UTM UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

SEMESTER I 2020/2021

DISCRETE STRUCTURE (SECI1013-03)

ASSIGNMENT 4

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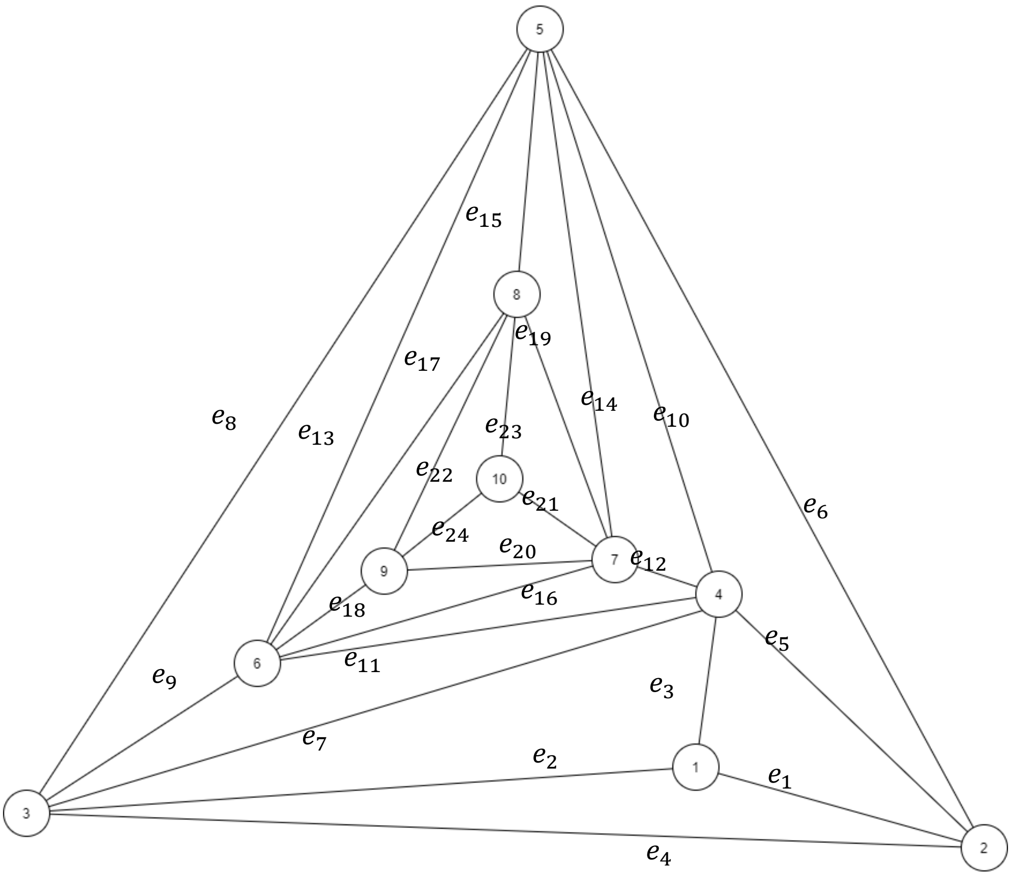
SECI1013: DISCRETE STRUCTURE

2020/2021 – Semester 1

ASSIGNMENT 4

1. Let G be a graph with $V(G) = \{1, 2 \ldots, 10\}$, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if $|v - w| \leq 3$. Draw the graph G and determine the numbers of edges, $e(G)$.

$$A_G = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$



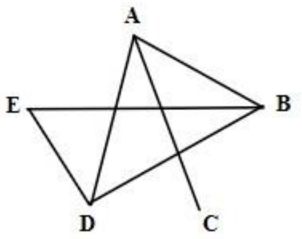
Number of edges, $e(G) = 24$

2. Model the following situation as graphs, draw each graph and gives the corresponding adjacency matrix.

(a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.

(Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)

∴ Edges corresponding to people that are friends = (A, B), (A, D), (A, C), (D, B), (D, E), (B, E)

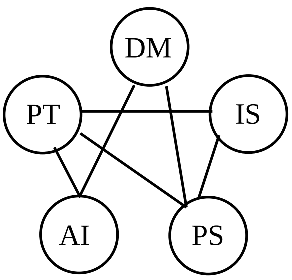


$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot: -

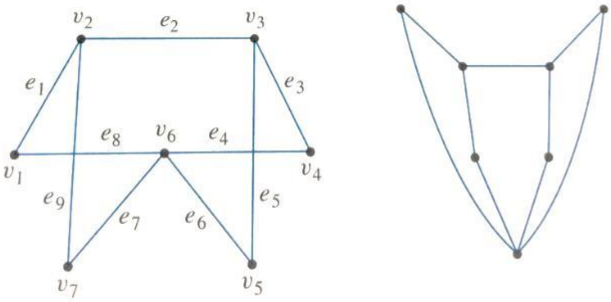
i. DM and IS
ii. DM and PT
iii. AI and PS
iv. IS and AI

∴ Edges that cannot be scheduled in the same time slot = (DM, IS), (DM, PT), (AI, PS), (IS, AI)

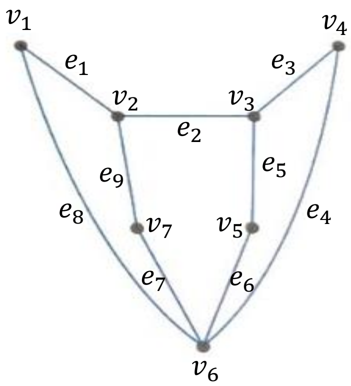


$$A = \begin{array}{c} \\ DM \\ PT \\ AI \\ PS \\ IS \end{array} \begin{array}{c} \begin{array}{ccccc} DM & PT & AI & PS & IS \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

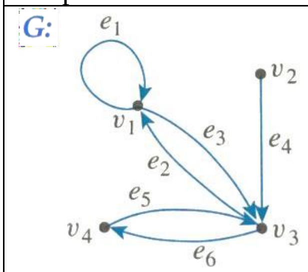
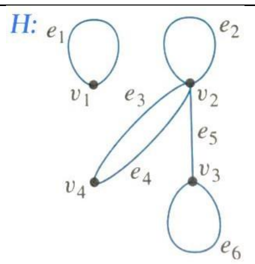
3. Show that the two drawing represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.



Answer:

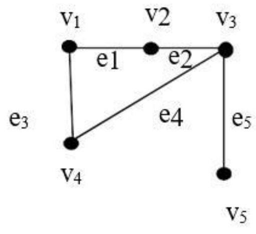


4. Find the adjacency and incidence matrices for the following graphs.

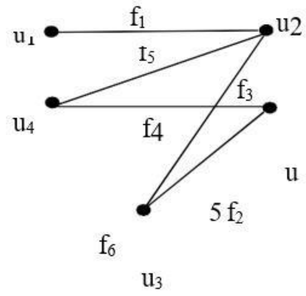
Graph	Adjacency Matrix	Incidence Matrix
G: 	$\begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ v_1 & 1 & 0 & 1 & 0 \\ v_2 & 0 & 0 & 1 & 0 \\ v_3 & 1 & 0 & 0 & 1 \\ v_4 & 0 & 0 & 1 & 0 \end{matrix}$	$\begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ v_1 & 1 & 0 & 1 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_3 & 0 & 1 & 0 & 0 & 0 & 1 \\ v_4 & 0 & 0 & 0 & 0 & 1 & 0 \end{matrix}$
H: 	$\begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ v_1 & 1 & 0 & 0 & 0 \\ v_2 & 0 & 1 & 1 & 2 \\ v_3 & 0 & 1 & 1 & 0 \\ v_4 & 0 & 2 & 0 & 0 \end{matrix}$	$\begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ v_1 & 2 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 2 & 1 & 1 & 1 & 0 \\ v_3 & 0 & 0 & 0 & 0 & 1 & 2 \\ v_4 & 0 & 0 & 1 & 1 & 0 & 0 \end{matrix}$

5. Determine whether the following graphs are isomorphic.

a)



G1

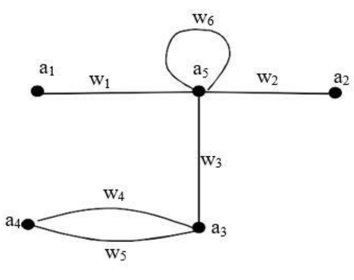


G_2

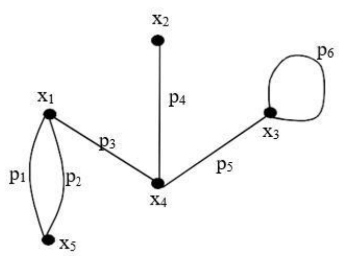
Yes, G_1 and G_2 are isomorphic because

- Vertices
  - G_1 = 5 vertices
  - G_2 = 5 vertices
- Edges
  - G_1 = 5 edges
  - G_2 = 5 edges
- Degree of each vertex
  - G_1 = a vertex having degree of 1, 3 vertices having degree of 2 and a vertex having degree of 3
  - G_2 = a vertex having degree of 1, 3 vertices having degree of 2 and a vertex having degree of 3
- Incident function, f: $G_1 \rightarrow G_2$, where $G_1 = \{v_1, v_2, v_3, v_4, v_5\}$ and $G_2 = \{u_1, u_2, u_3, u_4, u_5\}$ $f(v_1) = u_5$, $f(v_2) = u_4$, $f(v_3) = u_2$, $f(v_4) = u_3$, $f(v_5) = u_1$

b)



H_1

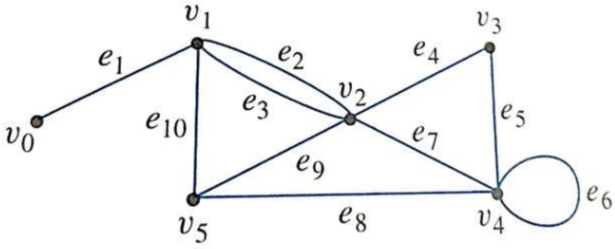


H_2

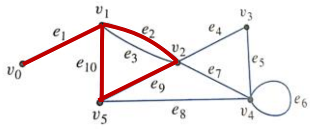
No, because degree of each vertex in both graphs are different

- H_1 = 2 vertices having degree of 1, a vertex having degree of 2, a vertex having degree of 3 and a vertex having degree of 5
- H_2 = a vertex having degree of 1, a vertex having degree of 2 and 3 vertices having degree of 3

6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.

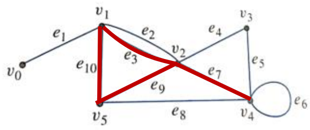


i. $v_0 e_1 v_1 e_{10} v_5 e_9 v_2 e_2 v_1$



trail because vertex is repeated and edge is not

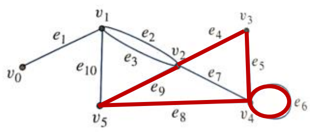
ii. $v_4 e_7 v_2 e_9 v_5 e_{10} v_1 e_3 v_2 e_9 v_5$



Just walk because vertex and edge are repeated

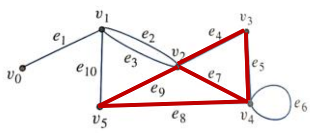
iii. v_2
Trivial walk because single vertex

iv. $v_5 e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4 e_8 v_5$



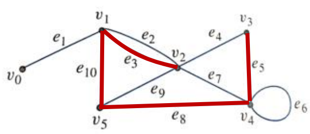
Circuit/cycle because start and end at the same vertex and not contain a repeated edge

v. $v_2 e_4 v_3 e_5 v_4 e_8 v_5 e_9 v_2 e_7 v_4 e_5 v_3 e_4 v_2$



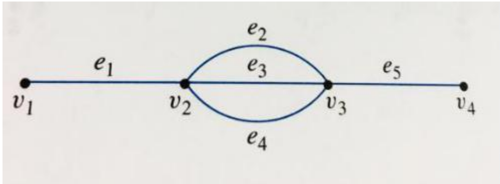
Closed walk because start and end at the same vertex and contain repeated edge

vi. $v_3 e_5 v_4 e_8 v_5 e_{10} v_1 e_3 v_2$



Path because not contain any repeated vertex and edge

7. Consider the following graph.



a) How many paths are there from $v_1 to v_4$?

1. (v1,e1,v2,e2,v3,e5, v4)
2. (v1,e1,v2,e3,v3,e5, v4)
3. (v1,e1,v2,e4,v3,e5, v4)

= 3 paths

b) How many trails are there from $v_1 to v_4$?

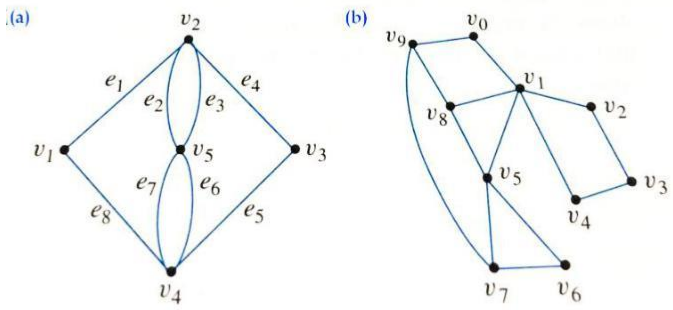
1. (v1,e1,v2,e2,v3,e5, v4)
2. (v1,e1,v2,e3,v3,e5, v4)
3. (v1,e1,v2,e4,v3,e5, v4)
4. (v1,e1,v2,e2,v3,e3,v2,e4,v3,e5,v4)
5. (v1,e1,v2,e4,v3,e3,v2,e2,v3,e5,v4)
6. (v1,e1,v2,e3,v3,e2,v2,e4,v3,e5,v4)
7. (v1,e1,v2,e3,v3,e4,v2,e2,v3,e5,v4)
8. (v1,e1,v2,e2,v3,e4,v2,e3,v3,e5,v4)
9. (v1,e1,v2,e4,v3,e2,v2,e3,v3,e5,v4)

= 9 trails

c) How many walks are there from $v_1 to v_4$?

undefined because edge and vertex are allowed to repeat.

8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



(a) has Euler circuit because every vertex has even degree

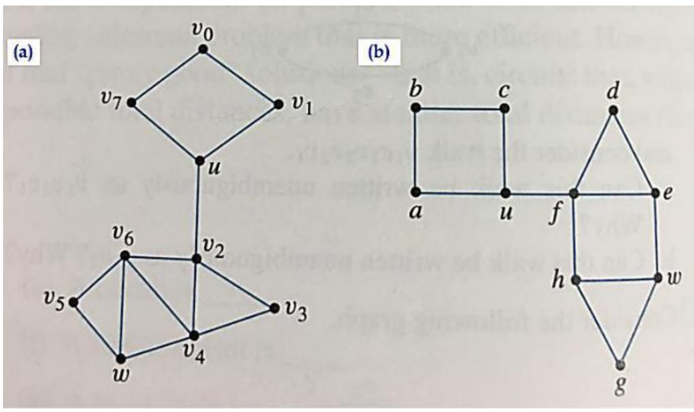
Vertex	v_1	v_2	v_3	v_4	v_5
Degree	2	4	2	4	4

$(v_1, e_1, v_2, e_2, v_5, e_3, v_2, e_4, v_3, e_5, v_4, e_6, v_5, e_7, v_4, e_8, v_1)$

(b) does not have Euler circuit because there are vertices have odd degree

Vertex	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
Degree	2	5	2	2	2	4	2	3	3	3

9. For each of graph in (a) – (b), determine whether there is an Euler path from to . If there is, find such a path.



a) Euler path for graph in (a)

$= u, v_1, v_0, v_7, u, v_2, v_3, v_4, v_2, v_6, v_4, w, v_5, v_6$

b) Euler path will not exist because

10. For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

a) There is no Hamiltonian circuit for graph in (a).
b) There is no Hamiltonian circuit for graph in (b).

11. How many leaves does a full *3-ary* tree with 100 vertices have?

$$l = \frac{[(m-1)n]+1}{3}$$

$$= \frac{[(3-1)100]+1}{3}$$

$$= 67$$

12. Find the following vertex/vertices in the rooted tree illustrated below.

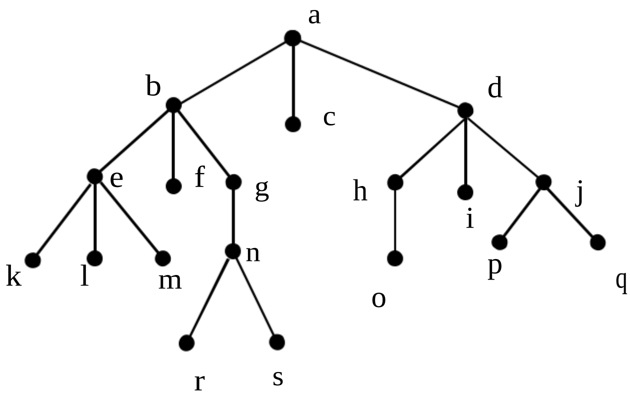


Figure 1

a) Root = a

b) Internal vertices = b, e, g, n, d, h, j

c) Leaves = c, f, I, k, l, m, o, p, q, r, s

d) Children of *n* = r and s

e) Parent of *e* = b

f) Siblings of *k* = l and m

g) Proper ancestors of *q* = j, d, a

h) Proper descendants of *b* = e, f, g, k, l, m, n, r, s

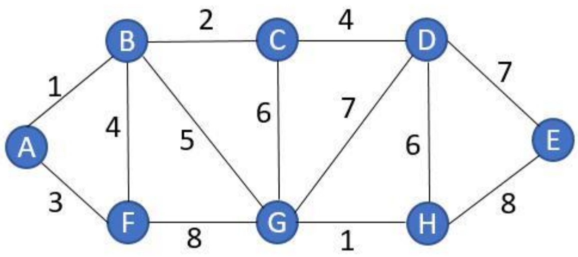
13. In which order are the vertices of ordered rooted tree in **Figure 1** is visited using *preorder*, *inorder* and *postorder*.

Preorder = a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

Inorder = k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

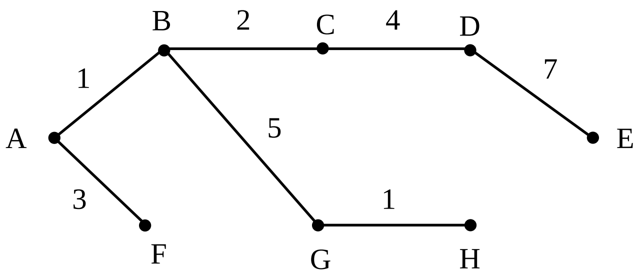
Postorder = k, l, m, e, f, r, s, n, g, b, c, o, h, I, p, q, j, d, a

14. Find the minimum spanning tree for the following graph using Kruskal's algorithm.

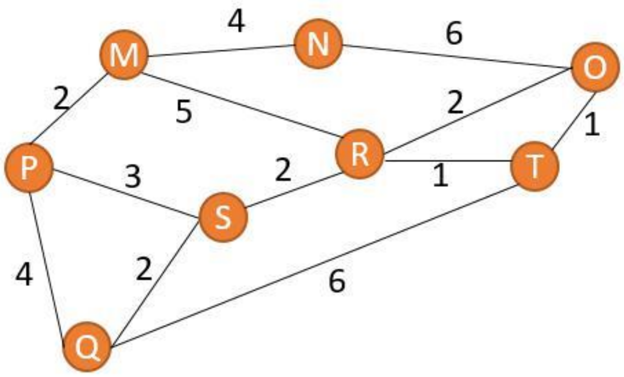


Edge	Weight	Will adding edge make circuit?	Action taken	Cumulative weight of subgraph
$e_1 = (A,B)$	1	No	Added	1
$e_2 = (G,H)$	1	No	Added	2
$e_3 = (B,C)$	2	No	Added	4
$e_4 = (A,F)$	3	No	Added	7
$e_5 = (B,F)$	4	Yes	Not added	7
$e_6 = (C,D)$	4	No	Added	11
$e_7 = (B,G)$	5	No	Added	16
$e_8 = (C,G)$	6	Yes	Not added	16
$e_9 = (D,H)$	6	Yes	Not added	16
$e_{10} = (G,D)$	7	Yes	Not added	16
$e_{11} = (D,E)$	7	No	Added	23
$e_{12} = (F,G)$	8	Yes	Not added	23
$e_{13} = (H,E)$	8	Yes	Not added	23

The minimum spanning tree:



15. Use Dijkstra's algorithm to find the shortest path from **M** to **T** for the following graph.



Iteration	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	{}	{M, N, O, P, Q, R, S, T}	∞	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N, O, P, Q, R, S, T}	0	4	∞	2	∞	5	∞	∞
2	{M, P}	{N, O, Q, R, S, T}	0	4	∞	2	6	5	5	∞
3	{M, P, N}	{O, Q, R, S, T}	0	4	10	2	6	5	5	∞
4	{M, P, N, R}	{O, Q, S, T}	0	4	7	2	6	5	5	6
5	{M, P, N, R, S}	{O, Q, T}	0	4	7	2	6	5	5	6
6	{M, P, N, R, S, Q}	{O, T}	0	4	7	2	6	5	5	6

∴ Shortest path: M→R→T and the length is 6.