SCHOOL OF COMPUTING
Faculty of Engineering

SEMESTER I 2020/2021

DISCRETE STRUCTURE (SECI1013-03)

ASSIGNMENT 3

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ASSIGNMENT 3

QUESTION 1 **[25 marks]**

a) Let $A=\{1, 2, 3, 4, 5, 6, 7, 8\}$, $B=\{2, 5, 9\}$, and $C=\{a, b\}$. Find each of the following: (9 marks)

i. $A-B = \{1, 3, 4, 6, 7, 8\}$
ii. $(A\cap B) \cup C = \{2, 5, a, b\}$
iii. $A\cap B\cap C = \{\}$
iv. $B\times C = \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$
v. $P(C) = \{\{\}, \{a\}, \{b\}, \{a, b\}\}$

b) By referring to the properties of set operations, show that: (4 marks)

$P = (P \cap ((P' \cup Q)')) \cup (P \cap Q)$
$= (P \cap (P \cap Q')) \cup (P \cap Q)$ (De Morgan's Laws)
$= ((P \cap P) \cap Q') \cup (P \cap Q)$ (Associative Laws)
$= (P \cap Q') \cup (P \cap Q)$ (Idempotent Laws)
$= P \cap (Q' \cup Q)$ (Distributive Laws)
$= P \cap U$ (Complement Laws)
$= P$

c) Construct the truth table for, $\mathbf{A} = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$. (4 marks)

p	q	(¬p ∨ q)	(q → p)	(¬p ∨ q) ↔ (q → p)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then $(x+2)^2$ is odd" (4 marks)

Let,
P(x) = x is an odd integer
Q(x) = $(x+2)^2$ is an odd integer

Symbolically;
$\forall x$ (P(x) →Q(x)

Let a be an odd integer. Then,
a = 2n+1 for some integer n
$(a+2)^2 = (2n+1+2)^2$
$(a+2)^2 = (2n+3)^2$
$(a+2)^2 = 4n^2+12n+9$
$(a+2)^2 = 4(n+3n)+9$
$(a+2)^2 = 4m+9$, m = n+3n is an integer
$(a+2)^2$ is an odd integer

Therefore, for all integer x, if x is odd, then $(x+2)^2$ is odd.

e) Let $P(x,y)$ be the propositional function $x \geq y$. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE. (4 marks)

i. $\exists x \, \exists y \, P(x, y)$

- The above statement is true because it is possible to find at least one real number x that is bigger than or equal to y to make the proposition true. Let $x > 0$ and $y > 0$, if $x = 1$, $y = 0$ the statement is true.

ii. $\forall x \, \forall y \, P(x, y)$

- The above statement is false because when the value of y is bigger than x, the proposition function $x \geq y$ will become false. Let $x > 0$ and $y > 0$, if $x = 0$, $y = 1$ the statement is false for all x and y.

QUESTION 2

a) Suppose that the matrix of relation R on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

relative to the ordering 1, 2, 3. (7 marks)

i. Find the domain and the range of R.

Domain = {1,2,3}

Range = {1,2,3}

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

- The relation is not irreflexive since $(x, x) \in R$ for some x in R. The matrix does have value 0 on its main diagonal but it has loops on two of its vertexes.
- The relation is antisymmetric because for all (x, y) $\in R$ and $x \neq y, then$ $(y, x) \notin R$. For example, $(1, 2) \in R$ but $(2, 1) \notin R$.

b) Let S={(x,y)| x+y ≥9}is a relation on X={2, 3, 4, 5}. Find: (6 marks)

i. The elements of the set S.

S = {(5,4), (4,5), (5,5)}

ii. Is S reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.

$$S_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad S_R^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- S is not reflexive because the relation does not have the value 1 on its main diagonal.
- S is symmetric because $S_R = S_R^T$

$$S_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes S_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- S is not transitive because $S_R \otimes S_R \neq S_R$
- Therefore, S is not an equivalence relation because S is not reflexive and transitive.

c) Let X= {1, 2, 3}, Y= {1, 2, 3, 4}, and Z= {1, 2}. (6 marks)

i. Define a function $f: X \rightarrow Y$ that is one-to-one but not onto.

f: X→Y

f(x)= {(1,1), (2,2), (3,3)}

ii. Define a function *g: X→Z* that is onto but not one-to-one.

g(x)= {(1,1), (2,1), (3,2)}

iii. Define a function *h: X→X* that is neither one-to-one nor onto.

h(x)= {(1,1), (2,1), (3,2)}

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

$m(x) = 4x+3$, $n(x) = 2x-4$ (6 marks)

i. Find the inverse of *m*.

Let y = 4x +3

y - 3 =4x

$x = \frac{y-3}{4}$

$m^{-1}(x) = \frac{x-3}{4}$

ii. Find the compositions of $n \circ m$.

$n \circ m = 2(4x+3) - 4$
$= 8x + 6 - 4$
$= 8x+2$

QUESTION 3 **[15 marks]**

a) Given the recursively defined sequence.

$a_k = a_{k-1} - 2k$, for all integers k ≥ 2, $a_1 = 1$

i. Find the first three terms. (2 marks)

$a_2 = a_1 + 2k$
$= 1+2(2)$
$a_2 = 5$

$a_3 = a_2 + 2k$
$= 5+2(3)$
$a_3 = 11$

$a_4 = a_3 + 2k$
$= 11+2(4)$
$a_4 = 19$

ii. Write the recursive algorithm. (5 marks)

Input = *k*

Output = *f(k)*

f(k)

{

if (*k*=1)

return 1

return *f(k*-1) +2*k*

}

b) A certain computer algorithm executes twice as many operations when it is run with an input of size *k* as it is run with an input of size *k*-1 (where *k* is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size *k*. Find a recurrence relation for $r_1,\ r_2$, ... r_k.

$r_1 = 7$

$r_k = 2r_{k-1}, \qquad k \geq 2$

$r_2 = 2(7) = 14$

$r_3 = 2(14) = 28$

Thus, the recurrence relation are 7, 14, 28, ...

c) Given the recursive algorithm:

Input: *n*

Output: *S (n)*

S(n) {

if (*n*=1)

return 5

return 5**S*(*n*-1)

}

Trace *S*(4). (4 marks)

Input: *n*

Output: *S(n)*

S(4){	*S*(4) = 625
if(*n*=4)	return 5*125
return 5**S*(3)	
}	
S(3){	*S*(3) = 125
if(*n*=3)	return 5*25
return 5**S*(2)	

```
    }
S(2){
        if(n=2)                                         S(2)= 25
                return 5*S(1)            <-- |          return 5*5
    }
S(1){
        if(n=1)
                return 5
    }
```

QUESTION 4 **[25 marks]**

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

(4 marks)

Choose first digit : 9 ways (3, 4, 5, 6, 7, 8, 9, A, B)
Choose second digit : 16 ways
Choose third digit : 16 ways
Choose fourth digit : 11 ways (5, 6, 7, 8, 9, A, B, C, D, E, F)

$\therefore$ 9×16×16×11 = 25,344 hexadecimal numbers

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

(4 marks)

Possible letters in alphabet = 26
Possible digit in numbers = 10

Choose first letter : 1 way (A)
Choose second letter : 26 ways
Choose third letter : 26 ways
Choose fourth letter : 26 ways

Choose first digit : 10 way
Choose second digit : 10 way
Choose third digit : 1 way (0)

$\therefore$ 1×26×26×26×10×10×1 = 1,757,600 license plates

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?

(5 marks)

$P(8,1) = 8$

$P(8,2) = 56$
$P(8,3) = 336$
Total = 8 + 56 + 336 = 400

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

(4 marks)

Choose 4 women : $C(7,4)$
Choose 3 men : $C(6,3)$

$\therefore C(7,4)\times C(6,3) = 700$

e) How many distinguishable ways can the letters of the word *PROBABILITY* be arranged?

(4 marks)

$P(11) = \dfrac{11!}{2!\times 2!} = 9{,}979{,}200$ ways

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

(4 marks)

Select $r = 10$ pastries from $n = 6$ kinds of pastry.
$C(6+10-1) = C(15,10)$
$= 3{,}003$

QUESTION 5 **[10 marks]**

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names. (4 marks)

$k = 3$
Let, pigeon = persons = 18
pigeonhole = names = 5

$\therefore \lceil \frac{18}{5} \rceil = 3$

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

(3 marks)

Integers that are odd = 10, therefore at most, 10 even integers will be picked out from them. So, if we pick at least one more after picking the even integers, there would definitely be an odd integer.
∴ In total, we need to pick 20-10+1 = 11 integers from 0 through 20 to be sure of getting at least one odd integer.

a) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

(3 marks)

Integers that are divisible by 5 = 20 integers, so at most, 80 non-divisible by 5 integers will be picked out. After picking all of them, there would definitely be an integer divisible by 5 if we picked one more.

∴ In total, we need to pick 100-20+1 = 81 integers from 1 to 100 to be sure of getting at least one integer divisible by 5.