TOPIC:

ASSIGNMENT 1 DISCRETE STRUCTURE

SECTION:

SECTION 02 (2020/2021)

GROUP:

GROUP 10

LECTURER'S NAME :

DR NOR AZIZAH ALI

GROUP MEMBERS:

NAME	**MATRIC NO**
CHONG KHAI ZE	A20EC0186
NUR IZZAH MARDHIAH BINTI RASHIDI	A20EC0116
AMIRAH RAIHANAH BINTI ABDUL RAHIM	A20EC0182

ASSIGNMENT DISCRETE STRUCTURE.

1. Let the universal set be the set R of all real numbers and let A={x∈R | 0 < x ≤ 2}, B={x∈R | 1 ≤ x < 4} and C={x∈R | 3 ≤ x < 9}. Find each of the following:

A = {1, 2}

B = {1, 2, 3}

C = {3, 4, 5, 6, 7, 8}

a) A ∪ C

A ∪ C = {1 ,2, 3, 4, 5, 6, 7, 8}

b) (A ∪ B)′

A ∪ B = {1, 2, 3}

(A ∪ B)' = {4, 5, 6, 7, 8}

c) A′ ∪ B′

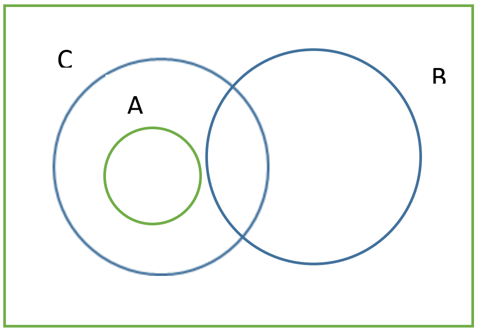
A' = {3, 4, 5, 6, 7, 8}

B' = {4, 5, 6, 7, 8}

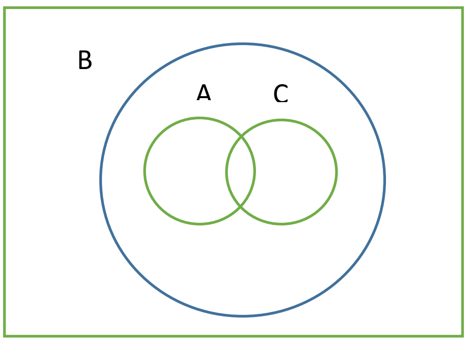
A' ∪ B' = {3, 4, 5, 6, 7, 8}

2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

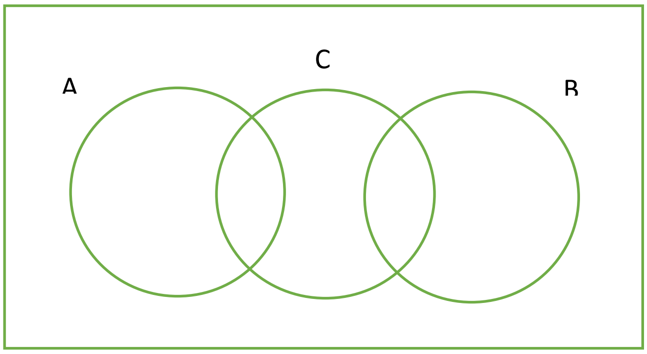
a) A ∩ B = ∅, A ⊆ C, C ∩ B ≠ ∅



b) A ⊆ B, C ⊆ B, A ∩ C ≠ ∅



c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subset B$, $C \not\subset B$



3. Given two relations S and T from A to B,

$S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in \mathrm{T}\}$
$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in \mathrm{T}\}$
Let $A=\{-1, 1, 2, 4\}$ and $B=\{1,2\}$ and defined binary relations S and T from A to B as follows:

For all $(x,y) \in A \times B$, $x\ S\ y \leftrightarrow |x| = |y|$
For all $(x,y) \in A \times B$, $x\ T\ y \leftrightarrow x-y$ is even
State explicitly which ordered pairs are in $A \times B$, S, T, $S \cap T$, and $S \cup T$.

A={-1, 1, 2, 4}

B={1, 2}

A X B={(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)}

S ={ (x, y) ∈ A×B, x & y ↔ |x| =|y|

Since,

I. (-1,1) ∈ A × *B*
|-*1*|=*1*

II. *(*1,1*)* ∈ A × B
1 =1

III. (2,2) ∈ A × B
|2|=|2|

S={(-1, 1), (1, 1), (2,2)}

T={(x, y) ∈ A × B × T y ↔ x - y is even

Thus,

(-1, 1): -1 -1 = -2 is even
(-1, 2): -1 -2 = -3 is odd
(1, 1): 1-1 =0 is even
(1, 2): 1- 2 =-1 is odd

(2, 1): 2- 1 =1 is odd
(2, 2): 2- 2 =0 is even
(4, 1): 4- 1 = 3 is odd
(4, 2): 4- 2 =2 is even

→ T={(-1, 1), (1, 1), (2, 2), (4,2)

→ S ∩ T={(-1, 1), (1, 1), (2, 2)

→ S ∪ T={(-1, 1), (1, 1), (2, 2), (4, 2)}

4. Show that $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$	
$= \neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q) \vee (p \wedge q)$	**De Morgan's Law**
$= (\neg\neg p \vee \neg q) \wedge (\neg\neg p \vee \neg\neg q) \vee (p \wedge q)$	**De Morgan's Law**
$= (p \vee \neg q) \wedge (p \vee q) \vee (p \wedge q)$	**Double Negation Law**
$= p \vee (\neg q \wedge q) \vee (p \wedge q)$	**Distributive Law**
$= p \vee (q \wedge \neg q) \vee (q \wedge p)$	**Commutative Law**
$= p \vee q \wedge (\neg q \vee p)$	**Distributive Law**
$= p \vee (q \wedge \neg q) \vee p$	**Associative Law**
$= p \vee (\emptyset) \vee p$	**Complement Law**
$= p \vee p$	**Identity Law**
$= p$	**Idempotent Law**

5. R1={(x,y)| x+y ≤6}; R1 is from X to Y; R2={(y,z)| y>z}; R2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5.

Find:

a) The matrix A1 of the relation R1 (relative to the given orderings)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b) The matrix A2 of the relation R2 (relative to the given orderings)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

c) Is R1 reflexive, symmetric, transitive, and/or an equivalence relation?

Reflexive : R1 is not reflexive because (a,a) ∉ R.
Symmetric : R1 is symmetric because $\forall\ a, b\ \in R\ then\ b, a \in R$
Transitive : R1 is not transitive because product Boolean is not the same.
= No, it is not equivalence relation since it does not satisfy the reflexive and transitive relation.

d) Is R2 reflexive, antisymmetric, transitive, and/or a partial order relation?

Reflexive : It is not reflexive because (a,a) ∉ R.
Antisymmetric : It is antisymmetric because $\forall\ a, b\ \in R\ then\ b, a \notin\ R$
Transitive : It is not transitive because product Boolean is not the same.
= **No, it is not partial order because it does not satisfy reflexive and transitive relation.**

6. Suppose that the matrix of relation R_1 on {1, 2, 3} is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R_2 on {1, 2, 3} is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

R1 = {(1,1), (2,2), (2,3), (3,1), (3,3) }

R2 = {(3,1), (1,2), (2,2), (3,3)}

R1 $\cup$ R2 = {(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)}

R1 $\cap$ R2 = {(2,2), (3,1), (3,3)}

7. If f :**R**→ **R** and g:**R**→ **R** are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

Let say, $x1 = x2$

If $f\ and\ g\ are\ f(x), then$;

$f(x1) = \ f(x2)$	$g(x1) = g(x2)$
$x1 = x2$	$x1 = x2$

This also means that;

$(f + g)(x1) = (f + g)(x2)$

$f(x1) + g(x1) = f(x2) + g\ (x2)$

Then we can say that $\boldsymbol{f + g}$ is one-to-one since $\boldsymbol{x1 = x2}$ since we have proven above.

8.With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let cn be the number of different ways to climb the staircase. Find a recurrence relation for $c1$, $c2$, …., cn.

When n=1, c(1)=1
When n=2, c(2)=2
When n>=3, the ways of climbing a staircase can divided into two groups based on the last step taken is either a single stair or two stairs together. If the last step is a single stair, the number of ways is c(n-1). If the last step is two stairs together, there are c(n-2) ways.
Therefore, c(n)=c(n-1)+c(n-2).