DISCRETE STRUCTURE SECI1013

2020/2021/1

ASSIGNMENT 4

LECTURER'S NAME:

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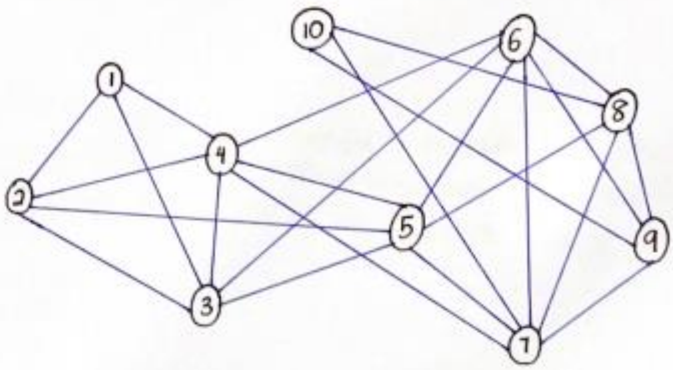
Question 1

ASSIGNMENT 4

1. Let G be a graph with $V(G) = \{1, 2, \dots, 10\}$, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if $|v-w| \leq 3$. Draw the graph G and determine the number of edges, $e(G)$.

Solution ;

$V(G) = \{(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (2,5), (3,1), (3,2), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,5), (4,6), (4,7), (5,2), (5,3), (5,4), (5,6), (5,7), (5,8), (6,3), (6,4), (6,5), (6,7), (6,8), (6,9), (7,4), (7,5), (7,6), (7,8), (7,9), (7,10), (8,5), (8,6), (8,7), (8,9), (8,10), (9,6), (9,7), (9,8), (9,10), (10,7), (10,8), (10,9)\}$



$\therefore e(G) = 24$ edges

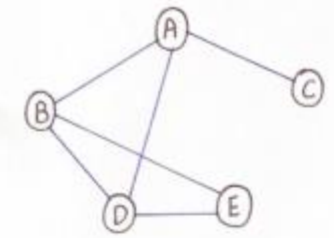
$$G = \begin{array}{cc} & \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Question 2

2. Model the following situation as graphs, draw each graphs and give the corresponding adjacency matrix.

(a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan are all friends.
(Note that you may use the representation of A = Ahmad, B = Bakri, C = Chong, D = David, E = Ehsan)

Solution :

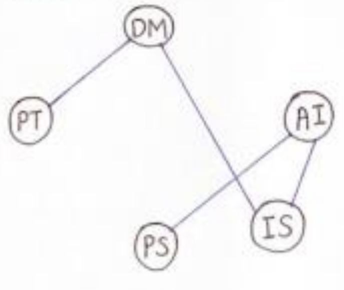


$$A_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(b) There are 5 subjects to be scheduled in the exam week ; Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistics (PS), and Information System (IS). The following subjects cannot be scheduled in the same time slot :-

i - DM and IS
ii - DM and PT
iii - AI and PS
iv - IS and AI

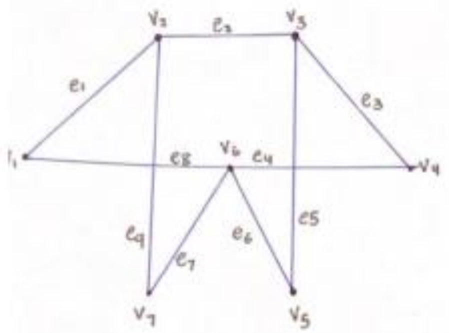
Solution :

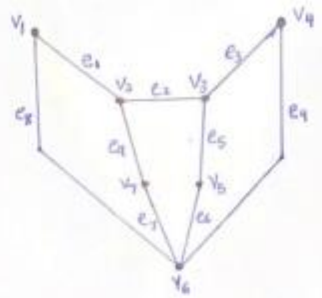


$$A_G = \begin{array}{c} \\ DM \\ PT \\ AI \\ PS \\ IS \end{array} \begin{array}{c} \begin{array}{ccccc} DM & PT & AI & PS & IS \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Question 3

3. Show that the two drawing represent the same by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.





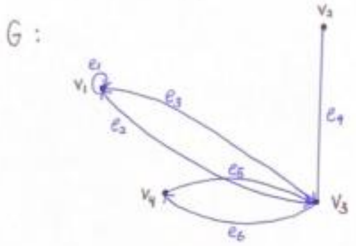
$\deg(v_1) = 2$
$\deg(v_2) = 3$
$\deg(v_3) = 3$
$\deg(v_4) = 2$
$\deg(v_5) = 2$
$\deg(v_6) = 4$
$\deg(v_7) = 2$

$\deg(v_1) = 2$
$\deg(v_2) = 3$
$\deg(v_3) = 3$
$\deg(v_4) = 2$
$\deg(v_5) = 2$
$\deg(v_6) = 4$
$\deg(v_7) = 2$

Question 4

4. Find the adjacency and incidence matrices for the following graphs.

G :



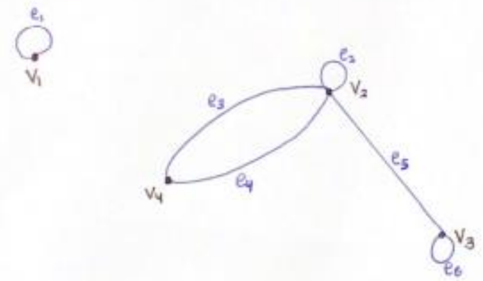
For adjacency matrix :

$$A_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

For incidence matrix ;

$$I_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

H :



For adjacency matrix ;

$$A_H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

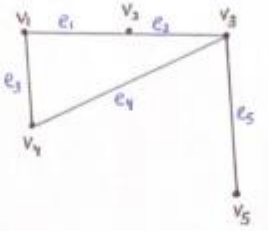
For incidence matrix ;

$$I_H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

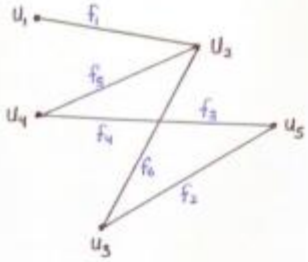
Question 5

5. Determine whether the following graphs are isomorphic.

(a)



G_1



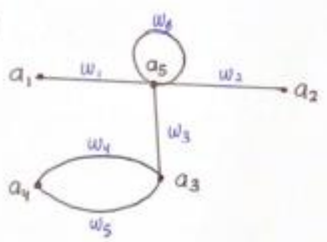
G_2

$$A_{G_1} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

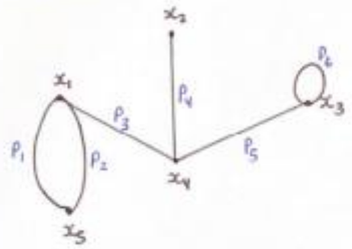
$$A_{G_2} = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$\therefore$ It is not isomorphic.

(b)



H_1



H_2

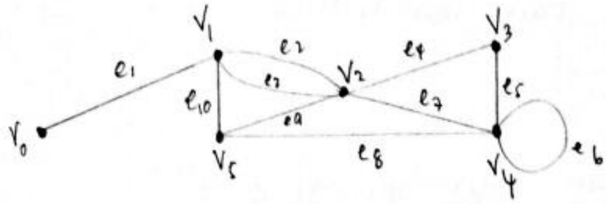
$$A_{H_1} = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$$A_{H_2} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$\therefore$ Not isomorphic

Question 6

(6) In the graph below, determine the following walks are trail, path, closed walk, circuit /cycles, simple circuits or just walk



a) $v_0 e_1 v_1 e_{10} v_5 e_9 v_2 e_2 v_1$ – Trail – do not have repeated edges, vertex of last and first not same

b) $v_4 e_7 v_2 e_9 v_5 e_{10} v_1 e_3 v_2 e_9 v_5$ – walk – vertex and edges are repeated, vertex of start and end are not same.

c) v_2 – walk only one vertex

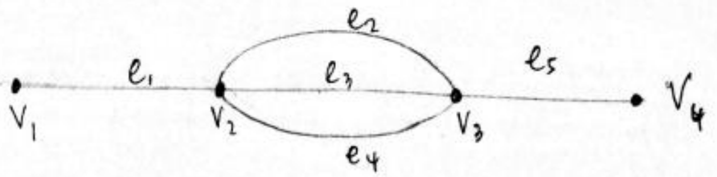
d) $v_5 e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4 e_8 v_5$ – circuit/ cycle. – vertex are repeated but not edges. vertex of start and end are same

e) $v_2 e_4 v_3 e_5 v_5 e_8 v_5 e_9 v_2 e_7 v_4 e_5 v_3 e_4 v_2$ – closed walk – vertex and edges are repeated ; vertex of start and end are same.

f) $v_3 e_5 v_4 e_8 v_5 e_{10} v_1 e_3 v_2$
– edges and vertex are not repeated.

Question 7

7)



a) How many path are there from V_1 to V_4?
- 3 path

b) How many trails are there from V_1 to V_4?
- 6 trails

c) How many walks are there from V_1 to V_4?
- infinity

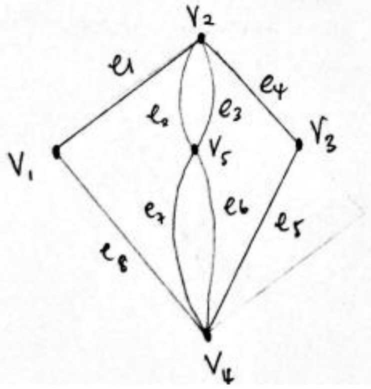
Question 8

euler Circuit.

(8) Determine which of the graph in (a) — (b) have euler Circuit. If the graph does not have a Euler Circuit, explain why not. If it does have a euler circuit, describe one.

$v \rightarrow$ vertex
$d \rightarrow$ degree.

a)



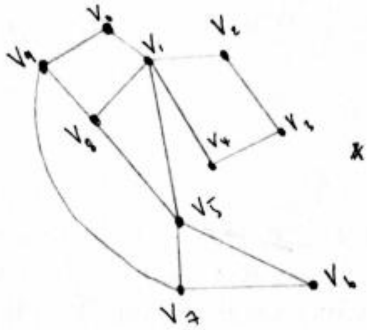
V	V_1	V_2	V_3	V_4	V_5
d	2	4	2	4	4

- degree of every vertex is even number, so graph is euler circuit.

$\therefore V_1 e_1 V_2 e_2 V_5 e_7 V_4 e_6 V_5 e_3 V_2 e_4 V_3 e_5 V_4 e_8 V_1$

$V \rightarrow$ vertex
$d \rightarrow$ degree

(8) (b)

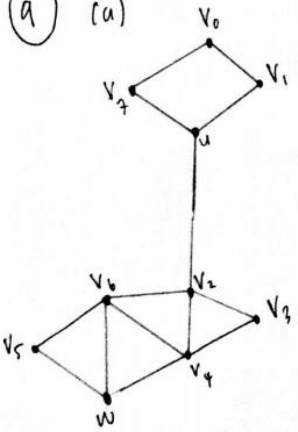


V	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9
d	2	5	2	2	2	4	2	3	3	3

* V_1, V_7, V_8, V_9 has odd degree number. so it does not euler circuit.

Question 9

9 (a)



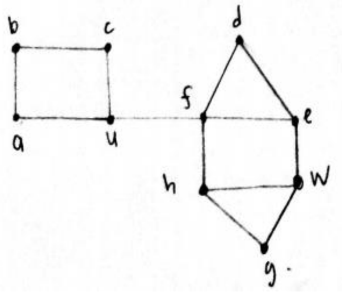
V → vertex
d → degree.

V	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7
d	2	2	4	2	4	2	4	2

∴ every vertex has even degree number.
it is euler path.

$U, V_1, V_0, V_7, U, V_2, V_3, V_4, V_6, V_2, V_4, W, V_5, V_6, W$

9 (b)



V – vertex
d – degree

V	a	b	c	d	e	f	g	h
d	2	2	2	2	3	4	2	3

∴ V_e and V_h hass odd number of degree

- no euler path

Question 10

graph (a)

10) Hamiltonian Path means there must have only one cycle, so, in graph (a), shows that there are more than one cycle, so, there is no exist Hamiltonian path in the circuit.

graph (b)

∴ There are more than one cycle in the graph (b), since Hamiltonian Path means to have only one cycle. so, there is no exist Hamiltonian path in the circuit.

Question 11

11. How many leaves does a full 3-ary tree with 100 vertices have?

$$l = [(m-1)n+1]/m$$

when $m = 3$, $n = 100$,

$$l = \frac{[(3-1)100+1]}{3}$$

$$= \frac{(2)100+1}{3}$$

$$= \frac{201}{3}$$

$$= 67$$

$\therefore$ A full 3-ary tree with 100 verticles have 67 leaves.

Question 12

12. Find the following vertex / vertices in the rooted tree illustrated below.

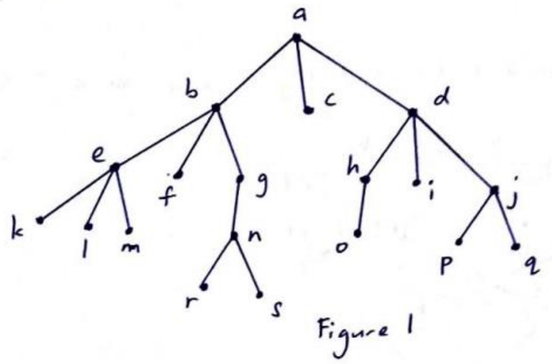


Figure 1

a) Root : a

b) Internal vertices : a, b, e, g, n, d, h, j

c) Leaves : c, f, k, l, m, r, s, o, i, p, q.

d) Children of n : r, s.

e) Parent of e : b

f) Siblings of k : l, m

g) Proper ancestors of q : a, d, j.

h) Proper descendants of b : e, f, g, k, l, m, n, r, s.

Question 13

13. In which order are the vertices of ordered rooted tree in Figure 1 is visited using preorder, inorder and postorder.

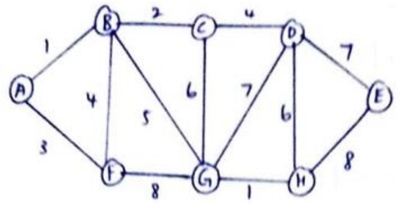
Preorder : a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q.

Inorder : k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q.

Postorder : k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a.

Question 14

14. Find the minimum spanning tree for the following graph using kruskal's algorithm.

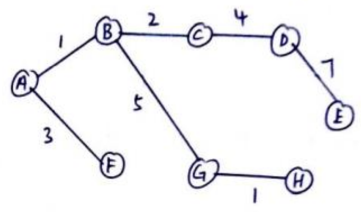


AB	1
AF	3
BC	2
BF	4
BG	5
CD	4
CG	6
DH	6
DG	7
DE	7
EH	8
FG	8
GH	1

Select the shortest edge :

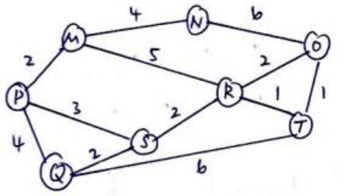
AB	1
GH	1
BC	2
AF	3
CD	4
BG	5
DE	7
total weight	23

The minimum spanning tree :



Question 15

15. Use Dijsktra's algorithm to find the shortest part from M to T for the following graph.



Iteration	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	{ }	{M, N, O, P, Q, R, S, T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N, O, P, Q, R, S, T}	0	4	∞	2	∞	5	∞	∞
2	{M, N}	{O, P, Q, R, S, T}	0	4	10	2	∞	5	∞	∞
3	{M, N, O}	{P, Q, R, S, T}	0	4	10	2	∞	5	∞	11
4	{M, N, O, P}	{Q, R, S, T}	0	4	10	2	6	5	5	11
5	{M, N, O, P, Q}	{R, S, T}	0	4	10	2	6	5	5	11
6	{M, N, O, P, Q, R}	{S, T}	0	4	7	2	6	5	5	6
7	{M, N, O, P, Q, R, T}	{S}	0	4	7	2	6	5	5	6

Hence, the shortest part from M to T is $M \to R \to T$.