

SECI1013-06 STRUKTUR DISKRIT

TUTORIAL 2

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# Tutorial #2 SECI1013 –Discrete Structure

**Chapter 3**

**Due date: 1 Jan 2021**

1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7
	1. How many numbers are there?

$$6 ×6 ×6 =216$$

* 1. How many numbers are there if the digits are distinct?

$$6×5×4 = 120$$

* 1. How many numbers between 300 to 700 are only odd digits allowed?

$$2×3×3 = 18$$

1. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table
	1. Men insist to sit next to each other

 $5! ×\left(6-1\right)! =5! ×5!=14400 $ways

* 1. The couple insisted to sit next to each other

 $2! × (9-1)! = 2!×8! = 80640$ ways

* 1. Men and women sit in alternate seat

$(5-1)! × 5! = 4! × 5! = 2880$ ways

* 1. Before her friend left, Anita wanted to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

 $2! × 11! = 79833600$ ways

1. In a school sport day, five sprinters are competing in a 100-meter race. How many ways are there for the sprinter to finish?
	1. If no ties

$$5 ×4 ×3 ×2 ×1=5 !=120 ways$$

* 1. Two sprinters tie

 $C \left(5 ,2\right)×P\left(4,4\right)= \frac{5 !}{2!\left( 5-2\right)! }×24=240 ways$

* 1. Two group of two sprinters tie

 $C \left(5 ,2\right)× C \left(3 ,2\right)×P\left(3,1\right)=\frac{5!}{2!\left(5-2\right)! } × \frac{3!}{2!\left(3-2\right)!} ×3= 90 ways $

1. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose?
	1. a dozen croissants?

$C ( 6 + 12 -1, 12) =\frac{(6+12 -1)!}{12!(6-1)!} = \frac{17!}{12!5!} = 6188$ ways

* 1. two dozen croissants with at least two of each kind?

 For one type only = 6 ways

$C ( 6 + 24 -1, 24)-6 =\frac{(6+24 -1)!}{24!(6-1)!} -6 = \frac{29!}{24!5!} - 6 = 118749$ ways

* 1. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

 $C\left(1+5-1,5\right)×C\left(1+3-1,3\right)×C\left(6+16-1,16\right)$

$\frac{5!}{5!} × \frac{3!}{3!} × \frac{(6+16-1)!}{16!(6-1)!} = \frac{21!}{16!5!}=20349$ ways

1. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.
	1. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

The order of wins/losses or ties does not matter for the k rounds and next other round need to have result either a win or a tie:

2 wins among 4 rounds and 1 ties/wins: $C(4,2)×C\left(3,1\right)×2=6×C\left(3,1\right)×2=36$

1 win among 3 rounds and 3 ties/wins: $C\left(3,1\right)×C\left(4,3\right)×2^{3}=3 ×C\left(4,3\right)×2^{3}=96 $

Because there are 2 team can have the wins:

 Number of scenarios $=(36+96)×2$

 $=264$ scenarios

* 1. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

$1024-264=760$ scenarios

 760 ways the scenarios result in the game not being settled in the first round of 10 penalty

 kicks.Then, the game settled in the second round of 10 penalty kicks:

 First round: 760 scenarios

 Second round: 264 scenarios

 Number of scenarios $=264 × 760=200640 $scenarios

* 1. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

 If first and second round not finish

First round = 760 scenarios

Second round = 760 scenarios

Sudden Death = $2+2+2+2+2=10 scenarios$

Number of scenarios $=760 ×760 ×10=5776000$ scenarios

1. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor’s class in order to guarantee that at least three answer sheets must be identical?

(Assume that no answers are left blank.)

Total number of questions $= 10$

Total number of answer choices $= 4$

Hence, the maximum number of students in professor’s class in order to guarantee that at least two answers sheets must be identical are $(4^{10 }×2 ) = 2097152$ students but the $2097153rd$ student’s sheet would have to be identical to at least with two other student. Therefore, you would need a minimum of $2097153$ students in order to guarantee that at least three answer sheets are identical.

1. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

$$P(H) = 0.75 P(H’) = 0.25$$

$$P(M) = 0.65 P(M’) = 0.35$$

$P\left(M∩H\right)=0.5 P(M∩N)' = P (M' ∪ N') = 0.5$

$$P\left(M^{'}∩H^{'}\right)=P\left(M^{'}\right)+P\left(H^{'}\right)-P\left(M^{'}∪H^{'}\right)$$

$$P(M' ∩ H') = 0.35 + 0.25 - 0.5 = 0.1$$

$$\frac{35}{X} = 0.1$$

$$X = 350$$

350 of candidates sit for the exam.

1. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.
* 300 to 699, inclusive without 1 as digit

$$\{3,4,5,6\}$$

$$\{0,2,3,4,5,6,7,8,9\}$$

$$\{0,2,3,4,5,6,7,8,9\}$$

$$4×9×9 = 324 ways$$

* 700 to 779, inclusive without 1 as digit

$$\{0,2,3,4,5,6,7,8,9\}$$

$$\{0,2,3,4,5,6,7\}$$

$$\{7\}$$

 $1×7×9 = 63$ ways

* 780

$$\{0\}$$

$$\{8\}$$

$$\{7\}$$

 1 ways

Without 1 as digit $= 324 + 63 + 1 = 388$ ways

From 300 to 780, inclusive

$780- 300 + 1 = 481$ ways

The probability that the number is chosen will have 1 as at least one digit is :

$\frac{481-388}{481} = 0.1933$

1. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are **not distinguishable**, and the parking lots are chosen at random.
	1. In how many ways can the cars be parked in the parking lots?

Blue cars = 2 , Yellow cars = 4 , Empty lots = 4

$\frac{10!}{2!4!4!} = 3150$ ways

* 1. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

$$\frac{4!}{4!}×\frac{7!}{2!4!} = 105 ways$$

Probability: $\frac{105}{3150} = \frac{1}{30} = 0.03333$

1. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4, 0.1 and 0.5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or hand phone are 0.6,0.8 and 1 respectively
	1. Find the probability the trainee receives the message

E : Email , L : Letter, H : Handphone, R : Receive

0.4

0.1

0.5

E

L

H

R

R

R

R'

R'

R'

0.6

0.8

1

0.4

0.2

0.0

 $P(R) = P(E∩R) + P(L∩R) + P(H∩R)$

$=(0.4×0.6)+(0.1×0.8)+(0.5×1)$

$=0.24+0.08+0.5$

$=0.82$

* 1. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

$$P\left(E | R\right)=\frac{P(R\bigcap\_{}^{}E)}{P(R)}=\frac{0.4×0.6}{0.82}=\frac{0.24}{0.82}=\frac{12}{41}=0.2927$$

1. In recent News, it was reported that light trucks, which include SUV’s , pick-up trucks and minivans, accounted for **40% of all personal vehicles** on the road in 2012. Assume the rest are cars. Of every **100,000 cars accidents, 20** involve a fatality; of every **100,000 light trucks accidents, 25** involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

A – Light Trucks A’ – Cars

B – Fatal Accident B’ – Not Fatal Accident

 $P(B|A')=\frac{20}{100000} = 0.0002$

 $P(B|A)=\frac{25}{100000} = 0.00025$

 $P\left(A\right)=0.4 $

 $P\left(A'\right)=1-P\left(A\right)=1-0.4=0.6$

$$P\left(A|B\right)=\frac{P\left(B|A\right) ∙P(A)}{P\left(B|A\right) ∙P\left(A\right)+P\left(B|A'\right) ∙ P(A')}$$

$$= \frac{0.00025 ×0.4}{\left(0.00025 ×0.4\right)+\left(0.0002×0.6\right)}$$

$$= 0.4545$$

1. There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contains at least 1 letter?

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| PATTERN | GROUPING | ARRANGEMENT | TOTAL |
| (6)(1)(1)(1) | $$C(9,6)×C(3,1)×C(2,1)×C(1,1)= 504$$ | $$\frac{4!}{3!}=4$$ | $$504×4=2016$$ |
| (5)(2)(1)(1) | $$C(9,5)×C(4,2)×C(2,1)×C(1,1)=1512$$ | $$\frac{4!}{2!}=12$$ | $$1512×12=18144$$ |
| (4)(3)(1)(1) | $$C(9,4)×C(5,3)×C(2,1)×C(1,1)=2520$$ | $$\frac{4!}{2!}=12$$ | $$2520×12=30240$$ |
| (4)(2)(2)(1) | $C(9,4)×C(5,2)×C(3,2)×C(1,1)=3780$  | $$\frac{4!}{2!}=12$$ | $$3780×12=45360$$ |
| (3)(3)(2)(1) | $$C(9,3)×C(6,3)×C(3,2)×C(1,1)=5040$$ | $$\frac{4!}{2!}=12$$ | $$5040×12=60480$$ |
| (3)(2)(2)(2) | $$C(9,3)×C(6,2)×C(4,2)×C(2,2)=7560$$ | $$\frac{4!}{3!}=4$$ | $$7560×4=30240$$ |

Total ways $=2016+18144+30240+45360+60480+30240$

$ =186480$ ways