

SECI1013-06 STRUKTUR DISKRIT

TUTORIAL 1

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DISCRETE STRUCTURE (SECI 1013)

TUTORIAL 1

1. Let the universal set be the set R of all real numbers and let A = { $x$ $\in $ R | 0 < $x$ ≤ 2 },

B = { $x$ $\in $ R | 1 ≤$ x$ < 4} and C = { $x$ $\in $ R | 3 ≤ $x$ < 9 }. Find each of the following:

a) A $∪$ C

B

A

4

3

2

0

$$A ∪C=\{ x\in R | 0<x\leq 2 or 3\leq x<9 \}$$

b) (A ∪ B)′

B

A

4

2

1

0

$$A∪B=\{ x\in R | 0<x<4 \}$$

$$( A∪B )'=\{ x\in R | x\leq 0 or x\geq 4 \}$$

c) A′ ∪ B′

$$A'∪B'=( A∩B )'$$

B

A

4

2

1

0

$$A∩B=\{ x\in R | 1\leq x \leq 2 \}$$

$$( A∪B )'=A'∪B'=\{ x \in R | x<1 or x>2 \}$$

2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

a) A ∩ B = ∅, A ⊆ C, C ∩ B ≠ ∅

B

A

C

b) A ⊆ B, C ⊆ B, A ∩ C ≠ ∅

C

A

B

c) A ∩ B ≠ ∅, B ∩ C ≠ ∅, A ∩ C = ∅, A ⊄ B, C ⊄ B

B

C

A

3. Given two relations S and T from A to B,

S ∩ T = {(x,y) ∈A×B | (x,y) ∈ S and (x,y) ∈ T}

S ∪ T = {(x,y) ∈A×B | (x,y) ∈ S or (x,y) ∈ T}

Let A={−1, 1, 2, 4} and B={1,2} and defined binary relations S and T from A to B as follows:

For all (x,y) ∈A×B, x S y ↔ |x| = |y|

For all (x,y) ∈A×B, x T y ↔ x− y is even

State explicitly which ordered pairs are in A×B, S, T, S ∩ T, and S ∪ T

$$A ×B=\{ \left(-1,1\right), \left(-1,2\right), \left(1,1\right), \left(1,2\right), \left(2,1\right), \left(2,2\right), \left(4,1\right), \left(4,2\right) \}$$

$$S=\{ \left(-1,1\right), \left(1,1\right), \left(2,2\right) \}$$

$$T=\left\{ \left(-1,1\right),\left(1,1\right), \left(2,2\right),\left(4,2\right) \right\}$$

$$S∩T=\{ \left(-1,1\right), \left(1,1\right), \left(2,2\right) \}$$

$$S∪T=\left\{ \left(-1,1\right),\left(1,1\right), \left(2,2\right),\left(4,2\right) \right\}$$

4. Show that ¬ ((¬p∧q) ∨ (¬p∧¬q)) ∨ (p∧q) ≡ p. State carefully which of the laws are used at each stage.

Answer:

$p ≡ ¬ \left( \left( ¬ p∧q \right)∨\left(¬ p∧¬q \right)\right) ∨( p∧q )$

$p ≡ ¬ \left( ¬ p∧q \right) ∧ ¬ \left(¬ p∧¬q \right) ∨( p∧q )$ (De Morgan’s Laws)

$p ≡(¬ ¬ p∨¬ q) ∧ ¬ ¬ \left(p∨q \right) ∨( p∧q )$ (De Morgan’s Laws)

$p ≡(p∨¬ q) ∧ \left(p∨q \right) ∨( p∧q )$ (Double negation Laws)

$p ≡p ∨ (¬ q ∧ q) ∨( p∧q )$ (Distributive Laws)

$p ≡p ∨ F ∨( p∧q )$ (Negation Laws)

$p ≡p ∨( p∧q )$ (Identity Laws)

$p ≡p$ #Proven (Absorption Laws)

5. $R\_{1}=\{(x,y)| x+y \leq 6\};$ $R\_{1}$ is from $X$ to $Y$; $R\_{2} =\{(y,z)| y>z\};$ $R\_{2}$ is from $Y$ to $Z$; ordering of

$X, Y$, and $Z$: 1, 2, 3, 4, 5.

Find:

a) The matrix $A\_{1}$of the relation $R\_{1}$ (relative to the given orderings)

 1 2 3 4 5

1

2

3

4

5

$$\left[\begin{array}{c}\begin{matrix}1&1&1\\1&1&1\\1&1&1\end{matrix} \begin{matrix}1&1\\1&0\\0&0\end{matrix}\\\begin{matrix}1&1&0\end{matrix} \begin{matrix}0&0\end{matrix}\\\begin{matrix}1&0&0\end{matrix} \begin{matrix}0&0\end{matrix}\end{array}\right]$$

b) The matrix $A\_{2}$ of the relation $R\_{2}$ (relative to the given orderings)

 1 2 3 4 5

1

2

3

4

5

$$\left[\begin{array}{c}\begin{matrix}0&0&0\\1&0&0\\1&1&0\end{matrix} \begin{matrix}0&0\\0&0\\0&0\end{matrix}\\\begin{matrix}1&1&1\end{matrix} \begin{matrix}0&0\end{matrix}\\\begin{matrix}1&1&1\end{matrix} \begin{matrix}1&0\end{matrix}\end{array}\right]$$

c) Is $R\_{1}$ reflexive, symmetric, transitive, and/or an equivalence relation?

$R\_{1}$ have 1 and 0 on main diagonal and $R\_{1}$is not reflexive.

$R\_{1}= R\_{1}^{T}$ , $R\_{1}$ is symmetric.

$$\left[\begin{array}{c}\begin{matrix}1&1&1\\1&1&1\\1&1&1\end{matrix} \begin{matrix}1&1\\1&0\\0&0\end{matrix}\\\begin{matrix}1&1&0\end{matrix} \begin{matrix}0&0\end{matrix}\\\begin{matrix}1&0&0\end{matrix} \begin{matrix}0&0\end{matrix}\end{array}\right] ⊗ \left[\begin{array}{c}\begin{matrix}1&1&1\\1&1&1\\1&1&1\end{matrix} \begin{matrix}1&1\\1&0\\0&0\end{matrix}\\\begin{matrix}1&1&0\end{matrix} \begin{matrix}0&0\end{matrix}\\\begin{matrix}1&0&0\end{matrix} \begin{matrix}0&0\end{matrix}\end{array}\right]= \left[\begin{array}{c}\begin{matrix}1&1&1\\1&1&1\\1&1&1\end{matrix} \begin{matrix}1&1\\1&1\\1&1\end{matrix}\\\begin{matrix}1&1&1\end{matrix} \begin{matrix}1&1\end{matrix}\\\begin{matrix}1&1&1\end{matrix} \begin{matrix}1&1\end{matrix}\end{array}\right]$$

$M\_{R\_{1}}⊗ M\_{R\_{1}}\ne M\_{R\_{1}}.$ So, $R\_{1}$ is not transitive.

Thus, R is not equivalence relation because it is a not reflexive, symmetric and not transitive.

d) Is $R\_{2} $ reflexive, antisymmetric, transitive, and/or a partial order relation?

$R\_{2} $have 0 on its main diagonal. So, $R\_{2} $ is irreflexive

$\left(1,2\right)\in R\_{2} ∧x\ne y$ but $\left(2,1\right)\notin R\_{2.}$ So, $R\_{2.} $is antisymmetric.

$$\left[\begin{array}{c}\begin{matrix}0&0&0\\1&0&0\\1&1&0\end{matrix} \begin{matrix}0&0\\0&0\\0&0\end{matrix}\\\begin{matrix}1&1&1\end{matrix} \begin{matrix}0&0\end{matrix}\\\begin{matrix}1&1&1\end{matrix} \begin{matrix}1&0\end{matrix}\end{array}\right] ⊗\left[\begin{array}{c}\begin{matrix}0&0&0\\1&0&0\\1&1&0\end{matrix} \begin{matrix}0&0\\0&0\\0&0\end{matrix}\\\begin{matrix}1&1&1\end{matrix} \begin{matrix}0&0\end{matrix}\\\begin{matrix}1&1&1\end{matrix} \begin{matrix}1&0\end{matrix}\end{array}\right]= \left[\begin{array}{c}\begin{matrix}0&0&0\\0&0&0\\1&0&0\end{matrix} \begin{matrix}0&0\\0&0\\0&0\end{matrix}\\\begin{matrix}1&1&0\end{matrix} \begin{matrix}0&0\end{matrix}\\\begin{matrix}1&1&1\end{matrix} \begin{matrix}0&0\end{matrix}\end{array}\right]$$

$M\_{R\_{2}}⊗ M\_{R\_{2}}\ne M\_{R\_{2}}. $So, $R\_{2} $is not transitive.

Thus, $R\_{2} $ is not partial order because it is an irreflexive, antisymmetric and not transitive.

6. Suppose that the matrix of relation $R\_{1}$ on {1, 2, 3} is

$$\left[\begin{matrix}1&0&0\\0&1&1\\1&0&1\end{matrix}\right]$$

relative to the ordering 1, 2, 3, and that the matrix of relation $R\_{2} $on {1, 2, 3} is

$$\left[\begin{matrix}0&1&0\\0&1&0\\1&0&1\end{matrix}\right]$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation $R\_{1}∪R\_{2}$

$$R\_{1}=\{ \left(1,1\right), \left(2,2\right), \left(2,3\right), \left(3,1\right), \left(3,3\right)\}$$

$$R\_{2}=\{ \left(1,2\right), \left(2,2\right), \left(3,1\right), \left(3,3\right)\}$$

$$R\_{1}∪R\_{2}=\{ \left(1,1\right),\left(1,2\right), \left(2,2\right), \left(2,3\right), \left(3,1\right), \left(3,3\right)\}$$

$$\left[\begin{matrix}1&1&0\\0&1&1\\1&0&1\end{matrix}\right]$$

b) The matrix of relation $R\_{1}∩R\_{2}$

$R\_{1}∩R\_{2}= ${$\left(2,2\right), \left(3,1\right), \left(3,3\right)\}$

$$\left[\begin{matrix}0&0&0\\0&1&0\\1&0&1\end{matrix}\right]$$

7. If $f$ : **R→ R** and $g $: **R→ R** are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

Answer:

When the function notation $f$ : **R→ R** and $g $: **R→ R**, its mean that $f$ and $g $are function from the real numbers to the real numbers.So, if $f$ is defined as $f\left(x\right)=x$ and $g$ as $g\left(x\right)= -x$, where $x\in R,$ this are one-to-one function.

However, $\left(f+g\right)\left(x\right)=0$ for all $x$, hence $f+g$ is not one-to-one function. This shows that adding two one-to-one functions does not necessarily produce a function that is one-to-one.

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer n≥1, if the staircase consists of n stairs, let $c\_{n}$ be the number of different ways to climb the staircase. Find a recurrence relation for $c\_{1}, c\_{2}, …., c\_{n}$.

Answer:

1 way

2 ways

$$c\_{2}$$

$$c\_{1}$$

3 ways

$$c\_{3}$$

$c\_{1}=1$,

$c\_{2}=2$,

$c\_{n}=c\_{n-1}+c\_{n-2} $ , $n \geq 3$

$$c\_{3}=c\_{2}+ c\_{1}$$

$$c\_{3}=2+1=3$$

9. The Tribonacci sequence ($t\_{n}$,) is defined by the equations,

$t\_{1}= t\_{2}= t\_{3}=1, t\_{n}= t\_{n-1}+ t\_{n-2}+ t\_{n-3} $for all n≥4.

a) Find $t\_{7}$.

$$t\_{7}= t\_{6}+ t\_{5}+ t\_{4}$$

$$t\_{4}= t\_{3}+ t\_{2}+ t\_{1}$$

$$t\_{4}= 1+1+1=3$$

$$t\_{5}= t\_{4}+ t\_{3}+ t\_{2}$$

$$t\_{5}= 3+ 1+ 1=5$$

$$t\_{6}= t\_{5}+ t\_{4}+ t\_{3}$$

$$t\_{6}=5+3+1=9$$

$$t\_{7}=9+5+3$$

$$t\_{7}=17$$

b) Write a recursive algorithm to compute $t\_{n}$, n≥1.

Answer:

Input : $n$ integer positive

Output : $t(n)$

$$t(n)$$

{ if ( $n=1 or n=2 or n=3 )$

 return 1

 return $t\left(n-1\right)+t\left(n-2\right)+t(n-3)$

}

# References

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