SECI 1013-03

STRUKTUR DISKRIT (DISCRETE STRUCTURE)

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ASSIGNMENT 3

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<u>ANSWER</u>

<u>Question 1</u>

a) i) A-B = {1, 3, 4, 6, 7, 8}

ii) (A ∩ B) ∪ C

(A ∩ B) = {2, 5}

(A ∩ B) ∪ C = {2, 5, a, b}

iii) A ∩ B ∩ C = ∅

iv) B x C = {2, a}, {2, b}, {5, a}, {5, b}, {9, a}, {9, b}

v) P(C) = {{}, {a}, {b}, {a, b}}

b) (P ∩ ((P' ∪ Q)')) ∪ (P ∩ Q) = P

= (P ∩ (P ∪ Q')) ∪ (P ∩ Q) Complement law

= (P ∩ (P ∪ Q)) ∪ (P ∩ Q) Absorption law

= P ∪ (P ∩ Q) Second Absorption law

= P

c) A = (¬ p V q) ↔ (q → p)

p	q	¬ p	¬ p V q	q → p	(¬ p V q) ↔ (q → p)
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) All integer x, if x is odd, then $(x + 2)^2$ is odd

P(x) = x is odd;

Q(x) = $(x + 2)^2$ is odd

$\forall$ (P(x) → Q(x))

a = 2n + 1

$a^2 = (2n + 1)^2$

$a^2 = 4n^2 + 4n + 1$

$a^2 = 2\,(2n^2 + 2n) + 1$

$a^2 = \underline{2m + 1}$, where $m = 2n^2 + 2n$

↑ odd

therefore, if x is odd, then $(x + 2)^2$ is odd.

e) $P(x, y)$: $x \geq y$ for all positive integers:

I. $\exists x\, \exists y\, P(x,y)$

Let x = 3, y = 1

- $3 \geq 1$
- This statement is true
- Because there are exist value of x and exist value of y which x is more or equal than y.

II. $\forall x\, \forall y\, P(x,y)$

Let x = 3, y = 7

- $3 < 7 \;\neq\; x \geq y$
- The statement is false
- Because not for every x and for every y which x is more or equal than y

QUESTION 2

a) i) Domain = {1, 2, 3}

Range = {1, 2}

ii) It is antisymmetric, because for all a, b ∈ A, if (a, b) ∈ R then a = b.

b) i) S = {(4, 5), (5, 4), (5, 5)}

ii)

$$\begin{array}{c} \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \end{matrix} \end{array}$$

Not reflexive because main diagonal contains 0 on it.

$$M_R{}^T = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Symmetric because transpose matrix is the same as original matrix.

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

It is not transitive because product of Boolean is not the same as original matrix.

c) i) f = {(1, 1), (2, 3), (3, 4)}

ii) f = {(1, 1), (2, 1), (3, 2)}

iii) f = {(1, 2), (2, 2), (3, 1)}

d) i) Find $m^{-1}(x)$

$m(x) = 4x + 3$

let $m(x) = y$

$4x + 3 = y$

$y = \frac{x-3}{4}$

$m^{-1}(x) = \frac{x-3}{4}$

ii) $n \circ m$

$n(m(x)) = 2(4x + 3) - 4$

$= 8x + 6 - 4$

Question 3

a. .

i. $a_1 = 1$
$a_2 = a_1 + 2(2) = 1 + 4 = 5$
$a_3 = a_2 + 2(3) = 3 + 6 = 9$
$a_4 = a_3 + 2(4) = 9 + 8 = 17$

ii. Input: k
Output: a(k)
a(k)

iii. {
if (k = 1)
return 1

return a (k – 1) + 2k

}

b. $r_1 = 7$
$r_2 = 2 \times 7$
$r_3 = 2\ (2 \times 7)$
$r_4 = 2\ (2\ (2 \times 7))$
$r_k = 7\ (2^{k-1})$

c. S (4)
n = 4
n ≠ 1
return 5 x S (3)

n = 3
n ≠ 1
return 5 x S (2)

n = 2
n ≠ 1
return 5 x S (1)

n = 1
return 5

So S (4) = 625
return 5 x 125 = 625

So S (3) = 125
return 5 x 25 = 125

So S (2) = 25
return 5 x 5 = 25

So S (1) = 5
return 5

Answer = 625

QUESTION 4

a. P (9,1) x 16^2 x P (11,1) = 9 ways x 256 ways x 11 ways
= 25,344 ways.

b. Possible combination: A _ _ _ 1-9 _ 0

For letters = 1 way * 26^3 ways

For words = 9 ways x 10 ways * 1 way

Combined = 1way x 26^3 ways x 9 ways x 10 ways * 1 way
= 1,581,840 ways.

c. If a row contains only one letter: C (8,1) = 8 ways

If a row contains two letters: P (8,2) = 56 ways

If a row contains three letters: P (8,3) = 336 ways

8 ways + 56 ways + 336 ways = 400 ways.

d. C (7,4) ways x C (6,3) ways

35 ways x 20 ways = 700 ways.

e. $\frac{11!}{2!2!}$ ways = 9,979,200 ways.

f. Repetition allowed: C (n+r-1, r)
r=10, n=6
C(6+10-1, 10) = C(15,10) = $\frac{(n+r-1)!}{r!((n-1)!}$ =$\frac{(6+10-1)!}{10!((6-1)!}$=$\frac{(15)!}{10!((5)!}$

= 3,003 ways

QUESTION 5

a. Possible ways of same first and last name: {Ali Daud, Ali Elyas, Bahar Daud, Bahar Elyas, Carlie Daud, Carlie Elyas} = 6 ways

$$\frac{18}{6} = \lceil 3 \rceil$$

Thus, at least 3 persons have the same first and last names.

b. Odd from 1 through 20: {1,3,5,7,9,11,13,15,17,19}
10 even numbers and 10 odd numbers.
Take the worst case, which is pick all 10 even numbers, there will be at least one more integer must be picked in order to get odd.
Thus, total integers should be taken will be all 10 (even) + 1 (whichever from odd) = 11 integers.

c. Divisible from 1 through 100:
{5,10,15,20,25,30,35,40,45,50,55,60,65,70,75,80,85,90,95,100}
There are 20 integers divisible by 5 and 100-20 = 80 ways integers that aren't divisible by 5.
Take the worst case, which is pick all 80 integers not divisible by 5, there will be at least one more integer must be picked in order to get integer divisible by 5.
Thus, total integers should be taken will be all 80 (not divisible by 5) + 1 (whichever divisible by 5) = 81 integers.