UTM
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CHAPTER 1

SET THEORY
[Part 2: Operation on Set]

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Union

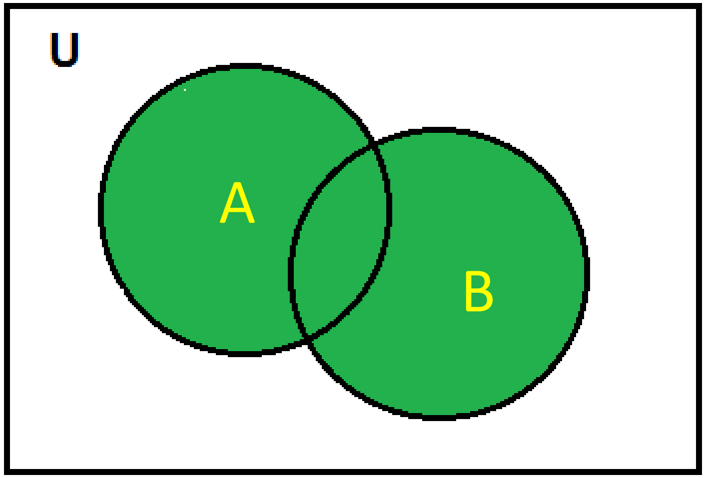
- The union of two sets **A** and **B**, denoted by **A** $\cup$ **B**, is defined to be the set

$$\mathbf{A} \cup \mathbf{B} = \{ x \mid x \in \mathbf{A} \text{ or } x \in \mathbf{B} \}$$

- The union consists of all elements belonging to either **A** or **B** (or both)

Union (cont'd)

- Venn diagram of $\mathbf{A} \cup \mathbf{B}$



- If **A** and **B** are finite sets, the cardinality of $\mathbf{A} \cup \mathbf{B}$,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Example:

A = {1, 2, 3, 4, 5}; **B** = {2, 4, 6} and **C** = {8, 9}

A ∪ B =

A ∪ C =

B ∪ C =

A ∪ B ∪ C =

Intersection

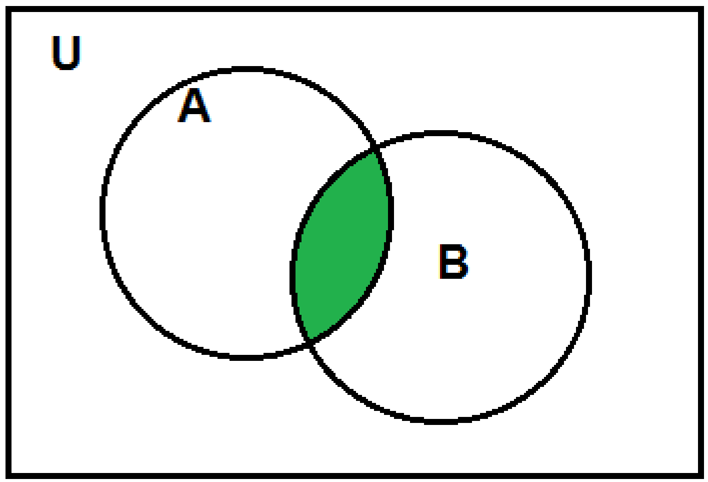
- The intersection of two sets **A** and **B**, denoted by **A** $\cap$ **B**, is defined to be the set

$$\mathbf{A} \cap \mathbf{B} = \{ x \mid x \in \mathbf{A} \text{ and } x \in \mathbf{B} \}$$

- The intersection consists of all elements belonging to both **A** and **B**.

Intersection (cont'd)

- Venn diagram of **A** $\cap$ **B**



Example:

W = {1, 2, 3, 4, 5, 6}; **X** = {2, 4, 6, 8, 10} ; **Y**={1, 2, 8, 10}

W ∩ X =

W ∩ Y =

Y ∩ X =

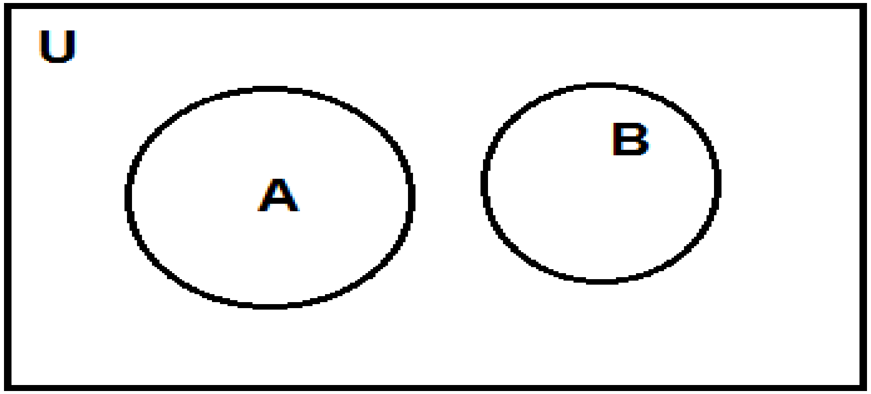
W ∩ X ∩ Y =

Disjoint

- Two sets **A** and **B** are said to be disjoint if,

$$\mathbf{A} \cap \mathbf{B} = \varnothing$$

- Venn diagram:



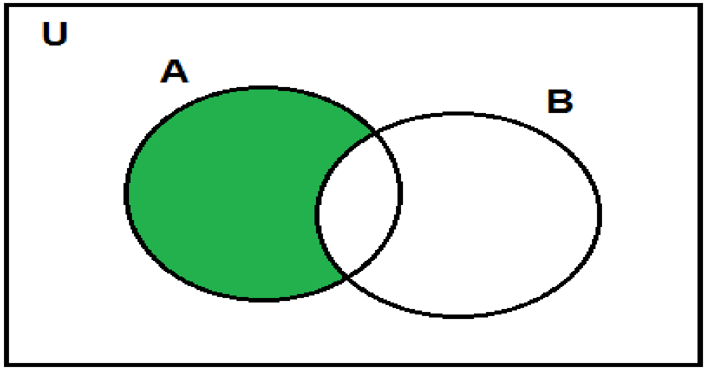
Example:

$$\mathbf{A} = \{1, 3, 5, 7, 9, 11\}; \mathbf{B} = \{2, 4, 6, 8, 10\}$$

$$\mathbf{A} \cap \mathbf{B} = \emptyset$$

Difference

- Let, **A** and **B** be sets. The difference of **A** minus **B**, denoted **A** – **B**, is the set of all elements in **A** that are not in **B**.
- Formally: $\mathbf{A} - \mathbf{B} = \{x \mid x \in \mathbf{A} \text{ and } x \notin \mathbf{B}\}$
- Venn diagram:



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Example:

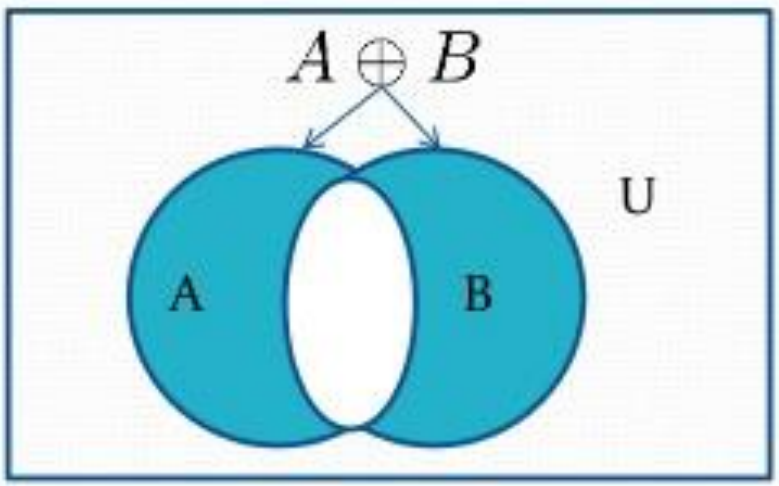
A= {1, 2, 3, 4, 5, 6, 7, 8}; **B**= {2, 4, 6, 8, 9}

$\mathbf{A} - \mathbf{B} =$

$\mathbf{B} - \mathbf{A} =$

Symmetric Difference

The symmetric difference of set $\mathbf{A}$ and set $\mathbf{B}$, denoted by $\mathbf{A} \oplus \mathbf{B}$ is the set $(\mathbf{A} - \mathbf{B}) \cup (\mathbf{B} - \mathbf{A})$



Venn Diagram

Example:

Let, **U**, **A** and **B** be sets.

$$\mathbf{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\mathbf{A} = \{1, 2, 3, 4, 5\}; \quad \mathbf{B} = \{4, 5, 6, 7, 8\}$$

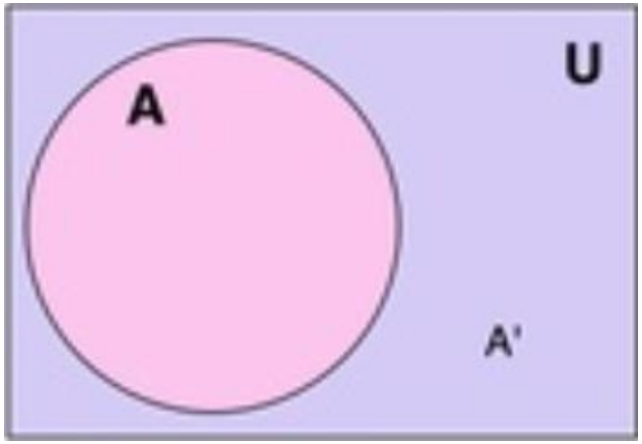
$$\mathbf{A} \otimes \mathbf{B} = (\mathbf{A} - \mathbf{B}) \cup (\mathbf{B} - \mathbf{A}) =$$

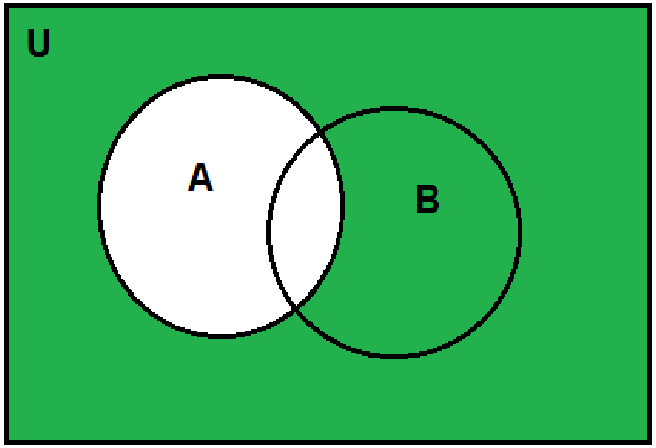
Complement

- The complement of a set **A** with respect to a universal set **U**, denoted by **A′** is defined to be

$$\mathbf{A}' = \{x \in \mathbf{U} \mid x \notin \mathbf{A}\}; \ \mathbf{A}' = \mathbf{U} - \mathbf{A}$$

- Venn diagram:





Example:

Let **U** be a universal set, and **A** is a subset of **U**.

$\mathbf{U} = \{ 1, 2, 3, 4, 5, 6, 7 \}$; $\mathbf{A} = \{ 2, 4, 6 \}$

$\mathbf{A}' = \mathbf{U} - \mathbf{A} =$

Set Identities

(or Properties of Set)

Let all sets referred to below be subsets of a universal set, **U**.

1) Commutative laws: For all sets **A** and **B**,

a) $\mathbf{A} \cup \mathbf{B} = \mathbf{B} \cup \mathbf{A}$

b) $\mathbf{A} \cap \mathbf{B} = \mathbf{B} \cap \mathbf{A}$

Set Identities (cont'd)

2) Associative laws: For all sets **A**, **B** and **C**,

a) $(A \cup B) \cup C = A \cup (B \cup C)$

b) $(A \cap B) \cap C = A \cap (B \cap C)$

Set Identities (cont'd)

3) Distributive laws: For all sets **A**, **B** and **C**,

a) $\mathbf{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$

b) $\mathbf{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

Set Identities (cont'd)

4) Identity laws: For all sets **A**,

a) $\mathbf{A} \cup \varnothing = \mathbf{A}$

b) $\mathbf{A} \cap \mathbf{U} = \mathbf{A}$

5) Complement laws: For all sets **A**,

a) $\mathbf{A} \cup \mathbf{A}' = \mathbf{U}$

b) $\mathbf{A} \cap \mathbf{A}' = \varnothing$

Set Identities (cont'd)

6) Double Complement law: For all sets **A**,

$$(\mathbf{A}')' = \mathbf{A}$$

7) Idempotent laws: For all sets **A**,

a) $\mathbf{A} \cup \mathbf{A} = \mathbf{A}$

b) $\mathbf{A} \cap \mathbf{A} = \mathbf{A}$

8) Universal bound laws: For all sets **A**,

a) $\mathbf{A} \cup \mathbf{U} = \mathbf{U}$

b) $\mathbf{A} \cap \varnothing = \varnothing$

Set Identities (cont'd)

9) De Morgan's laws: For all sets **A** and **B**,

a) $(\mathbf{A} \cup \mathbf{B})' = \mathbf{A}' \cap \mathbf{B}'$

b) $(\mathbf{A} \cap \mathbf{B})' = \mathbf{A}' \cup \mathbf{B}'$

10) Absorption laws: For all sets **A** and **B**,

a) $\mathbf{A} \cup (\mathbf{A} \cap \mathbf{B}) = \mathbf{A}$

b) $\mathbf{A} \cap (\mathbf{A} \cup \mathbf{B}) = \mathbf{A}$

Set Identities (cont'd)

11) Complements of **U** and $\varnothing$:

a) $\varnothing' = \mathbf{U}$

b) $\mathbf{U}' = \varnothing$

12) Set difference laws: For all sets **A** and **B**,

$$\mathbf{A}-\mathbf{B} = \mathbf{A} \cap \mathbf{B}'$$

13) Properties of empty set. For all set **A**,

a) $\mathbf{A} \cup \varnothing = \mathbf{A}$

b) $\mathbf{A} \cap \varnothing = \varnothing$

Example:

Let **A**, **B** and **C** denote the subsets of a set **S** and let **C′** denote a complement of **C** in **S**. If $\mathbf{A} \cap \mathbf{C} = \mathbf{B} \cap \mathbf{C}$ and $\mathbf{A} \cap \mathbf{C}' = \mathbf{B} \cap \mathbf{C}'$, then prove that **A = B.**

Example:

Let, **A** and **B** be sets. By referring to the set identities, show that:

$$\mathbf{A} - (\mathbf{A} \cap \mathbf{B}) = \mathbf{A} - \mathbf{B}$$

Generalized Unions & Intersection

Given sets $A_0, A_1, A_2, \ldots$ that are subsets of a universal set **U** and given a nonnegative integer, n.

$$\bigcup_{i=0}^{n} A_i = \{x \in \mathbf{U} \mid x \in A_i \text{ for at least one } i = 0, 1, 2, \ldots, n\}$$

$$\bigcup_{i=0}^{\infty} A_i = \{x \in U \mid x \in A_i \text{ for at least one nonnegative integer } i\}$$

Union

$$\bigcap_{i=0}^{n} A_i = \{x \in \mathbf{U} \mid x \in A_i \text{ for all } i = 0, 1, 2, \ldots, n\}$$

$$\bigcap_{i=0}^{\infty} A_i = \{x \in \mathbf{U} \mid x \in A_i \text{ for all nonnegative integer } i\}$$

Intersection

Example:

For each positive integer i, let $A_i = \left\{x \in \mathbf{R} \mid -\frac{1}{i} < x < \frac{1}{i}\right\} = A_i = \left(-\frac{1}{i}, \frac{1}{i}\right)$.

Find:

a) $\mathrm{A}_1 \cup \mathrm{A}_2 \cup \mathrm{A}_3$

b) $\mathrm{A}_1 \cap \mathrm{A}_2 \cap \mathrm{A}_3$

Example:

For each positive integer i, let $A_i = \left\{x \in \mathbf{R} \middle| -\frac{1}{i} < x < \frac{1}{i}\right\} = A_i = \left(-\frac{1}{i}, \frac{1}{i}\right)$.

Find:

a) $A_1 \cup A_2 \cup A_3$

b) $A_1 \cap A_2 \cap A_3$

Cartesian Product

- Given sets **A** and **B**, the Cartesian product of **A** and **B**, denoted **A**×**B** (read "A cross B") is the set of all ordered pairs (*a*, *b*), where *a* is in **A** and *b* is in **B**.

- Symbolically:

$$\mathbf{A}\times\mathbf{B} = \{(a, b) \mid a \in \mathbf{A} \text{ and } b \in \mathbf{B}\}$$

Example:

Let, **A** and **B** be sets.

A = {*a*, *b*}; **B** = {1, 2}

A×**B** =

B×**A** =

Cartesian Product (cont'd)

Notation:

An ordered pair of elements $a \in$ **A** and $b \in$ **B** written as (a, b) is a listing of the elements a and b in a specific order.

- The ordered pair (a, b) specifies that a is the first element and b is the second element.
- An ordered pair (a, b) is considered distinct from ordered pair (b, a), unless $a = b$.
- Example: $(1, 2) \neq (2, 1)$

Cartesian Product (cont'd)

- For any set **A**, $\mathbf{A} \times \emptyset = \emptyset \times \mathbf{A} = \emptyset$
- If $\mathbf{A} \neq \mathbf{B}$, then $\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$
- If $|\mathbf{A}| = m$ and $|\mathbf{B}| = n$, then $|\mathbf{A} \times \mathbf{B}| = m * n$

Cartesian Product (cont'd)

- The Cartesian product of sets $A_1, A_2, \ldots., A_n$ is defined to be the set of all n-tuples

  $(a_1, a_2,\ldots a_n)$ where $a_i \in A_i$ for $i=1,\ldots,n$;

- It is denoted $A_1 \times A_2 \times \ldots. \times A_n$

  $|A_1 \times A_2 \times \ldots. \times A_n| = |A_1|.|A_2| \ldots. |A_n|$

Example:

Let, **A**, **B** and **C** be sets.

$\mathbf{A} = \{a, b\}$; $\mathbf{B} = \{1, 2\}$, $\mathbf{C} = \{x, y\}$

$\mathbf{A} \times \mathbf{B} \times \mathbf{C} =$

$|\mathbf{A} \times \mathbf{B} \times \mathbf{C}| =$

Example:

Let, **C**, **D** and **E** be sets.

$\mathbf{C} = \{1, 3\}$; $\mathbf{D} = \{2, 4, 6\}$; $\mathbf{E} = \{a, b\}$

Find:

a) $\mathbf{C}\times\mathbf{D} = ?$

b) $\mathbf{D}\times\mathbf{C} = ?$

c) Is $\mathbf{C}\times\mathbf{D} = \mathbf{D}\times\mathbf{C}$?

d) $(\mathbf{C}\times\mathbf{D}) \times \mathbf{E} = ?$

e) $|\mathbf{C}\times\mathbf{D}| = ?$

f) $|\mathbf{C}\times\mathbf{D}\times\mathbf{E}| = ?$

Example - Solution:

a) $\mathbf{C} \times \mathbf{D} =$

b) $\mathbf{D} \times \mathbf{A} =$

c) Since $\mathbf{C} \neq \mathbf{D}$, therefore $\mathbf{C} \times \mathbf{D} \neq \mathbf{D} \times \mathbf{C}$

d) $(\mathbf{C} \times \mathbf{D}) \times \mathbf{E} =$

e) $|\mathbf{C}| = 2$, $|\mathbf{D}| = 3$, $|\mathbf{C} \times \mathbf{D}| =$

f) $|\mathbf{C} \times \mathbf{D} \times \mathbf{E}| =$

EXERCISES

Exercise #1

Given sets,

$\mathbf{U} = \{ a, b, c, d, e, f , g, h, i, j, k, l, m \}$

$\mathbf{A} = \{ a, c, f, m\}$

$\mathbf{B} = \{ b, c, g, h, m \}$

Find:

i) $\mathbf{A} \cup \mathbf{B}$

ii) $\mathbf{A} \cap \mathbf{B}$

iii) $|\mathbf{A} \cup \mathbf{B}|$

iv) $\mathbf{A} - \mathbf{B}$

v) $\mathbf{A}'$

Exercise #2

Let the universal set, $\mathbf{U}=\{1, 2, 3, 4, \ldots, 10\}$.

Let a set $\mathbf{A}=\{1, 4, 7, 10\}$, $B=\{1, 2, 3, 4, 5\}$ and $\mathbf{C}=\{2, 4, 6, 8\}$

List the elements of each set:

a) $\mathbf{U}'$

b) $\mathbf{B}' \cap (\mathbf{C} - \mathbf{A})$

c) $(\mathbf{A} \cup \mathbf{B}) \cup (\mathbf{C} - \mathbf{B})$

d) $(\mathbf{A} \cap \mathbf{B}) \cup (\mathbf{B} - \mathbf{C})$

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Exercise #3

Let **A**, **B** and **C** be sets such that

$$\mathbf{A} \cap \mathbf{B} = \mathbf{A} \cap \mathbf{C} \text{ and } \mathbf{A} \cup \mathbf{B} = \mathbf{A} \cup \mathbf{C}$$

Prove that **B** = **C**

Exercise #4

Given sets **A**= {w, x}; **B**={1, 2} and **C**={KB, SD, PS}.

a) Determine the following set,

i) **A×B**

ii) **B×C**

iii) **A×C**

iv) **A×B×C**

v) **(B×C)×A**

vi) **A×B× (A×C)**

b) Find:

i. |**A×B**|

ii. |**B×C**|

iii. |**A×C**|

iv. |**A×B×C**|

v. |**B×C×A**|

vi. |**A×B×A×C**|