UNIVERSITI TEKNOLOGI MALAYSIA

DISCRETE STRUCTURE SECI1013

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ASSIGNMENT 2

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1. Consider 3-digit number whose digit are either 2, 3, 4, 5, 6 or 7

a. How many numbers are there?

Solution:

n1 n2 n3

n1 = 6 ways

n2 = 6 ways

n3 = 6 ways

∴ Number of ways = 6 x 6 x 6
= 216

b. How many numbers are there if the digits are instinct?

Solution:

n1 n2 n3

n1 = 6 ways

n2 = 5 ways

n3 = 4 ways

∴ Number of ways = 6 x 5 x 4
= 120

c. How many numbers between 300 to 700 is only odd digits allow?

Solution:

From 300 to 700 so the range is 301-699.

Only odd digits allowed: {3,5,7}

n1 n2 n3

n1 = 2 ways

n2 = 3 ways

n3 = 3 ways

∴ Number of ways = 2 x 3 x 3
= 18

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table

a. Men insist to sit next to each other

Solution:

Number of ways = (6-1)!

= 120

For the men, they can change seats = 5!

= 120

∴ Number of ways = 120 x 120

= 14400 ways

b. The couple insisted to sit next to each other

Solution:

Number of ways = (9-1)!

= 40320

Since the couple can interchange,

∴ Number of ways = 40320 x 2

= 80640 ways

c. Men and women sit in alternate seat

Solution:

First, let the 5 men seat in alternate chairs,
Number of ways = (5-1)!
= 24
Then, there are 5 seats that are available left for women,
Number of ways = 5!
= 120
∴ Number of ways = 120 x 24
= 2880 ways

d. Before her friend left, Anita wants to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other?

Solution:

Number of ways = 11!
= 39 916 800
Since Anita and her husband can interchange,
∴ Number of ways = 11! x 2
= 79 833 600 ways

3. In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish

a. If no ties

Solution:

∴ Number of ways = 5!
= 120 ways

b. Two sprinters tie

Solution:

$^5C_2$
_ _ _ **[_ _]**
act as 1

$^4P_4$

∴ Number of ways = $^4P_4$ x $^5C_2$
= 240 ways

c. Two group of two sprinters tie

Solution:

$^3C_2$ $^5C_2$
_ **[_ _]** **[_ _]**
act as 1 act as 1

$^3P_3$

∴ Number of ways = $^5C_2$ x $^3C_2$ x $^3P_3$
= 180 ways

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a. a dozen croissants?

Solution:

n = 6, r = 12

C (n+r-1, r) = $\frac{(n+r-1)!}{r!(n-1)!}$

∴ C (17, 12) = $\frac{17!}{12!\,5!}$

= 6188 ways

b. two dozen croissants with at least two of each kind?

Solution:

n = 6, r = 24

Let each croissant have 2 each for the first dozen so 6 x 2 = 12 croissants so this is fixed.

Now, need to pick another 12 where n = 6, r = 12

∴ C (17, 12) = $\frac{17!}{12!\,5!}$

= 6188 ways

c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

Solution:

n = 6, r = 16

∴ C (21, 16) = $\frac{21!}{16!\,5!}$

= 20349 ways

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

Solution:

Cannot equal:

1. When there are 2 (r) wins in 4 (n) games;

   $C(4, 2) = \frac{4!}{2!\,2!}$

   = 6 scenarios

2. When there is 1 (r) win in 3 (n) games;

   $C(3, 1) = \frac{3!}{1!\,2!}$

   = 3 scenarios

After that, it’s either a win or a tie, there are 2 options for each game.

1. When 2 wins, 1 tie; C (4,2) x C (3,1) x 2

   = 36 scenarios

2. When 1 win, 3 ties; C (3,1) x C (4,3) x 2^3

   = 96 scenarios

∴ Number of scenarios = 2 x (36 + 96)

= 264 scenarios

b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

Solution:

For 10 penalty kicks (2 outcomes per penalty kick) so the total number of scenarios is $2^{10} = 1024$ scenarios.

But in (a) 264 scenarios were settled so,

Number of scenarios not settled = 1024 – 264

= 760 scenarios

First round = 760 scenarios

Second round (settles so same as in a) = 264 scenarios

∴ Number of scenarios = 760 x 264

= 200 640 scenarios

c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Solution:

First and second round is not settled;

First round = 760 scenarios

Second round = 760 scenarios

There are 10 additional kicks so each team kicks twice for each round so there are 5 rounds,

Sudden death (either A or B wins) = 2 + 2 + 2 + 2 + 2

= 10 scenarios

∴ Number of scenarios = 760 x 760 x 10

= 5 776 000 scenarios

6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

Solution:

k = pigeonhole = 4^{10} (possible answer sheets)

m = at least how much = 3

∴ n = k (m-1) + 1

$= 4^{10}$ (3-1) + 1

$= 4^{10}$ (2) + 1

= 2 097 153 students

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Solution:

P(H) = 0.7, P(M) = 0.65, P (H ∩ M) = 0.5, n(failed) = 35

P (FH) = 0.3, P(FM) = 0.35, P (FH ∩ FM) = 0.5

P (FH ∪ FM) = P(FH) + P(FM) – P (FH ∩ FM)

= 0.3 + 0.35 – 0.5

= 0.15

= 15%

Let total number of students = x,

$$\frac{15}{100} \times x = 35$$

$$\therefore \quad x = 233.33$$

≈ 233 students

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Solution:

From 300 to 780, there are 481 numbers.

When there's only 1 digit as 1;

The 300's; 301, 310, 312, 313, 314,315,316,317,318,319,321,331,341,351,361,371,381,391 [total=18]

The 400's: 401, 410, 412, 413, 414,415,416,417,418,419,421,431,441,451,461,471,481,491[total=18]

The 500's: 501, 510, 512, 513, 514,515,516,517,518,519,521,531,541,551,561,571,581,591[total=18]

The 600's: 601, 610, 612, 613, 614,615,616,617,618,619,621,631,641,651,661,671,681,691[total=18]

The 700's: 701, 710, 712, 713, 714,715,716,717,718,719,721,731,741,751,761,771, [total=16]

Number when only 1 digit as 1 = 88

When there's 2 digits as 1 = 5

Total number that has at least one digit as 1 = 88 + 5

= 93

∴ P (have at least one digit as 1) = 93 / 481

= 0.19

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same colour are not distinguishable, and the parking lots are chosen at random.

a. In how many ways can the cars be parked in the parking lots?

Solution:

Number of slots = 10, Number of cars = 6

Number of ways = $^{10}C_6$

= 210 ways

The cars can be arranged in 6! Ways

∴ Number of ways = $^{10}C_6 \times 6!$

= 151200 ways

b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

Solution:

Let the four vacant spots as 1,

∴ Number of ways when empty lots are next to one another

= 7!

= 5040 ways

∴ P (empty lots are next to one another)

= 5040 / 151200

= 1/30

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0.5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively

a. Find the probability the trainee receives the message

Solution:

P(SE) = probability send use email = 0.4

P(SL) = probability send use letter = 0.1

P(SH) = probability send use handphone = 0.5

P(RE) = probability receive use email = 0.6

P(RL) = probability receive use letter = 0.8

P(RH) = probability receive use handphone = 1

∴ P(R) = P(SE)P(RE) + P(SL)P(RL) + P(SH)P(RH)

= 0.4(0.6) + 0.1(0.8) + 0.5(1)

= 0.82

b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

Solution:

∴ P (E | R)

$= \frac{P(E \cap \mathrm{R})}{\mathrm{P(R)}}$

$= \frac{0.4 \times 0.6}{0.82}$

= 0.29

11. In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

Solution:

Let A = light truck, A' = cars

B = fatal, B' = not fatal

$$P(B|A') = \frac{20}{100000} \quad , \; P(B|A) = \frac{25}{100000}$$

P(A) = 0.4

P(A') = 1 – P(A)

= 0.6

$$\therefore P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|A')P(A')}$$

$$= \frac{(0.00025)(0.4)}{(0.00025)(0.4)+(0.00020)(0.6)}$$

= 0.4545

= 45.45%

12. There are 9 letters having different colours (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

Solution:

Total ways to distribute

$= 4^9$

= 262 144 ways

Let

T = tetrahedron has no letters,

C = cube has no letters,

P = polyhedron has no letters,

D = dodecahedron has no letters

$|T| = |C| = |P| = |D| = 2^9$

Since the letters can put in one of the two other boxes,

$|T \cup C| = |C \cup P| = |P \cup D| = |D \cup T| = 1^9 = 1$

$| T \cap C \cap P \cap D | = 0$

$| T \cup C \cup P \cup D | = |T| + |C| + |P| + |D| - |T \cup C| - |C \cup P| - |P \cup D| - |D \cup T| + | T \cap C \cap P \cap D |$

$= 2^9 + 2^9 + 2^9 + 2^9 - 1 - 1 - 1 - 1 + 0$

$= 2044$

∴ Number of ways that each box contains at least 1 letter

$= 4^9 - 2044$

= 260 100 ways