SECI1013: DISCRETE STRUCTURE
2020/2021 – SEMESTER 1
ASSIGNMENT# 4

Group	**:**	6	
Name	**:**		**Question**
		1. Rubathrah a/p Veerakkumaran (B20EC0048)	1, 2, 3
		2. Singthai Srisoi (A20EC0147)	4, 5, 6, 7
		3. Muhammad Dinie Hazim Bin Azali (A20EC0084)	8, 9, 10, 11
		4. Lee Ming Qi (A20EC0064)	12,13,14,15
Section	**:**	03	
Date	**:**	21th January 2021	

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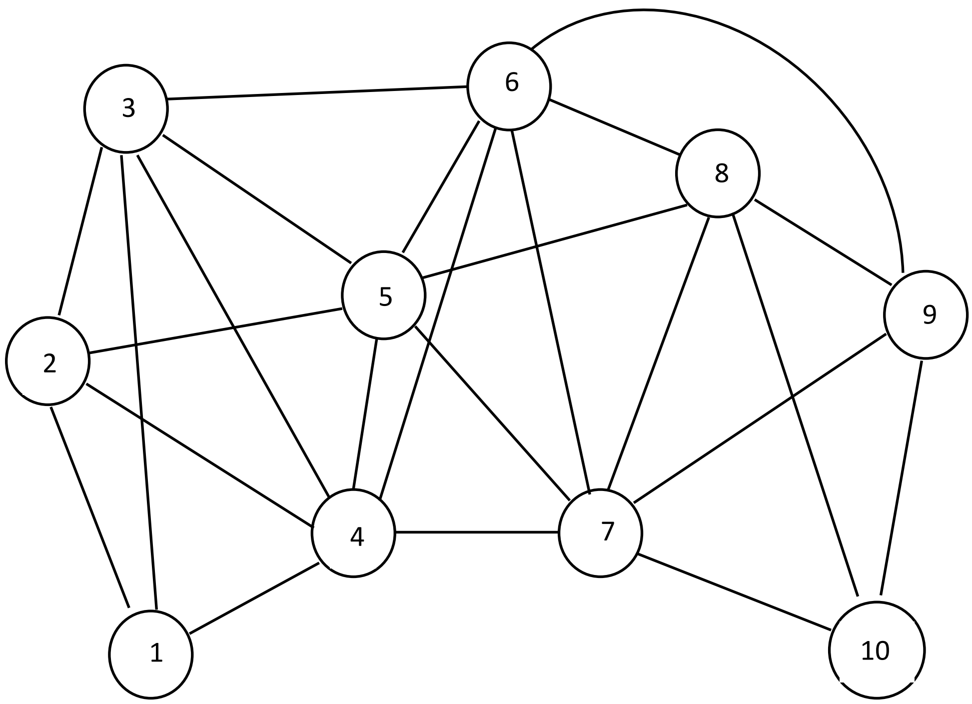
1. Let G be a graph with $V(G) = \{1, 2, \ldots., 10\}$, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if $|v - w| \leqslant 3$. Draw the graph G and determine the numbers of edges, $e(G)$.

Answer:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	0	1	1	1	0	0	0	0	0	0
2	1	0	1	1	1	0	0	0	0	0
3	1	1	0	1	1	1	0	0	0	0
4	1	1	1	0	1	1	1	0	0	0
5	0	1	1	1	0	1	1	1	0	0
6	0	0	1	1	1	0	1	1	1	0
7	0	0	0	1	1	1	0	1	1	1
8	0	0	0	0	1	1	1	0	1	1
9	0	0	0	0	0	1	1	1	0	1
10	0	0	0	0	0	0	1	1	1	0



2. Model the following situation as graphs, draw each graph and gives the corresponding adjacency matrix.

(a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.

(Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)

(b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot: -

i. DM and IS

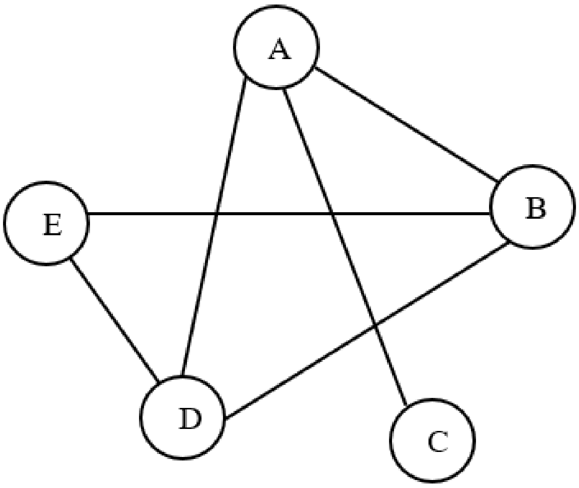
ii. DM and PT

iii. AI and PS

iv. IS and AI

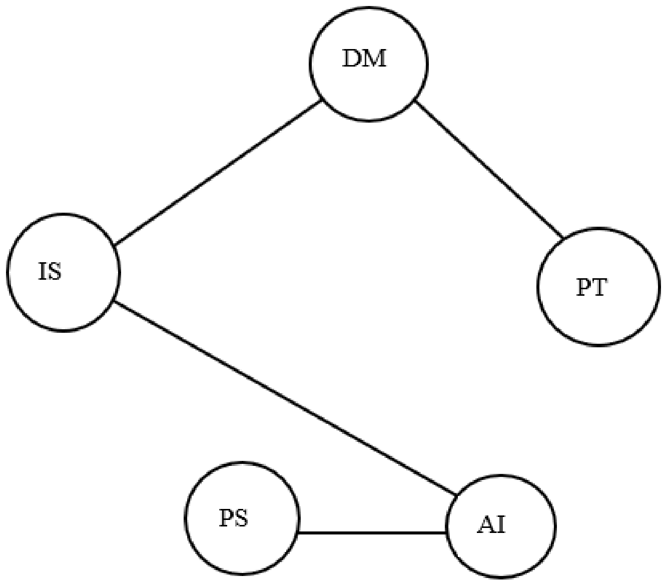
Answer:

a)



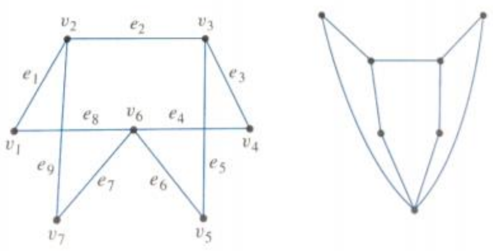
$$M_{5X5} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

b)

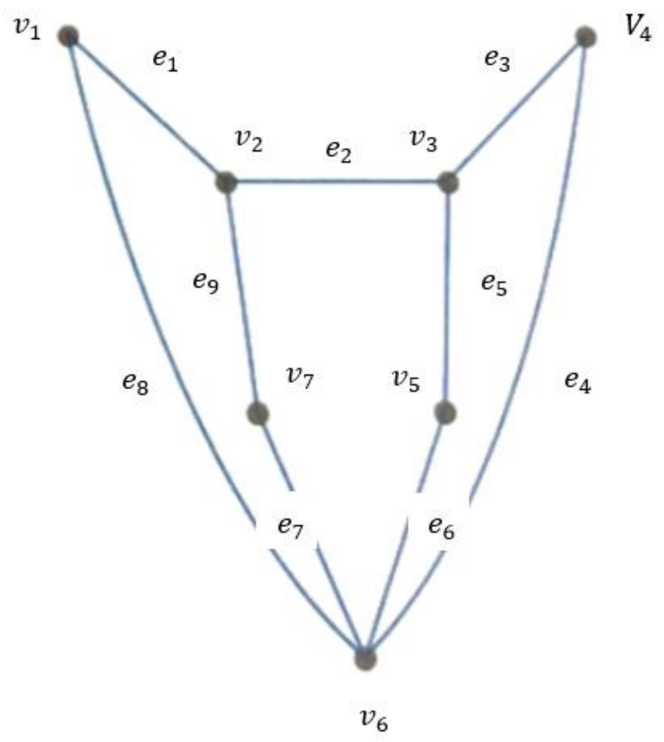


$$M_{5X5} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

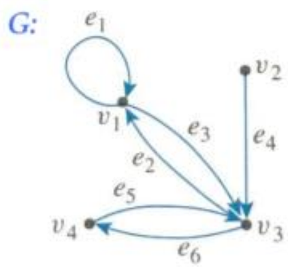
3. Show that the two drawing represent the same graph by labelling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.

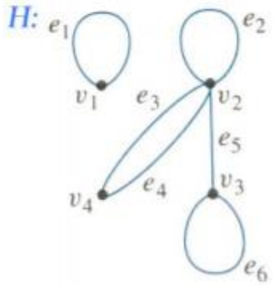


Answer:



4. Find the adjacency and incidence matrices for the following graphs.





Answer:

Adjacency matrices

G =

	V1	V2	V3	V4
V1	1	0	2	0
V2	0	0	1	0
V3	2	1	0	2
V4	0	0	2	0

Incidence matices

G =

	e1	e2	e3	e4	e5	e6
V1	2	1	1	0	0	0
V2	0	0	0	1	0	0
V3	0	1	1	1	1	1
V4	0	0	0	0	1	1

Adjacency matrices

H =

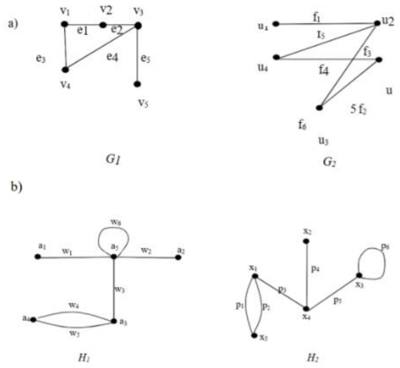
	V1	V2	V3	V4
V1	1	0	0	0
V2	0	1	1	2
V3	0	1	1	0
V4	0	2	0	0

Incidence matrices

H =

	e1	e2	e3	e4	e5	e6
V1	2	0	0	0	0	0
V2	0	2	1	1	1	0
V3	0	0	0	0	1	2
V4	0	0	1	1	0	0

5. Determine whether the following graphs are isomorphic.



Answer:

(a)

Adjacency matrix of G1&G2

G1 =

	V1	V2	V3	V4	V5
V1	0	1	0	1	0
V2	1	0	1	0	0
V3	0	1	0	1	1
V4	1	0	1	0	0
V5	0	0	1	0	0

G2 =

	U1	U2	U3	U4	U5
U1	0	1	0	0	0
U2	1	0	1	1	0
U3	0	1	0	0	1
U4	0	1	0	0	1
U5	0	0	1	1	0

G1 and G2 have the same number of vertices and edges.

Define f: G1→G2, where G1 = {V1, V2, V3, V4, V5} and G2 = {U1, U2, U3, U4, U5};

f(V1) = U5; f(V2) = U4; f(V3) = U2; f(V4) = U3; f(V5) = U1

By comparing the adjacency matrices of G1 and G2,

G1 =

	V1	V2	V3	V4	V5
V1	0	1	0	1	0
V2	1	0	1	0	0
V3	0	1	0	1	1
V4	1	0	1	0	0
V5	0	0	1	0	0

G2 =

	U5	U4	U2	U3	U1
U5	0	1	0	1	0
U4	1	0	1	0	0
U2	0	1	0	1	1
U3	1	0	1	0	0
U1	0	0	1	0	0

Hence, G1 and G2 are isomorphic.

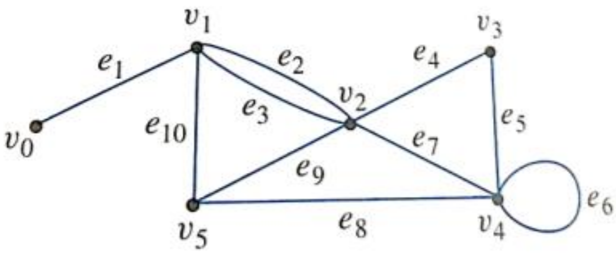
(b)

H1 and H2 have the same number of vertices and edges.

But pairs of connected vertices do not have the corresponding pair of vertices connected.

Hence H1 and H2 are not isomorphic.

6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



a) $v_0 e_1 v_1 e_{10} v_5 e_9 v_2 e_2 v_1$

b) $v_4 e_7 v_2 e_9 v_5 e_{10} v_1 e_3 v_2 e_9 v_5$

c) v_2

d) $v_5 e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4 e_8 v_5$

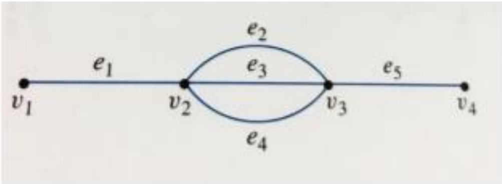
e) $v_2 e_4 v_3 e_5 v_4 e_8 v_5 e_9 v_2 e_7 v_4 e_5 v_3 e_4 v_2$

f) $v_3 e_5 v_4 e_8 v_5 e_{10} v_1 e_3 v_2$

Answer:

a)	Trail
b)	Trail
c)	Close walk
d)	Cycle
e)	Cycle
f)	Path

7. Consider the following graph.



a) How many paths are there from $v1$ to $v4$?

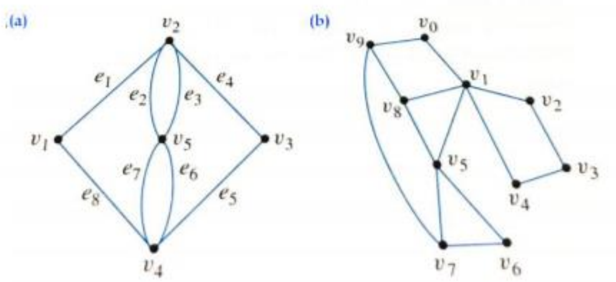
b) How many trails are there from $v1$ to $v4$?

c) How many walks are there from $v1$ to $v4$?

Answer:

a)	3
b)	9
c)	3

8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.

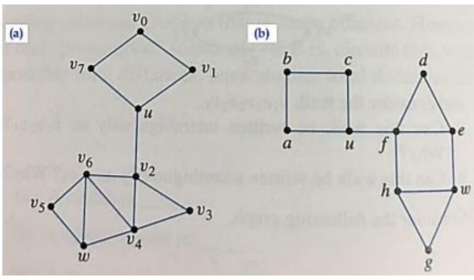


Answer:

Graph (a) have Euler circuit which is (v_1, e_1, v_2, e_2, v_5, e_3, v_2, e_4, v_3, e_5, v_4, e_6, v_5, e_7, v_4, e_8, v_1)

Graph (b) does not have Euler circuit because we can see that v_1, v_7, v_8, and v_9 have odd degree. So, we conclude that graph (b) does not have Euler circuit.

9. For each of graph in (a) – (b), determine whether there is an Euler path from u to w. If there is, find such a path



Answer:

Graph (a) have Euler path which is (u, v_7, v_0, v_1, u, v_2, v_3, v_4, v_6, v_5, w).

Graph (b) does not have Euler path because if we checked the condition for Euler path, only u and w vertices that can only having odd degree. However, in graph (b), h and e vertices also having odd degree. So, that's why it cannot have Euler path.

10. For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

Answer:

Both graph (a) and (b) do not have Hamiltonian circuit because they cannot form a cycle.

11. How many leaves does a full 3-ary tree with 100 vertices have?

Answer:

m = 3, n = 100

$l = \frac{(m-1)n+1}{m}$ leaves

$= \frac{(3-1)100+1}{3}$

$= \frac{(2)100+1}{3}$

$= \frac{201}{3}$

= 67 leaves

∴ A full 3-ary tree with 100 vertices has 67 leaves.

12. Find the following vertex/vertices in the rooted tree illustrated below.

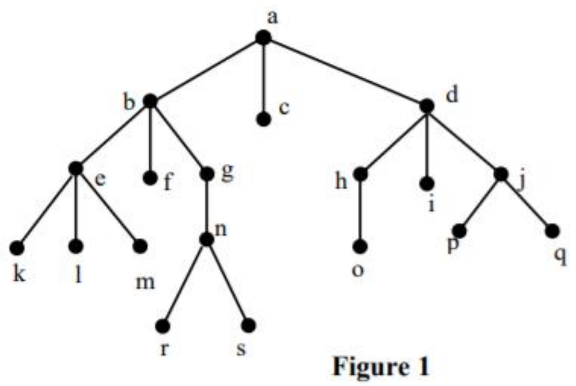


Figure 1

a) Root
b) Internal vertices
c) Leaves
d) Children of *n*
e) Parent of *e*
f) Siblings of *k*
g) Proper ancestors of *q*
h) Proper descendants of *b*

Answer:

a) a
b) a, b, d, e, g, h, j, n
c) c, f, i, k, l, m, o, p, q, r, s
d) r, s
e) b
f) l, m
g) a, d, j
h) e, f, g, k, l, m, n, r, s

13. In which order are the vertices of ordered rooted tree in Figure 1 is visited using preorder, inorder and postorder.

Answer:

Preorder:

a b e k l m f g n r s c d h o i j p q

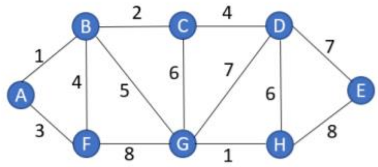
Inorder:

k l e m f b g r n s c a o h d i p j q

postorder:

k l m e f r s n g b c o h i p q j d a

14. Find the minimum spanning tree for the following graph using Kruskal's algorithm.

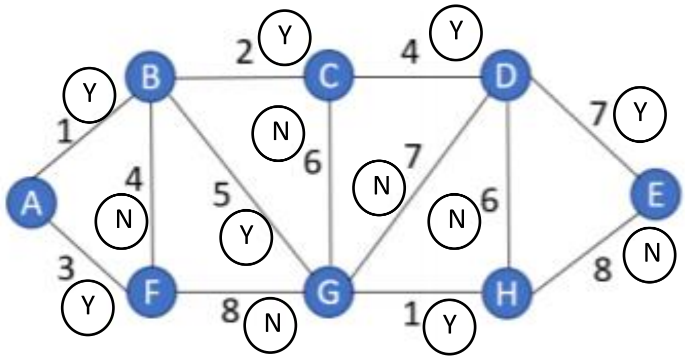


Answer:

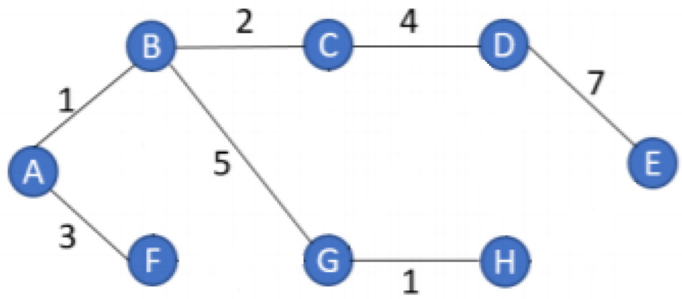
Edge	Size
AB	1
GH	1
BC	2
AF	3
BF	4
CD	4
BG	5
CG	6
DH	6
GD	7
DE	7
FG	8
HE	8

(Y) = Shortest edge that does not create cycle

(N) = Not the shortest edge that does not create cycle



After deleting the unused edge marked with (N) :



∴ The solution is

AB 1

GH 1

BC 2

AF 3

CD 4

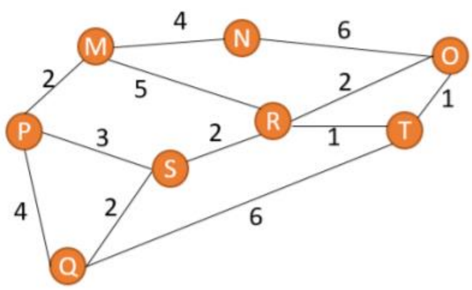
BG 5

DE 7

Total weight of tree = 1+1+2+3+4+5+7

= 23

15. Use Dijsktra's algorithm to find the shortest path from M to T for the following graph.



Answer:

Iteration	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	{}	{M, N, O, P, Q, R, S, T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N, O, P, Q, R, S, T}	0	4	∞	2	∞	5	∞	∞
2	{M, P}	{N, O, Q, R, S, T}	0	4	∞	2	6	5	5	∞
3	{M, P, N}	{O, Q, R, S, T}	0	4	10	2	6	5	5	∞
4	{M, P, N, R}	{O, Q, S, T}	0	4	7	2	6	5	5	6
5	{M, P, N, R, S}	{O, Q, T}	0	4	7	2	6	5	5	6
6	{M, P, N, R, S, Q}	{O, T}	0	4	7	2	6	5	5	6
7	{M, P, N, R, S, Q, T}	{O}	0	4	7	2	6	5	5	6

= shortest length in each iteration

= shortest path

Shortest length = 6

Shortest Path = {M, R, T}