SECI1013: DISCRETE STRUCTURE
2020/2021 – SEMESTER 1
ASSIGNMENT# 3

Group : 6

Name : **Question**

	Name	Question
1.	Rubathrah a/p Veerakkumaran (B20EC0048)	3 & 5
2.	Singthai Srisoi (A20EC0147)	2
3.	Lee Ming Qi (A20EC0064)	4
4.	Muhammad Dinie Hazim Bin Azali (A20EC0084)	1

Section : 03

Date : 7th January 2021

QUESTION 1 [25 marks]

a) Let A = {1, 2, 3, 4, 5, 6, 7, 8}, B = {2, 5, 9}, and C = {a, b}. Find each of the following:

i. $A - B$ (9 marks)

ii. $(A \cap B) \cup C$

iii. $A \cap B \cap C$

iv. $B \times C$

v. $P(C)$

b) By referring to the properties of set operations, show that: (4 marks)

$$(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$$

c) Construct the truth table for, $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$. (4 marks)

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then (x+2)2 is odd" (4 marks)

e) Let P(x,y) be the propositional function x ≥ y. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\exists x\ \exists y\ P(x, y)$ (4 marks)

ii. $\forall x\ \forall y\ P(x, y)$

Answer:

a) A = {1, 2, 3, 4, 5, 6, 7, 8}, B = {2, 5, 9}, and C = {a, b}

i. A – B = {1, 3, 4, 6, 7, 8}

ii. (A ∩ B) ∪ C = {2, 5, a, b}

iii. A ∩ B ∩ C = ø

iv. B×C = {(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)}

v. P(C) = {ø, {a}, {b}, {a, b}}

b) (P∩ ((P'∪Q)'')) ∪ (P∩Q)

= (P ∩ ((P')' ∩ Q')) ∪ (P ∩ Q) ➔ De Morgan's laws

= (P ∩ (P ∩ Q')) ∪ (P ∩ Q) ➔ Double complement laws

= ((P ∩ P) ∩ Q') ∪ (P ∩ Q) ➔ Associative laws

= (P ∩ Q') ∪ (P ∩ Q) ➔ Idempotent laws

= P ∩ (Q' ∪ Q) ➔ Distributive laws

= P ∩ U ➔ Complement laws

= P ➔ Identity laws

c) **A** = $(\neg p \lor q) \leftrightarrow (q \rightarrow p)$

p	q	¬p ∨ q	q → p	(¬p ∨ q) ↔ (q → p)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

d) Let $x = 2n + 1$

Thus, $((2n + 1) + 2)^2 = (2n + 3)^2$

$= 4n^2 + 12n + 9$

$= 2(2n^2 + 6n) + 9$

$= 2m + 9$; where $m = 2n^2 + 6n$

Thus, $(x + 2)^2$ is odd

e) i. True, if $(x, y) \neq 0$ and $x \geq y$

ii. False, if $(x, y) \neq 0$, it can be $x < y$

QUESTION 2 **[25 marks]**

a) Suppose that the matrix of relation R on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

relative to the ordering 1, 2, 3. (7 marks)

i. Find the domain and the range of R.

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

b) Let S={(x,y)| x+y ≥9}is a relation on X={2, 3, 4, 5}. Find: (6 marks)

i. The elements of the set *S*.

ii. Is *S* reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.

c) Let X={1, 2, 3}, Y={1, 2, 3, 4}, and Z={1, 2}. (6 marks)

i. Define a function $f: X \rightarrow Y$ that is one-to-one but not onto.

ii. Define a function $g: X \rightarrow Z$ that is onto but not one-to-one.

iii. Define a function $h: X \rightarrow X$ that is neither one-to-one nor onto.

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

$m(x) = 4x+3$, $n(x) = 2x-4$ (6 marks)

i. Find the inverse of *m*.

ii. ii. Find the compositions of *n o m*.

Answer:

a) i.

domain = {1, 2, 3}

range = {1, 2}

ii.

it is not irreflexive, because b ∈ X, but (b, b) ∉ R

(1, 1) and (2, 2) are an element of R

It is antisymmetric

Because $\forall a, b \in A, (a, b) \in R \wedge a \neq b \rightarrow (b, a) \notin R$

(1,2) is an element of R but (2,1) is not

(3,1) is an element of R but (1,3) is not

b) i.

S ={(4,5),(5,4),(5,5)}

ii.

Not reflexive, because (5,5) is an element of S

Symmetric, because (4,5) and (5,4) is an element of S and $M_S = M_S^T$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Not transitive, because $M_S \times M_S \neq M_S$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{x} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Hence, S is not an equivalence relation as it is not reflexive and not transitive.

c) i. $f(x) = x$

ii. $g(x) = x$

iii. $h(x) = x^2$

d) i.

$y = 4x + 3$

$y - 3 = 4x$

$x = (y - 3) / 4$

$m^{-1}(y) = (y - 3) / 4$

$m^{-1}(x) = (x - 3) / 4$

ii.

$n \text{ o } m = n(m(x))$

$= 2(4x + 3) - 4$

$= 8x + 6 - 4$

$= 8x + 2$

QUESTION 3 **[15 marks]**

a) Given the recursively defined sequence.

$$a_k = a_{k-1} + 2k, \text{ for all integers } k \geq 2, a_1 = 1$$

i) Find the first three terms. (2 marks)

ii) Write the recursive algorithm. (5 marks)

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k−1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let rk = the number of executes with an input size k. Find a recurrence relation for r1, r2, …. rk. (4 marks)

c) Given the recursive algorithm:

```
Input: n
Output: S (n)
S(n) {
        if (n=1)
            return 5
        return 5*S(n–1)
        }
```

Trace *S*(4). (4 marks)

Answer:

a)

i) k = 2, a2 = a2 -1 + 2 * 2

= a1 + 4

= 1 + 4

= 5

If k = 3, a3 = a3 -1 + 2 * 3

= a2 + 6

= 5 + 6

= 11

First three terms are 1, 5, 11

ii) Input: *k*

Output: *f (k)*

```
f (k) {
        if (k = 1)
                return 5
        return f (k – 1) + 2k
}
```

b) r_k = $2r_{k-1}$

r_1= 7

There for r_k = $2r_{k-1}$, k ≥ 2 with r_1= 7

c) S(1),

n = 1

because n =1

return 5,

S(1) = 5

S(2),

n = 2

because n is not equal to 1

return 5 * S(1),

return 5 * 5 = 25,

S(2) = 25

S(3),

n = 3

because n is not equal to 1

return 5 * S(2),

return 5 * 25 = 125,

S(3) = 125

S(4),

n = 4

because n is not equal to 1

return 5 * S(3),

return 5 * 125 = 625,

S(4) = 625

QUESTION 4 **[25 marks]**

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

(4 marks)

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

(4 marks)

c) How many arrangements in a row of no more than three letters can be formed using the letters of word COMPUTER (with no repetitions allowed)?

(5 marks)

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

(4 marks)

e) How many distinguishable ways can the letters of the word PROBABILITY be arranged?

(4 marks)

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

(4 marks)

Answer:

a) A = Number of digits for 1st digit = 9 (3, 4, 5, 6, 7, 8, 9, A, B)
B = Number of digits for 2nd digit = 16 (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)
C = Number of digits for 3rd digit = 16 (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)
D = Number of digits for 4th digit = 11 (5, 6, 7, 8, 9, A, B, C, D, E, F)

Total possible hexadecimal numbers = A x B x C x D
= 9 x 16 x 16 x 11
= 25344

Hence, there are 25344 hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long.

b) A = Number of letters for 1st letter = 1 (A)
B = Number of letters for 2nd letter = 26 (all alphabets)
C = Number of letters for 3rd letter = 26 (all alphabets)
D = Number of letters for 4th letter = 26 (all alphabets)

E = Number of digits for 1st digit = 10 (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
F = Number of digits for 2nd digit = 10 (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
G =Number of digits for 3rd digit = 1 (0)

Total possible license plate = A x B x C x D x E x F x G
= 1 x 26 x 26 x 26 x 10 x 10 x 1
= 1757600

Hence, there are 1757600 license plates could begin with A and end in 0.

c) A = Number of arrangements for 3 letters = ${}^{8}P_{3}$ = 336
B = Number of arrangements for 2 letters = ${}^{8}P_{2}$ = 56
C = Number of arrangements for 1 letters = ${}^{8}P_{1}$ = 8

Total number of arrangements = A + B + C
= 336 + 56 + 8
= 400

Hence, there are 400 arrangements in a row of no more than three letters can be formed using the letters of word COMPUTER (with no repetitions allowed).

d) A = Number of ways to choose 4 women = ${}^{7}C_{4}$ = 35
B = Number of ways to choose 3 men = ${}^{6}C_{3}$ = 20

Number of ways to choose group of 7 = A x B
= 35 x 20
= 700

Hence, there are 700 groups of seven can be chosen that contain four women and three men.

e) n = number of letters = 11
n_1 = number of letter 'P' = 1
n_2 = number of letter 'R' = 1
n_3 = number of letter 'O' = 1
n_4 = number of letter 'B' = 2
n_5 = number of letter 'A' = 1
n_6 = number of letter 'I' = 2
n_7 = number of letter 'L' = 1
n_8 = number of letter 'T' = 1
n_9 = number of letter 'Y' = 1

Total way = $\frac{n!}{n1!n2!n3!n4!n5!n6!n7!n8!n9!}$
= $\frac{11!}{1!1!1!2!1!2!1!1!1!}$
= 9979200

Hence, there are 9979200 distinguishable ways can the letters of the word PROBABILITY be arranged.

f) r = 10
n = 6

Ways of selections = ${}^{n+r-1}C_{r}$
= ${}^{6+10-1}C_{10}$
= ${}^{15}C_{10}$
= 3003

Hence, there are 3003 different selections of ten pastries.

QUESTION 5 **[10 marks]**

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

(4 marks)

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

(3 marks)

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

(3 marks)

Answer:

a) Each first name has 2 ways to form a name,

Number of names = 2 + 2 + 2 = 6

Pigeonhole = 6; pigeons =18

$k = \frac{18}{6} = 3$

b) Available odd integers = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

Odd integer = 10

Even integer = 20 – 10 =10

Pick in order to be sure of getting at least one that is odd is 10 + 1 = 11.

c) Integer divisible by 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

Number of integer divisible by 5 = 20

Number of integer not divisible by 5 = 100 – 20 = 80

Pick in order to be sure of getting one that is divisible by 5 is 80 + 1 = 81.