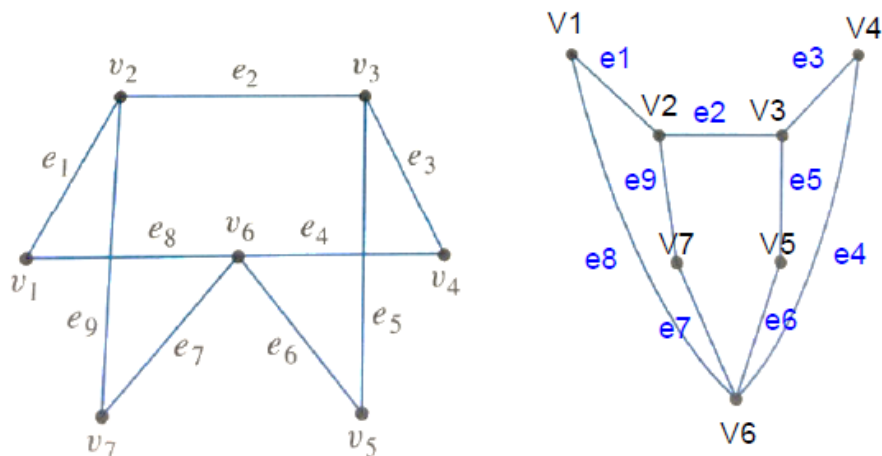
iii and iv) = AI — DM
PT

Final graph DM — PS
AI — PT — IS

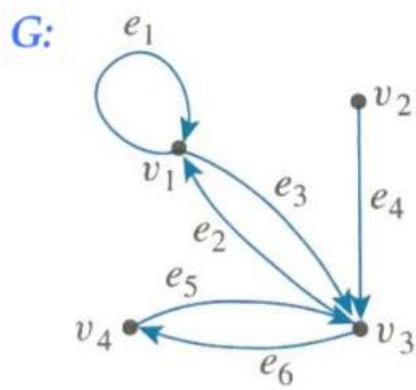
	DM	PT	AI	PS	IS
DM	0	0	1	1	0
PT	0	0	1	1	1
AI	1	1	0	0	1
PS	1	1	0	0	1
IS	0	1	1	1	0

3. Show that the two drawing repre

edges of the right-hand drawing to correspond to left-hand drawing.



4)

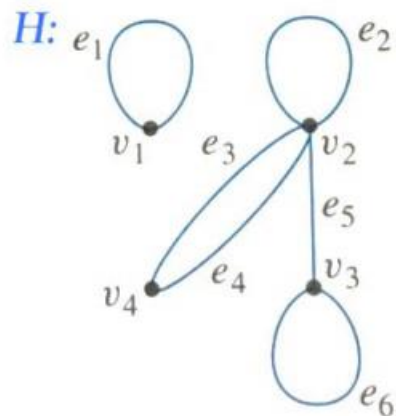


Adjacency matrices

	V1	V2	V3	V4
V1	1	0	1	0
V2	0	0	1	0
V3	1	0	0	1
V4	0	0	1	0

Incidence matrices

	e1	e2	e3	e4	e5	e6
V1	1	0	1	0	0	0
V2	0	0	0	1	0	0
V3	0	1	0	0	0	1
V4	0	0	0	0	1	0



Adjacency matrices

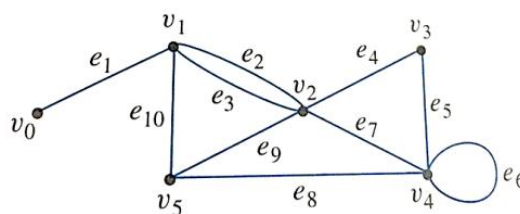
	V1	V2	V3	V4
V1	1	0	0	0
V2	0	1	1	1
V3	0	1	1	0
V4	0	1	0	0

Incidence matrices

	e1	e2	e3	e4	e5	e6
V1	2	0	0	0	0	0
V2	0	2	1	1	1	0
V3	0	0	0	0	1	2
V4	0	0	1	1	0	0

5. a) G_1 and G_2 are isomorphisms
 b) H_1 and H_2 are not isomorphisms

6) In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks



a) $v_0 e_1 v_1 e_{10} v_5 e_9 v_2 e_2 v_1$

Repeated vertices : Yes

Repeated edges : No

Open/Close : Open

$\therefore$ Trails

b) $v_4 e_7 v_2 e_9 v_5 e_{10} v_1 e_3 v_2 e_9 v_5$

Repeated vertices : Yes

Repeated edges : Yes

Open/Close : Open

$\therefore$ Walks

c) v_2

Repeated vertices : No

Repeated edges : No

Open/Close : Close

$\therefore$ Cycle

d) $v_5 e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4 e_8 v_5$

Repeated vertices : Yes

Repeated edges : No

Open/Close : Close

$\therefore$ Circuit

e) $v_2 e_4 v_3 e_5 v_4 e_8 v_5 e_9 v_2 e_7 v_4 e_5 v_3 e_4 v_2$

Repeated vertices : Yes

Repeated edges : Yes

Open/Close : Close

$\therefore$ Closed walks

f) $v_3 e_5 v_4 e_8 v_5 e_{10} v_1 e_3 v_2$

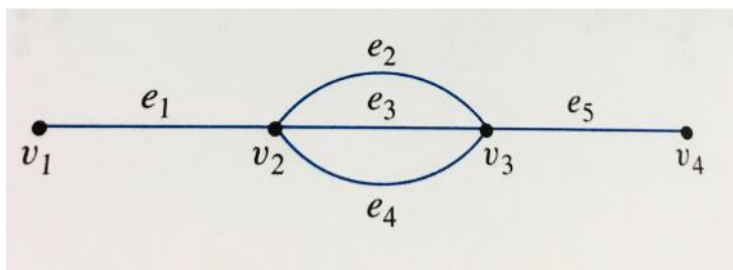
Repeated vertices : No

Repeated edges : No

Open/Close : Open

$\therefore$ Paths

7. Consider the following graph.



a) How many paths are there from v_1 to v_4 ?

= $v_1, e_1, v_2, e_2, v_3, e_5, v_4$

= $v_1, e_1, v_2, e_3, v_3, e_5, v_4$

= $v_1, e_1, v_2, e_4, v_3, e_5, v_4$

= total paths is 3

b) How many trails are there from v_1 to v_4 ?

$= v_1, e_1, v_2, e_2, v_3, e_5, v_4$
 $v_1, e_1, v_2, e_4, v_3, e_2, v_2, e_3, v_3, e_5, v_4$
 $= v_1, e_1, v_2, e_3, v_3, e_5, v_4$
 $= v_1, e_1, v_2, e_4, v_3, e_5, v_4$
 $= v_1, e_1, v_2, e_2, v_3, e_3, v_2, e_4, v_3, e_5, v_4$
 $= v_1, e_1, v_2, e_3, v_3, e_4, v_2, e_2, v_3, e_5, v_4$
 $= v_1, e_1, v_2, e_4, v_3, e_3, v_2, e_2, v_3, e_5, v_4$
 =total is 9 trails

c) How many walks are there from v_1 to v_4 ?
 = infinity because it can go through combination of edges and vertices repeatedly.

8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.

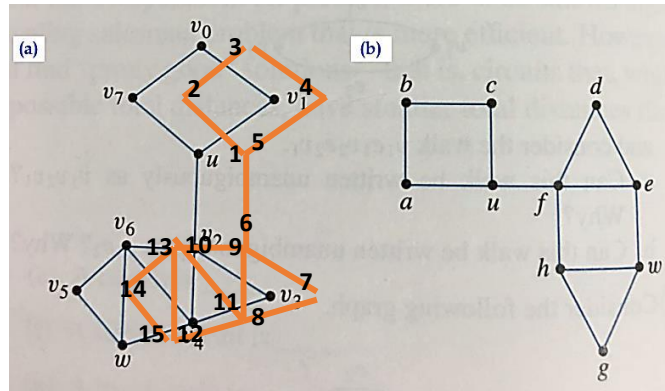
Diagram a has Euler circuit because every vertex has a positive even degree.

$\text{Deg}(V_1) = 2$
 $\text{Deg}(V_2) = 4$
 $\text{Deg}(V_3) = 2$
 $\text{Deg}(V_4) = 4$
 $\text{Deg}(V_5) = 4$

Diagram b does not have Euler circuit because is has an odd degree

$\text{Deg}(V_0) = 2$
 $\text{Deg}(V_1) = 5$
 $\text{Deg}(V_2) = 2$
 $\text{Deg}(V_3) = 2$
 $\text{Deg}(V_4) = 2$
 $\text{Deg}(V_5) = 4$
 $\text{Deg}(V_6) = 2$
 $\text{Deg}(V_7) = 3$
 $\text{Deg}(V_8) = 3$
 $\text{Deg}(V_9) = 3$

9) For each of graph in (a) – (b), determine whether there is an Euler path from u to w . If there is, find such a path.



- (a) Euler path = (u, v₇, v₀, v₁, u, v₂, v₃, v₄, v₂, v₆, v₄, w, v₆, v₅, w)
 (b) No Euler path

10)

For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

-Both graph in (a) and (b) could not form to Hamiltonion circuit because exist the only one degree on vertices, between vertex (u) and (v₂) for the graph in (a) and also between vertices (u) and (v) for the graph in (b), this happens to both graph unable to complete the circuit.

11. How many leaves does a full 3-ary tree with 100 vertices have?

$$m = 3$$

$$n = 100$$

$$i = (100-1)/3$$

$$= 33$$

$$l = [(3-1)100 + 1] / 33$$

$$= 6 \text{ leaves}$$

12. Find the following vertex/vertices in the rooted tree illustrated below.

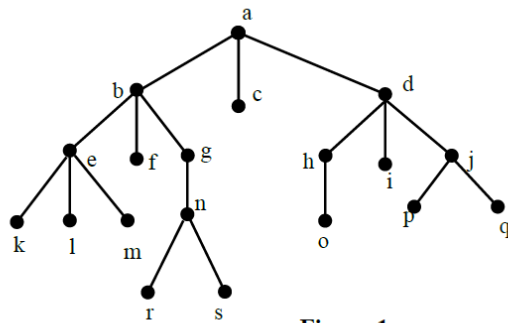
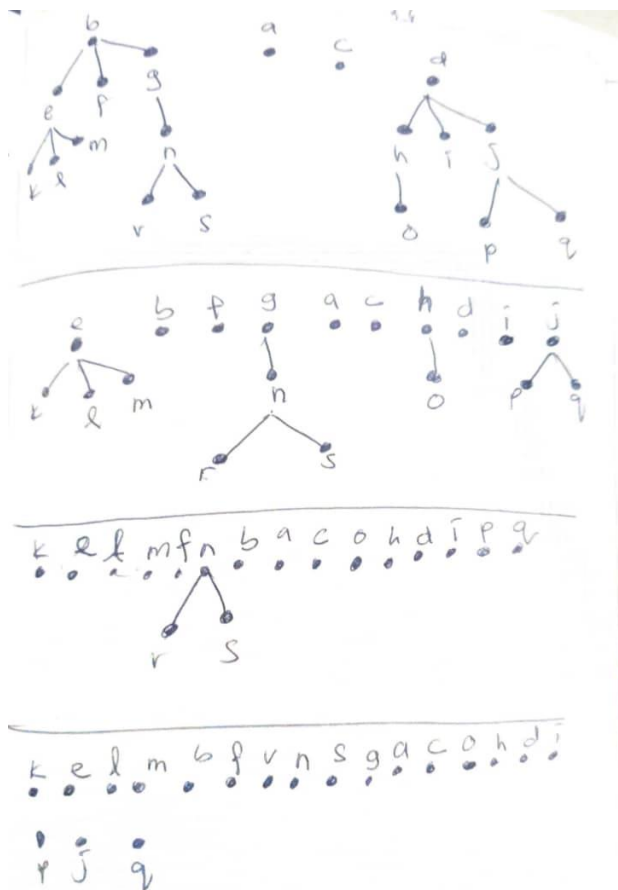


Figure 1

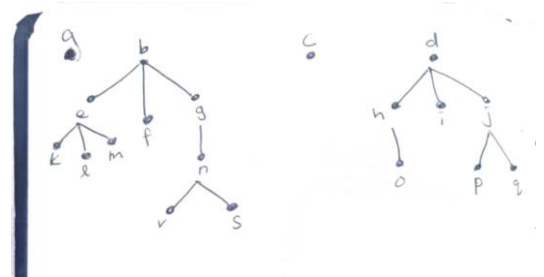
- a) Root = **a**
- b) Internal vertices = **b, d, e, g, n, h, j**
- c) Leaves = **c, f, i**
- d) Children of **n** = **r and s**
- e) Parent of **e** = **b**
- f) Siblings of **k** = **l and m**
- g) Proper ancestors of **q** = **a, d, j**
- h) Proper descendants of **b** = **e, k, l, m, f, g, n, r, s**

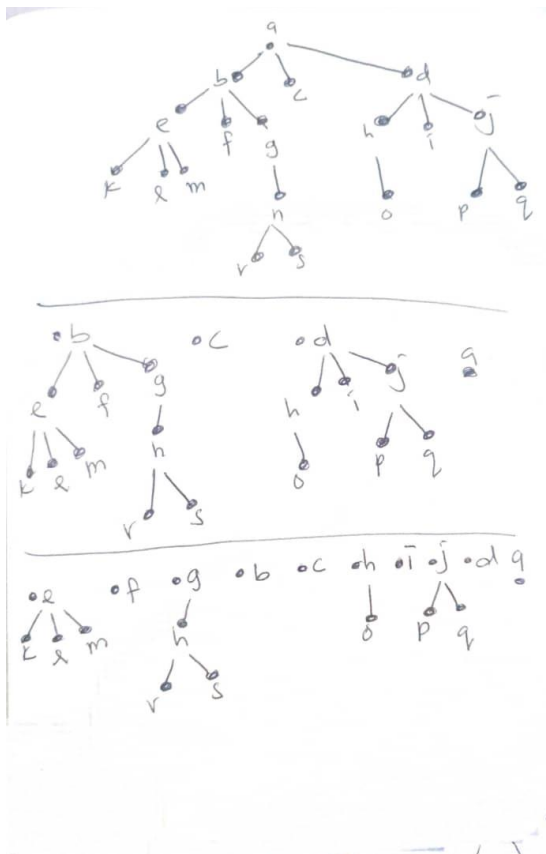
13. In which order are the vertices of ordered rooted tree in Figure 1 is visited using preorder, inorder and postorder.

- Preorder



- Inorder





postorder

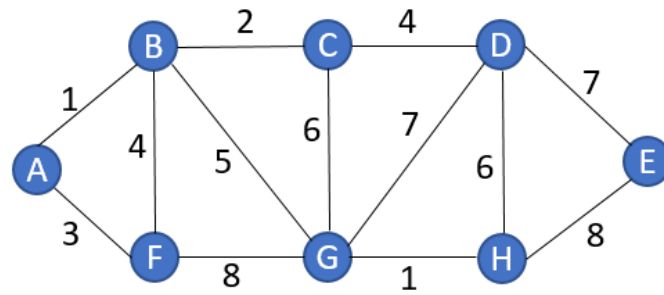
k l m e f h g b c o
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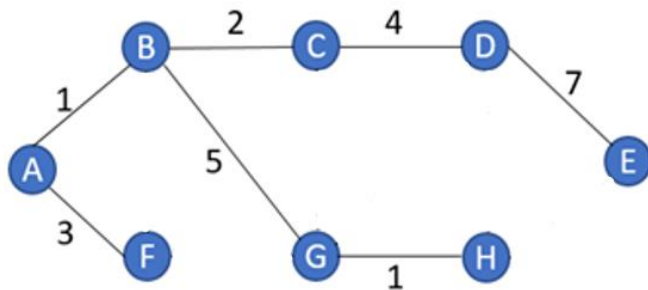
k l m e f ~~h~~ v s h g b c
• • • • •

o h i p q j d a
• • • • •

14. Find the minimum spanning tree for the following graph using Kruskal's algorithm.



EDGE	SIZE
GH	1
AB	1
BC	2
AF	3
BF	4
CD	4
BG	5
CG	6
DH	6
DG	7
DE	7
FG	8
HE	8



$$\begin{aligned}\text{Total weight} &= 7 + 5 + 4 + 3 + 2 + 1 + 1 \\ &= 23\end{aligned}$$

15.

iteration	S	N	L(M)	L(N)	L(P)	L(R)	L(S)	L(Q)	L(O)	L(T)
0	{}	{M,N,P,R,S,Q,O,T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N,P,R,S,Q,O,T}	0	4	2	5	∞	∞	∞	∞
2	{M,P}	{N,R,S,Q,O,T}	0	4	2	5	5	6	∞	∞
3	{M,P,N}	{R,S,Q,O,T}	0	4	2	5	5	6	10	∞
4	{M,P,N,R}	{S,Q,O,T}	0	4	2	5	7	6	7	6

$$\text{Shortest distance} = \{M,R,T\} = M \rightarrow R \rightarrow T = 5 + 1 = 6$$