FACULTY OF SOCIAL SCIENCES AND HUMANITIES

DISCRETE STRUCTURE (SECI 1013)

2020/2021 – SEMESTER 1

ASSIGNMENT# 1

GROUP 2

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1. Let the universal set be the set **R** of all real numbers and let $A=\{x\in\mathbf{R} \mid 0 < x \leq 2\}$, $B=\{x\in\mathbf{R} \mid 1 \leq x < 4\}$ and $C=\{x\in\mathbf{R} \mid 3 \leq x < 9\}$. Find each of the following:

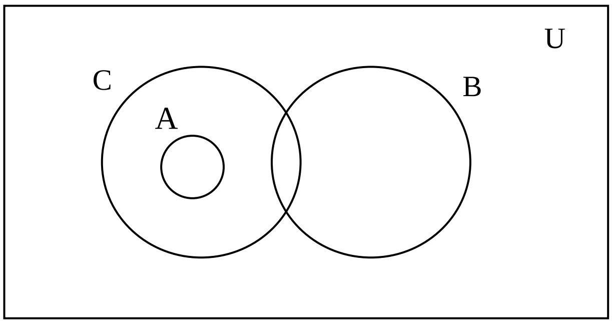
   a) $A \cup C = \{x \in \mathbf{R} \mid 0 < x < 9\}$.

   b) $(A \cup B)' = \{x \in \mathbf{R} \mid 0 \leq x \text{ and } x > 3\}$

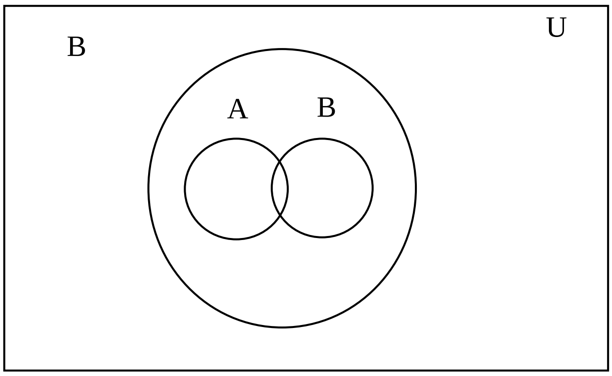
   c) $A' \cup B' = \{x \in \mathbf{R} \mid 2 \leq x \text{ and } x > 3\}$

2. Draw Venn diagrams to describe sets *A*, *B*, and *C* that satisfy the given conditions.

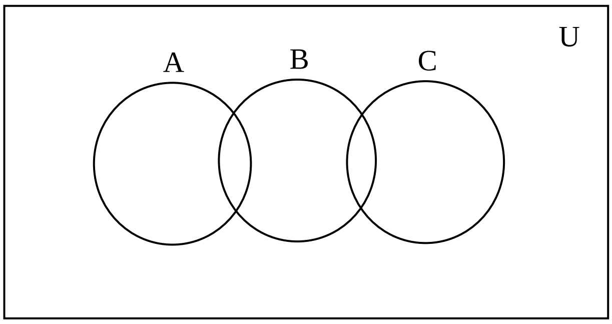
   a) $A \cap B = \emptyset, A \subseteq C, C \cap B \neq \emptyset$



   b) $A \subseteq B, C \subseteq B, A \cap C \neq \emptyset$



   c) $A \cap B \neq \emptyset, B \cap C \neq \emptyset, A \cap C = \emptyset, A \not\subset B, C \not\subset B$



3. Given two relations S and T from A to B,

   $S \cap T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ and } (x,y) \in \mathrm{T}\}$

   $S \cup T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ or } (x,y) \in \mathrm{T}\}$

   Let $A=\{-1, 1, 2, 4\}$ and $B=\{1,2\}$ and defined binary relations S and T from A to B as follows:

   For all $(x,y) \in A\times B$, $\quad x\ S\ y \leftrightarrow |x| = |y|$

   For all $(x,y) \in A\times B$, $\quad x\ T\ y \leftrightarrow x-y$ is even

   State explicitly which ordered pairs are in $A\times B$, S, T, $S \cap T$, and $S \cup T$.

- $A\times B= \{(-1, 1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$
- $S= \{(-1, 1), (1,1), (2,2)\}$
- $T= \{(-1, 1), (1,1), (2,2), (4,2)\}$

- $S \cap T= \{(-1, 1), (1,1), (2,2)\}$
- $S \cup T= \{(-1, 1), (1,1), (2,2), (4,2)\}$

4.

Show that $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

From LHS:

$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$

$= ((\neg\neg p \vee \neg q) \wedge q) \wedge \neg(\neg p \wedge \neg q)) \vee (p \wedge q))$ — De' Morgan's Law

$= ((\neg\neg p \vee \neg q) \wedge (\neg\neg p \vee \neg\neg q)) \vee (p \wedge q)$ — De' Morgan's Law

$= ((p \vee \neg q) \wedge (p \vee q)) \vee (p \wedge q)$ — Double negation

$= P \vee (\neg q \wedge q) \vee (p \wedge q)$ — Distributive law

$= P \vee (q \wedge \neg q) \vee (q \wedge p)$ — Commutative law

$= P \vee q \wedge (\neg q \vee p)$ — Distributive law

$= (p \vee q) \wedge (p \vee \neg q)$ — Commutative law

$= P \vee (q \wedge \neg q)$ — Distributive law

$= P \vee \emptyset$ — Complement law

$= p$ ∴ proven — Identity law

5. a) the matrix A_1 of the relation R_1

A_1= {(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)}

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b) the matrix A_2 of the relation R_2

A_2 = {(2,1),(3,1),(3,2),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(5,4)

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

c) is R_1 reflexive, symmetrix, transitive and/or equivalence relation?

-it's not reflexive because the diagonal does not all have value 1.

- its symmetrix for (x, y)∈ R, (y, x) ∈ R.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$M_R \neq M_R$ so it is not transitive.

∴ it is not equivalence relation because it is not reflexive and not transitive.

d) is reflexive , antisymmetric, transitive and/or a partial order relation.

∴ it is not reflexive because the diagonal does bot all have value 1

∴ it is not antisymmetric for a ≠ b (b, a) $not\ belong\ to$ R

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

∴ it is not transitive because $M_R \neq M_R$.

∴ it is not partial order relation because it is not reflexive and not transitive

6.

$$R_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

R_1 = {(1,1), (2,2), (2,3), (3,1), (3,3)}

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

R_2 = {(1,1), (2,2), (2,3), (3,1), (3,1)

(a) the matrix of relation R_1 U R_2

R_1 U R_2 = {(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)}

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(b) the matrix of relation $R_1 \cap R_2$

$R_1 \cap R_2$ = {(1,1), (2,2), (3,1), (3,3)}

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

7. If *f*: **R**→ **R** and *g*:**R**→ **R** are both one-to-one, is *f* + *g* also one-to-one? Justify your answer.

Solution:

(f + g)(x) = f(x) + g(x)

Let us assume;

(f + g)(a) = (f + g)(b) => for some arbitrary a, b ∈ **R**

f(a) + g(a) = f(b) + g(b)

f(a) = f(b) => because f is one-to-one

g(a) = g(b) => because g is one-to-one

a = b

∴ Since a,b ∈ R were arbitrary, this means ∀a,b ∈ R and a = b are proven.

Therefore, f+g is one-to-one

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase. Find a recurrence relation for $c_1, c_2, \ldots, c_n$.

Solution:

Let c_n = number of different ways

- ⇨ When n = 1, move up 1 stair
- ⇨ When n = 2, move up 2 stairs
- ⇨ When $n \geq 3$, move up more than 2 steps so use both 1 and 2 stairs

Let when last step is 1 stair, then c_{n-1} ways to arrive

Let when last step is 2 stairs, then c_{n-2} ways to arrive

Since c_n is number of different ways,

$c_1 = 1$, $c_2 = 2$

$c_n = c_{n-1} + c_{n-2}$, when $n \geq 3$

9. The Tribonacci sequence (t_n) is defined by the equations,
$t_0 = 0$,

$t_1 = t_2 = 1$,

$t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$.

(a) Find t_7

Solution:

$t_3 = t_2 + t_1 + t_0$

$= 1 + 1 + 0$

$= 2$

$t_4 = t_3 + t_2 + t_1$

$= 2 + 1 + 1$

$= 4$

$t_5 = t_4 + t_3 + t_2$

$= 4 + 2 + 1$

$= 7$

$t_6 = t_5 + t_4 + t_3$

$= 7 + 4 + 2$

$= 13$

$\therefore t_7 = t_6 + t_5 + t_4$

$= 13 + 7 + 4$

$= 24$

b) Write a recursive algorithm to compute t_n, $n \geq 3$.

Solution:

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Input : n , Output : t(n)

t(n)

{if (n = 1 or n = 2)

    return 1

    return t(n-1) + t(n-2) + t(n-3)}
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