UTM

UNIVERSITI TEKNOLOGI MALAYSIA

DISCRETE STRUCTURE (SECI 1013-03)

SEMESTER 1-2020/2021

ASSIGNMENT 3

GROUP 5

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Question 1

ai. $A-B = \{1, 3, 4, 6, 7, 8\}$

ii. $(A \cap B) \cup C = \{2, 5, a, b\}$

iii. $A \cap B \cap C = \emptyset$

iv. $B \times C = \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$

v. $P(C) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q)$

$= (P \cap (P \cup Q')) \cup (P \cap Q)$ (De Morgan's law)

$= P \cup (P \cap Q)$ (Absorption law)

$= P$ (Absorption law)

c) **A** $= (\neg p \vee q) \leftrightarrow (q \rightarrow p)$

p	q	$\neg p$	$(\neg p \vee q)$	$(q \rightarrow p)$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) let x = 3

$(3 + 2)^2 = 25$ (odd)

let x = 5

$(5 + 2)^2 = 49$ (odd)

let a = odd integer

$a = 2n + 1$

$(a + 2)^2 = ((2n + 1) + 2))^2$

$= (2n + 3)^2$

$= 4n^2 + 12n + 9$

$= 2((2n^2 + 6n + 4) + 1$

let $(2n^2 + 6n + 4) = m$

$(a + 2)^2 = 2m + 1$

$(a + 2)^2$ is an odd integer

ei) $\exists x \, \exists y P(x, y)$

There are some x, positive integer and some y, positive integer where $x \geq y$.

This statement is true.

ii) $\forall x \, \forall y P(x, y)$

For all x, positive integer and all y, positive integer where $x \geq y$.

This statement is false because when x = 5 and y = 6, $x < y$. There are at least one x is smaller than y.

Question 2

ai) Domain = {1, 2, 3}

Range = {1, 2}

ii) The relation is not irreflexsive because $(1,1), (2,2) \in$ R but $(3,3) \notin$ R.

The relation is antisymmetric because $(1,2) \in$ R but $(2,1) \notin$ R and $(1,1), (2,2) \in$ R but $(3,3) \notin$ R.

bi) set S = {(4,5), (5,4), (5,5)}

ii) S is not reflexive because $(5,5) \in$ R but $(1,1), (2,2), (3,3), (4,4) \notin$ R.

S is symmetric because $M_R = M_R^T$.

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad M_R^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

S is not transitive because $M_R \times M_R \neq M_R$.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

S is not an equivalence relation because S is symmetric but is not reflexive and not transitive.

ci) f = {(1,1), (2,2), (3,3)}

ii) g = {(1,1), (2,2), (3,2)}

iii) h = {(1,1), (2,1), (3,2)}

di) $m(x) = 4x+3$

let $y = 4x + 3$ $\qquad m^{-1}(x) = \frac{y-3}{4}$

$4x = y - 3$

$x = \frac{y-3}{4}$

ii) $nm(x) = n(4x + 3)$

$= 2(4x + 3) - 4$

$= 8x + 6 - 4$

$= 8x + 2$

Question 3

ai) $a_1 = 1$ $\quad a_2 = a_1 + 2(2)$ $\quad a_3 = a_2 + 2(3)$

$= 1 + 4$ $\quad = 5 + 6$

$= 5$ $\quad = 11$

ii) Input: k

Output: f(k)

f(k) {

if (k = 1)

return 1

return f(k − 1) + 2k

}

b) $r_1 = 7$ $\quad r_k = 2r_{k-1}\, where\ k > 1$

c) S(1) = 5

S(2) =5*(S(1)) S(3) = 5*(S(2)) S(4) = 5*(S(3))

= 5*5 = 5*25 = 5*125

= 25 = 125 = 625

Question 4

a) First digit: 3 through B = 9 ways

Second digit = 16 ways

Third digit = 16 ways

Last digit:5 through F = 11 ways

$$9 \times 16 \times 16 \times 11 = 25344$$

b) $1 \times 26^3 \times 10^2 \times 1 = 1757600$

c) Three letters = $8 \times 7 \times 6$ 8+336 + 56 = 400

$= 336$

Two letters = 8×7

$= 56$

One letter = 8

d) $7C4 \times 6C3 = 700$

e) $P(11) = \frac{11!}{2!2!}$

$= 9979200$

f) $C(10 + 6 - 1,10) = C(15,10)$

$= \frac{15!}{10!5!}$

$= 3003$

Question 5

a) $k = 2 \times 3 = 6$

$n = 18$

$\frac{n}{k} = \frac{18}{6} = 3$ people with same first name and last name

b) Odd number from 1 to 20 = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

Probability of getting an odd number = 0.5

10 + 1 = 11 (Pick at least 11 number to be sure of getting at least one that is odd)

c) There are 20 integers from 1 through 100 that are divisible by 5.

$100 - 20 = 80$

$80 + 1 = 81$ (Pick at least 81 to be sure of getting one that is divisible by 5)