TOPIC:

ASSIGNMENT 2 DISCRETE STRUCTURE

SECTION:

SECTION 03 (2020/2021)

GROUP:

GROUP 10

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ASSIGNMENT DISCRETE STRUCTURE.

1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7

   a. How many numbers are there?

      6! x 6! x 6! = 373 248 000

   b. How many numbers are there if the digits are instinct?

      6! x 5! x 4! = 2 073 600

   c. How many numbers between 300 to 700 is only odd digits allow?

      4! x 6! x 3! = 103 680

      Odd numbers = { 3, 5, 7 }

      Any numbers from given digit = { 2, 3, 4, 5, 6, 7 }

      First digit for range of 301 – 699 = { 3, 4, 5, 6 }

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner,

how many ways Anita can arrange them around a round dinner table

   a. Men insist to sit next to each other

      Since Anita need to arrange them around a round table, circular permutation is applied;

      Total = (n - 1)!
      = (10 – 1)!
      = 9!
      = 362 880

      Case 1: If men want to sit next to each other;

      M M M M M W W W W W
      1 2 3 4 5 6

      5!

      6!

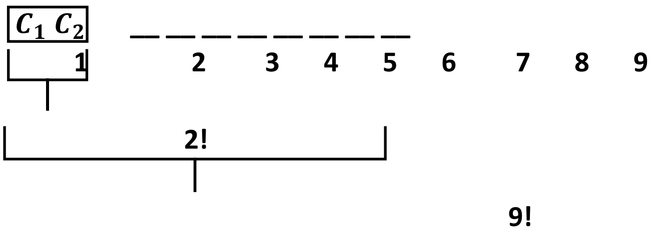
(6 – 1) ! x 5! = 14 400

Case 2: If men insist to sit next to each other;

Case 2 = Total – Case 1
= 362 880 – 14 400
= 348 480

b. The couple insisted to sit next to each other

Case 1: If the couple want to sit next to each other;

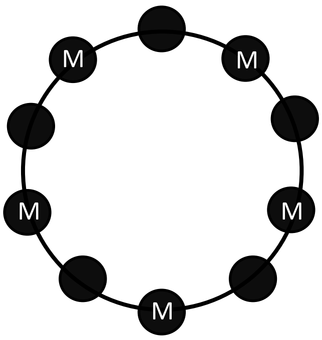


(9 – 1)! x 2! = 80 640

Case 2: If the couple insist to sit next to each other;
Case 2 = Total – Case 1
= 362 880 – 80 640
= 282 240

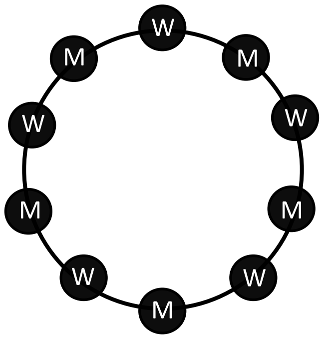
c. Men and women sit in alternate seat

Arrangement of men :



(5 – 1)! = 4!
= 24

Fill in women in between men :



5! = 120

Arrangement of men and women alternately :

Arrangement of men x Arrangement of women = 4! x 5!
= 2 880

d. Before her friend left, Anita want to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

$\boxed{A_1\ A_2}$ — — — — — — — — — —
1 2 3 4 5 6 7 8 9 10 11

2!

11!

Using line arrangement concept;
11! x 2! = 79 833 600

3. In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish

a. If no ties

5! = 120

b. Two sprinters tie

2*4*3*2*1= 48

c. Two group of two sprinters tie

3*2*1= 6

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a dozen croissants?

n=6 , r=12

C(n+r-1, r) = C(6+12-1,12)

=C(17,12)

=17!/(12!(17-12)!)

= 6188

b. two dozen croissants with at least two of each kind?

n=6 , r=12

C(n+r-1, r) = C(6+12-1,12)

=C(17,12)

=17!/(12!(17-12)!)

= 6188

c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

n=6 , r=16 (24-5-3)

C(n+r-1, r) =C(6+16-1, 16)

=C(21, 16)

=21!/(16!(21-16)!)

=20 349

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

=> the team loses :

2 wins when 4 games played : C(4,2) = 6

1 win when 3 games played :C (3,1) = 3

=> balance game of the team win or ties ;

2 wins and 1 ties/wins : C(4,2) . C(3,1). 2 = 36

1 win and 3 ties/wins : C(3,1) . C(4,3). 2^3 = 96

=> since there are two teams :

Total scenarios = (36 + 96) . 2

= 132 . 2

= 264

b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

=> since the game is not done in first round, so:

There are 2^10 = 1024 possibilities but based on a) it can be 264 scenarios to win,

Total first round = 1024 - 264

= 760

=> since the game settled in second round so;

Total scenario based on a) = 264

=> total scenario is :

= 760. 264

= 200640

c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

=> since first and second round no team wins, so:

First round = 760

Second round = 760

=> the game ends in less than 10 games:

= 2 + 2 + 2 + 2 + 2

= 10

=> total scenarios :

= 760 . 760 . 10

=5776000

6.A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

4^{10}=10 000 = unique answer sheets

10 000 * 2 = 20 000 (students)

By pigeonhole principle , 20 000+1 = 20 001

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics,
while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects,
what is the number of candidates sit for the exam?

Let H = the students have passed in history
Let M = the students have passed in Mathematics

Given that,
H = 0.75

M = 0.65
$H \cap M$ = 0.5
$H' \cap M'$ = 35 students

Students who passed :-

a. **History only**
   H - $(H \cap M)$ = 0.75 – 0.5
   = 0.25

b. **Mathematics only**
   M - $(H \cap M)$ = 0.65 – 0.5
   = 0.15

c. **Total students**
   History only + Mathematics only + Both History and Mathematics = H + M + $(H \cap M)$
   = 0.25 + 0.15 + 0.5
   = 0.9

Students who failed :

1 – total student who passed = 1 – 0.9
= 0.1

Let X = Number of candidates sit for the exam;

$$\frac{10}{100} \times X = 35$$

$$X = \frac{100 \times 35}{10}$$

$$= 350 \text{ students}$$

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

<u>300</u>≤x≤<u>700</u> <u>701</u>≤x≤<u>780</u>

Case A (_ _ 1) : (4 x 10 x 1) + (1 x 9 x 1) = 49

Case B (_ 1 _) : (4 x 1 x 10) + (1 x 1 x 10) = 50

Case C (_ 1 1) : (4 x 1 x 1) + (1 x 1 x 1) = 5

Case total ways = 49 + 50 + 5 + 104

Total selection = 780 - 300 + 1 = 481

P(Case total ways) = $\frac{104}{481}$ = $\frac{8}{37}$ = 0.2162

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.

a. In how many ways can the cars be parked in the parking lots?

10P6 = 151200 ways

b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

= 6! . 4! . 2!

= 34560/ 151200

= 0.2286

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively

a. Find the probability the trainee receives the message

let P(H) = probability trainee receives messages

P(H)= (0.4x0.6) + (0.1x0.8) + (0.5x1) = 0.24+0.08+0.5 = 0.82

b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

P(E|H)= P(H|E)P(E)/ 0.82 = (0.6x0.4)/0.82= 0.2927

11. In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality.

If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

60% - cars → P(C)=0.6

40% - SUV, pick up truck, minivans →P(A)= 0.4

100k cars accidents →20 fatal → P(H|C)=0.02

100k light trucks →25 fatal →P(H|A)= 0.025

P(H)= P(H|A) P(A) + P(H|C)P(C)= (0.025x0.4) + (0.02x0.6) = 0.01+0.012= 0.022

P(A|H) = P(H|A) P(A)/0.022= (0.025x0.4)/0.022 = 0.4545

12. There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

i) possible ways to place : 4^9 = 262144

ii) not possible : 4 x 3^9 = 78732

iii) not possible : 4 x 2^9 = 2048

iv) not possible : 4 x 1^9 = 4

so , 262144 - (78732 + 2048 + 4) = 181360 ways