SECI 1013-03

STRUKTUR DISKRIT (DISCRETE STRUCTURE)

SEMESTER 1, 2020/2021

ASSIGNMENT 1

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ANSWER

1. a) $A \cup C = \{x \in \mathbf{R} | \; 0 < x < 9\}$

   b) $(A \cup B)' =$

   $A = \{x \in \mathbf{R} | \; 0 < x \leq 2\}$

   $B = \{x \in \mathbf{R} | \; 1 \leq x < 4\}$

   $C = \{x \in \mathbf{R} | \; 3 \leq x < 9\}$

   $A \cup B = \{x \in \mathbf{R} | \; 0 < x < 4\}$

   $(A \cup B)' = \{x \in \mathbf{R} | \; 4 \leq x < 9\}$

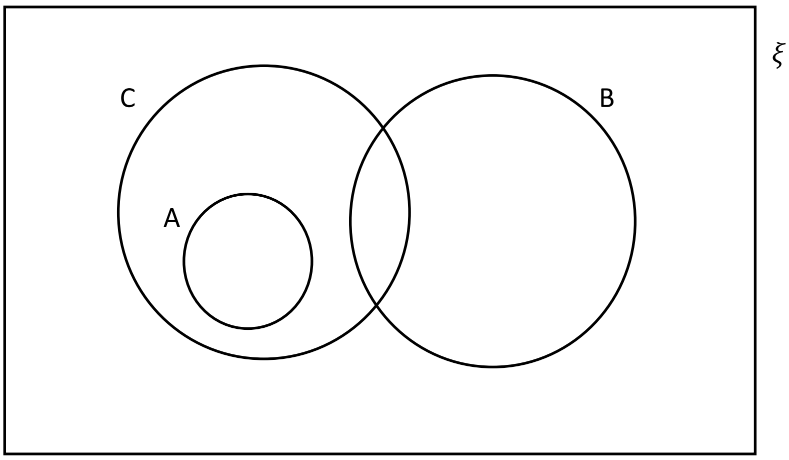
   c) $A' \cup B' =$

   $A' = \{x \in \mathbf{R} | \; 2 < x < 9\}$

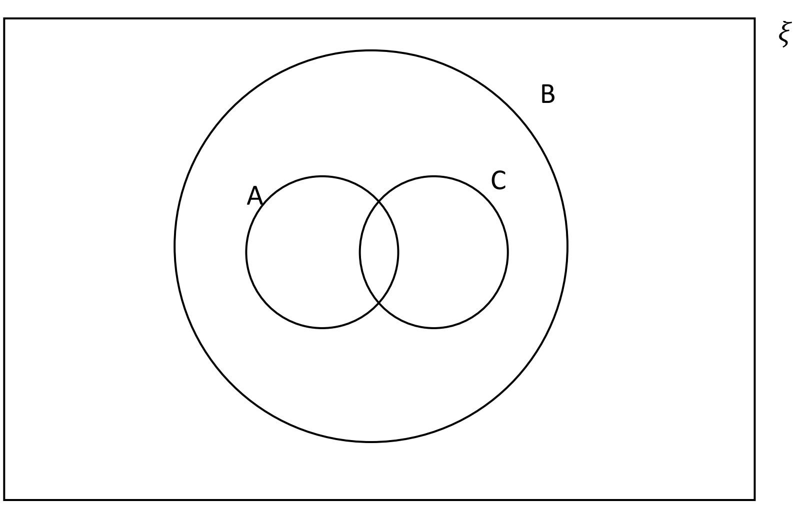
   $B' = \{x \in \mathbf{R} | \; 4 \leq x < 9\}$

   $A' \cup B' = \{x \in \mathbf{R} | \; 2 < x < 9\}$

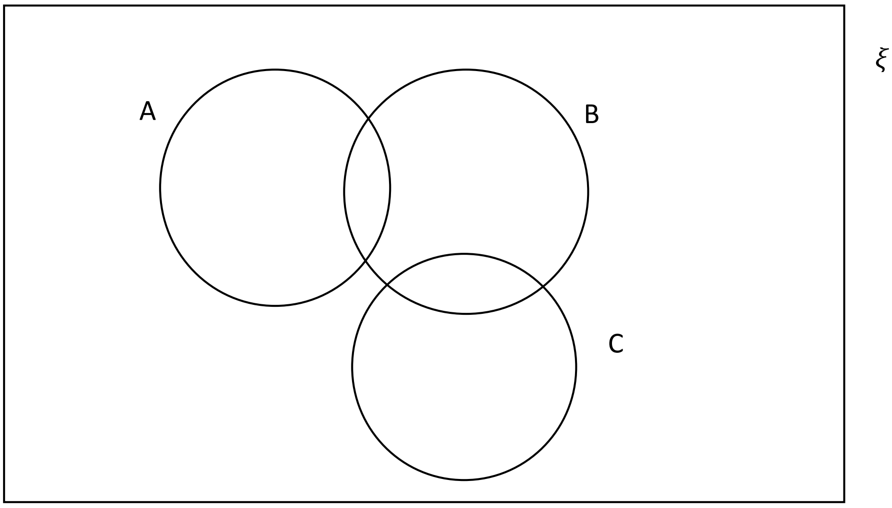
2. a)



   b)



c)



3. A = {-1, 1, 2, 4}
   B = {1, 2}
   A × B = {(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)}
   S = {(-1, 1), (1, 1), (2, 2)}
   T = {(2, 2), (4, 2)}
   $S \cap T$ = {(2, 2)}
   $S \cup T$ = {(-1, 1), (1, 1), (2, 2), (4, 2)}

4.

¬ (¬ (p Λ q) V (¬p Λ ¬q)) V (p Λ q) → De Morgan's Law

¬ (¬p Λ q) Λ ¬ (¬p Λ ¬q) V (p Λ q) → De Morgan's law

(¬¬p V ¬q) Λ (¬¬p V ¬¬q) V (p Λ q) → Double Complement law

(p V ¬q) Λ (p V q) V (p Λ q) → Distributive law

p V (¬q Λ q) V (p Λ q) → Complement Law

p V Ø V (p Λ q) → Empty Set Law

p V (p Λ q) → Absorption Law

p

¬ (¬ (p Λ q) V (¬p Λ ¬q)) V (p Λ q) ≡ p shown

5. a)

$$M_{R1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

b)

$$M_{R2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

c)

- R_1 is not reflexive
- for $\forall$ x $\in$ A, (x, x) $\in$ R, 4 $\in$ A but (4, 4) $\notin$ R and 5 $\in$ A but (5, 5) $\notin$ R
- R_1 is symmetric
- for $\forall$ a, b $\in$ A, (a, b) $\in$ R $\longrightarrow$ (b, a) $\in$ R
- R_1 is transitive
- $\forall$ a, b $\in$ A, (a, b) $\in$ R Λ (b, c) $\in$ R $\longrightarrow$ (a, c) $\in$ R
- $M_R \otimes M_R = M_R$
- $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

- Hence, R_1 is not an equivalence relation because it is symmetric and transitive but not reflexive d)

- R_2 is not reflexive
- for ∀ x ∈ A, (x, x) ∈ R, 1 ∈ A but (1, 1) ∉ R, 2 ∈ A but (2, 2) ∉ R, 3 ∈ A but (3, 3) ∉ R, 4 ∈ A but (4, 4) ∉ R and 5 ∈ A but (5, 5) ∉ R
- R_2 is antisymmetric
- ∀ a, b ∈ A, (a, b) ∈ R Λ a ≠ b ⟶ (b, a) ∉ R
- (2, 1) ∈ R but (1, 2) ∉ R which implies that a ≠ b
- R_2 is transitive
- ∀ a, b ∈ A, (a, b) ∈ R Λ (b, c) ∈ R ⟶ (a, c) ∈ R
- (5, 4) ∈ R Λ (4, 3) ∈ R ⟶ (5, 3) ∈ R
- Hence, R_2 is not partial order relation because it is antisymmetric and transitive but not reflexive

6. R_1= {(1,1),(2,2),(2,3),(3,1),(3,3)}

R_2= {(1,2),(2,2),(3,1),(3,3)}

a. $R_1 \cup R_2$= {(1,1), (2,2),(2,3),(3,1),(3,3),(1,2)}

$$M_{R_1 \cup R_2}= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

b. $R_1 \cap R_2$= {(2,2),(3,1),(3,3)}

$$M_{R_1 \cap R_2}= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

7. $f + g$ is also one-to-one.

For one-to-one function, if whenever f (a) = f (b) then a = b, no element of B is the image of more than one element in A. Thus, the function must be linear function so that only one output will be produced when one input number is plugged into the function.

Let $f(x) = 4x - 1$ and $g(x) = -x + 2$,

So,

$(f+g)(x) = (4x-1) + (-x+2)$

$= 4x - x - 1 + 2$

$= 3x + 1$

That is, when a x value is plugged into the final equation $(f+g)(x) = 3x + 1$, there will be only one different output for each x value plugged in that indicates it is a one-to-one function.

8. When

$n = 1,\ C_1 = 1$, that is, there will be only one possible way to climb the entire staircase.

$n = 2,\ C_2 = 2$, there is a maximum of two possible ways to climb the entire staircase either by one-step or two-step.

So the recurrence relation,

$$C_n = C_{n-1} + C_{n-2}\ , n \geq 3\ \text{, when;} C_1 = 1 \text{ and } C_2 = 2.$$

So, for

$C_3 = C_2 + C_1 = 2 + 1 = 3$

$C_4 = C_3 + C_2 = 3 + 2 = 5$

$C_5 = C_4 + C_3 = 5 + 3 = 8$

With sequence,

1,2,3,5,8,..

9. a)
$t_0 = 0$
$t_1 = 1$
$t_2 = 1$
$t_3 = t_2 + t_1 + t_0 = 1 + 1 + 0 = 2$
$t_4 = t_3 + t_2 + t_1 = 2 + 1 + 1 = 4$
$t_5 = t_4 + t_3 + t_2 = 4 + 2 + 1 = 7$
$t_6 = t_5 + t_4 + t_3 = 7 + 4 + 2 = 13$
$t_7 = t_6 + t_5 + t_4 = 13 + 7 + 4 = 24$
$t_7 = 24$

b)

Input = n positive integer
Output = f(n)

```
f(n)
{       if (n = 1 or n = 2 or n = 3)
                return 1
        return f (n – 1) + f (n – 2) + f (n – 3)
}
```