CHAPTER 3

[Part 1]

COUNTING METHODS

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Basic Counting Principles

- Counting principle is all about choices we might make given many possibilities.
- It is used to find the number of possible outcomes.
- It provides a basis of computing probabilities of discrete events.

Basic Counting Principles (cont'd)

Sample of counting problems:

- Problem 1: How many ways are there to seat n couples at a round table, such that each couple sits together?

- Problem 2: How many ways are there to express a positive integer n as a sum of positive integers?

Basic Counting Principles (cont'd)

- Problem 3: There are three boxes containing books. The first box contains 15 mathematics books by different authors, the second box contains 12 chemistry books by different authors, and the third box contains 10 computer science books by different authors. A student wants to take a book from one of the three boxes. In how many ways can the student do this?

- Problem 4: The department will award a free computer to either a CS student or a CS professor. How many different choices are there, if there are 530 students and 15 professors?

Basic Counting Principles (cont'd)

There are a number of basic principles that we can use to solve such problems:

1) **Addition Principle**

2) **Multiplication Principle**

Addition Principle

- Suppose that tasks T_1, T_2,… T_k can be done in n_1, n_2,… n_k ways, respectively.

- If all these tasks are **independent** of each other, then the number of ways to do one of these tasks is $n_1 + n_2 + \ldots + n_k$

- If a task can be done in n_1 ways and a second task in n_2 ways, and if these two tasks cannot be done at the same time, then there are $n_1 + n_2$ ways to do either task.

Example:

How many ways to find the number of integers between 5 and 50 that end with 1 or 7.

Solution:

Let T denote this task. We divide T into the following tasks.

T_1: Find all integers between 5 and 50 that end with 1.

11, 21, 31, 41 => 4 ways

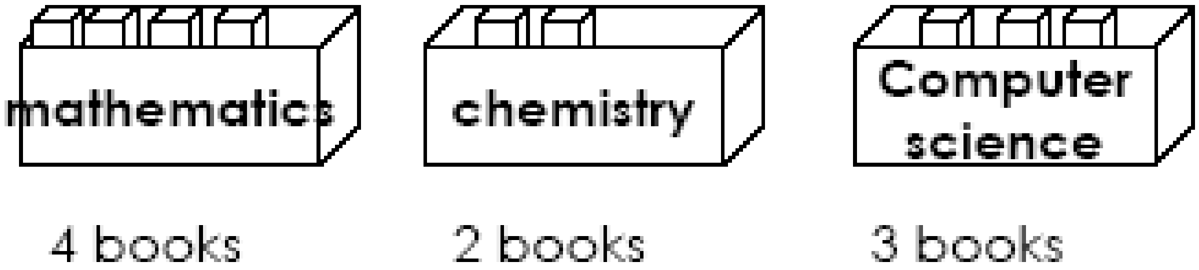
T_2: Find all integers between 5 and 50 that end with 7.

7, 17, 27, 37, 47 => 5 ways

$n_1 + n_2 = 4 + 5 =$ 9 ways

Example:

A student wants to take a book from one of the three boxes. In how many ways can the students do this?



Example - Solution

T_1: choose a mathematics book => 4 ways

T_2: choose a chemistry book => 2 ways

T_3: choose a computer science book => 3 ways

The number of ways to do one of these tasks is,

$$4 + 2 + 3 = 9 \text{ ways}$$

Example

There are 8 male students and 21 female students in Discrete Structure class. Among all of them, 7 students are Chinese and the rest are Malay.

a) In how many ways can we select a student - a boy or a girl?

b) In how many ways can we select a student - a Chinese or a Malay?

Example - Solution

a) In how many ways can we select a student - a boy or a girl?

- T_1: To select a boy = 8 ways
- T_2: To select a girl = 21 ways
- The number of ways:

8+21=29 ways

b) In how many ways can we select a student - a Chinese or a Malay?

- T_1:To select a Chinese =7 ways
- T_2:To select a Malay = 22 ways
- The number of ways:

7+22=29 ways

Multiplication Principle

Suppose that a procedure can be broken down into two successive tasks. If there are n_1 ways to do the first task and n_2 ways to do the second task after the first task has been done, then there are $n_1 \times n_2$ ways to do the procedure.

A task T can be completed in k successive steps.

Step 1 can be completed in n_1 different ways.

Step 2 can be completed in n_2 different ways.

Step *k* can be completed in n_k different ways.

Then the task T can be completed in $n_1 \times n_2 \times .. \times n_k$ different ways.

Example:

There are 8 male students and 21 female students in Discrete Structure class. Among all of them, 7 students are Chinese and the rest are Malay.

a) In how many ways can we select 2 students - a boy and a girl?

b) In how many ways can we select 2 students - a Chinese and a Malay?

Example - Solution

a) In how many ways can we select 2 students - a boy and a girl?

- T1: To select a boy = 8 ways
- T2: To select a girl = 21 ways
- The number of ways:

8 x 21 = 168 ways

b) In how many ways can we select 2 students - a Chinese and a Malay?

- T_1: To select a Chinese = 7 ways
- T2:To select a Malay = 22 ways
- The number of ways:

7 x 22 = 154 ways

Example:

Morgan is a lead actor in a new movie. She needs to shoot a scene in the morning in studio A and an afternoon scene in studio C. She looks at the map and finds that there is no direct route from studio A to studio C. Studio B is located between studios A and C. Morgan's friends Brad and Jennifer are shooting a movie in studio B. There are three roads, say A_1, A_2, and A_3, from studio A to studio B and four roads, say B_1, B_2, B_3, and B_4, from studio B to studio C. **In how many ways can Morgan go from studio A to studio C and have lunch with Brad and Jennifer at Studio B?**

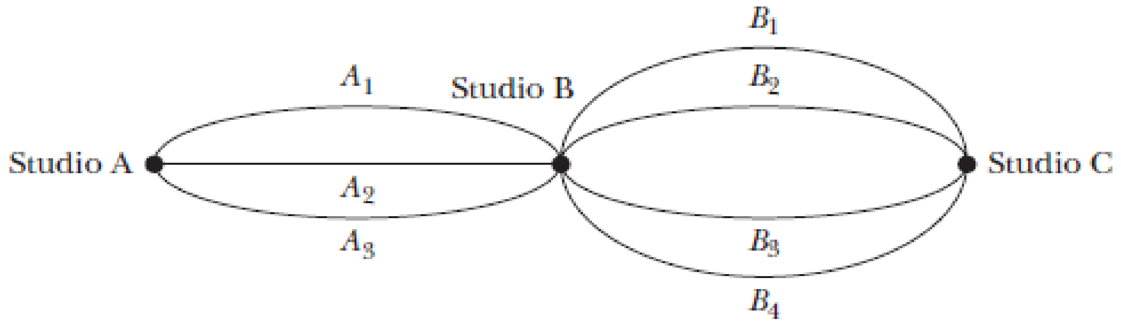


Figure A: Routes from studio A to studio C

Example- Solution

- T_1 = 3 ways (from studio A to B)
- T_2 = 4 ways (from studio B to C)

The number of ways to go from studio A to studio C via studio B is $3 \times 4 = 12$ ways.

Example:

The letters *A*, *B*, *C*, *D*, and *E* are to be used to form strings of length 4. How many strings can be formed if we do not allow repetitions? For example: *BADE*, *ACBD*, *AEBC* ..

Solution:

- T_1: choose the first letter ➔ 5 ways
- T_2: choose the second letter ➔ 4 ways
- T_3: choose the third letter ➔ 3 ways
- T_4: choose the fourth letter ➔ 2 ways

There are 5×4×3×2= 120 strings.

Example:

The letters *A, B, C, D*, and *E* are to be used to form strings of length 4. How many strings can be formed if we allow repetitions? For example: *BABB, AABB, ACEE* ..

Solution:

- T_1: choose the first letter ➔ 5 ways
- T_2: choose the second letter ➔ 5 ways
- T_3: choose the third letter ➔ 5 ways
- T_4: choose the fourth letter ➔ 5 ways

There are 5×5×5×5= 625 strings.

Example:

The letters A, B, C, D, and E are to be used to form strings of length 4. How many strings begin with A, if repetitions are not allowed? For example: *ADEC*, *ACBD*, *AEBC* ..

Solution:

- T_1: choose the first letter ➔ **A** = 1 way
- T_2: choose the second letter ➔ 4 ways
- T_3: choose the third letter ➔ 3 ways
- T_4: choose the fourth letter ➔ 2 ways

There are $1 \times 4 \times 3 \times 2 = 24$ strings.

Applied Basic Counting

- The counting problem that we have considered so far involved either the addition principle or the multiplication principle.
- Sometimes, however, we need to use both of these counting principles to solve a particular problem.

Example:

How many 8-bit strings begin either 101 or 111?

Solution:

$\underline{1}\ \underline{0}\ \underline{1}\ \_\ \_\ \_\ \_\ \_ : 2 \times 2 \times 2 \times 2 \times 2 = 32$

$\underline{1}\ \underline{1}\ \underline{1}\ \_\ \_\ \_\ \_\ \_ : 2 \times 2 \times 2 \times 2 \times 2 = 32$

$$32 + 32 = 64$$

Example:

The following items are available for breakfast:

3 types of cereal

4 types of juices

6 types of breads

How many ways a breakfast can be prepared if exactly 2 items are selected from 2 different groups?

Example – Solution:

A breakfast can be prepared in either of the following 3 ways:

- cereal & juice: $3 \times 4 = 12$
- cereal & bread: $3 \times 6 = 18$
- juice & bread: $4 \times 6 = 24$

Total ways: $12 + 18 + 24 = 54$ ways

Example:

A six-person committee composed of Aina, Wan, Chan, Tan, Syed and Helmi are to be selected to hold as a chairperson, secretary, and treasurer.

1) In how many ways this can be done?
2) In how many ways this can be done if either Aina or Wan must be chairperson?
3) In how many ways this can be done if Syed must hold one of the position?
4) In how many ways this can be done if Tan and Helmi must hold any position?

Example-Solution:

1) To select chairperson, secretary and treasurer:
   - Select the chairperson: 6 ways
   - Select the secretary: 5 ways
   - Select the treasurer: 4 ways

   Total ways: $6 \times 5 \times 4 = 120$ ways.

2) To select chairperson, secretary and treasurer with either Aina or Wan must be chairperson:
   - If Aina is chairperson: $5 \times 4 = 20$ ways
   - If Wan is chairperson: $5 \times 4 = 20$ ways

   Total ways: $20 + 20 = 40$

- Alternatively,
  - ✓ Select the chairperson: 2 ways
  - ✓ Select the secretary: 5 ways
  - ✓ Select the treasurer: 4 ways

  Total ways: $2 \times 5 \times 4 = 40$

Example-Solution:

3) To select chairperson, secretary and treasurer with Syed must hold one of the position:

- If Syed is chairperson: $1 \times 5 \times 4 = 20$
- If Syed is secretary: $1 \times 5 \times 4 = 20$
- If Syed is treasurer: $1 \times 5 \times 4 = 20$

Total ways: $20 + 20 + 20 = 60$ ways

- Alternatively,
  - Assign Syed for any position: 3 ways
  - Fill the highest remaining position: 5 ways
  - Fill the last position: 4 ways

  Total ways: $3 \times 5 \times 4 = 60$ ways

4) To select chairperson, secretary and treasurer with Tan and Helmi must hold any position?

- Assign a position to Tan: 3 ways
- Assign a position to Helmi: 2 ways
- Fill the remaining position: 4 ways

Total ways: $3 \times 2 \times 4 = 24$ ways

Exercises

Exercise #1

Danial, Kenny and Joseph are fighting over a turn to play a game that can only has 2 players at a time. The first player will have priority to start the game. In how many ways can we select 2 players at a time?

1) 3 ways ✓
2) 2 ways

3 x 2 = 6 ways

Exercise #2

Diana, Sherry, Devi and Mary are going to DSI using a motorcycle. In how many ways can we select 2 peoples to ride the motorcycle?

1) 4 ways ✓
2) 3 ways

4 x 3 =12 ways

Exercise # 3

A computer access password consists of from three to five letters chosen from the 26 in the alphabet with repetitions allowed. How many different passwords are possible?

3 letters = 26 x 26 x 26 = 17576
4 letters = 26 x 26 x 26 x 26 = 456976
5 letters = 26 x 26 x 26 x 26 x 26 = 11881376 ✓

==12355928

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Exercise # 4

Given three sets of integers; **A**={1, 3, 5}, **B** = {4, 6} and **C** = {0, 2,7,9}. How many ways are there to choose one integer from set **A**, **B**, or **C**?

a= 3 ways
b= 2 ways
c= 4 ways

3+2+4= 9 ways

✓

Exercise # 5

Suppose there are three roads from city A to city B and five roads from city B to C.

(i) How many ways is it possible to travel from city A to C via city B?

we are traveling from city A to city C via city B.
A to B = 3 ways
B to C = 5 ways

multiplication rule: 3x5 - 15 ways

✓

(ii) How many different round-trip routes are there from city A to B to C to B and back to A?

we are tarveling from city A to city B to C to B to A.
A to B = 3 ways
B to C = 5 ways
C to B = 5 ways
B to A = 3 ways

3x5x5x3 - 225 ways

✓

(iii) How many different routes are there from city A to B to C to B and back to A in which no road is traversed twice?

A to B = 3 ways; B to C = 5 ways; C to B= 4 ways; B to A= 2ways;
3x5x4x2=120 ways

✓