



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING

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SECI 1013 – DISCRETE STRUCTURE SECTION-08

ASSIGNMENT 4

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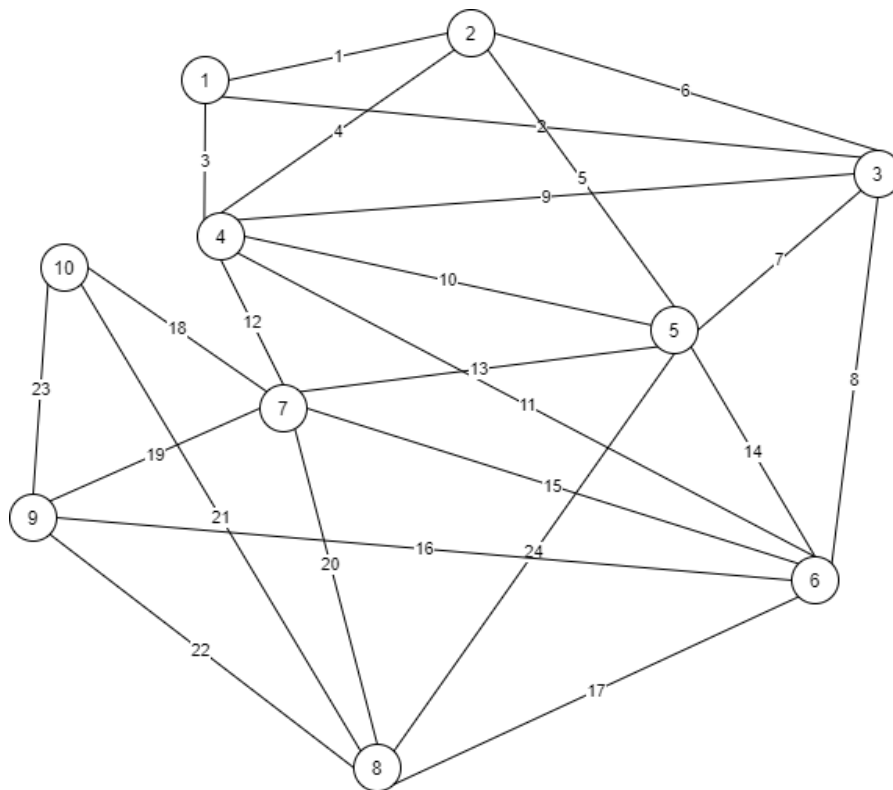
**ASSIGNMENT 4**

1. Let G be a graph with $V(G) = \{1, 2, \dots, 10\}$, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if $|v - w| \leq 3$. Draw the graph G and determine the numbers of edges, $e(G)$.

SOLUTION:

$$V(G) = \{(1,2), (2,1), (1,3), (3,1), (1,4), (4,1), (2,3), (3,2), (2,4), (4,2), (2,5), (5,2), (3,4), (4,3), (3,5), (5,3), (3,6), (6,3), (4,5), (5,4), (4,6), (6,4), (4,7), (7,4), (5,6), (6,5), (5,7), (7,5), (5,8), (8,5), (6,7), (7,6), (6,8), (8,6), (6,9), (9,6), (7,8), (8,7), (7,9), (9,7), (7,10), (10,7), (8,9), (9,8), (8,10), (10,8), (9,10), (10,9)\}$$

	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	0	0	0	0	0	0
2	1	0	1	1	1	0	0	0	0	0
3	1	1	0	1	1	1	0	0	0	0
4	1	1	1	0	1	1	1	0	0	0
5	0	1	1	1	0	1	1	1	0	0
6	0	0	1	1	1	0	1	1	1	0
7	0	0	0	1	1	1	0	1	1	1
8	0	0	0	0	1	1	1	0	1	1
9	0	0	0	0	0	1	1	1	0	1
10	0	0	0	0	0	0	1	1	1	0



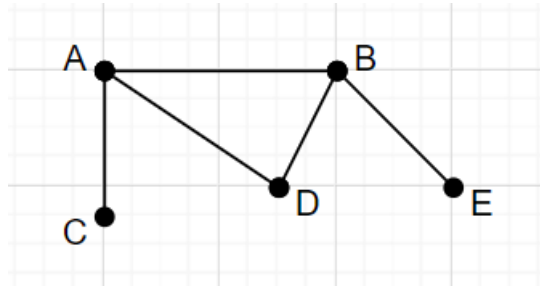
Thus, the number of edges, $e(G) = 24$

2. Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.

- (a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.

(Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)

SOLUTION:



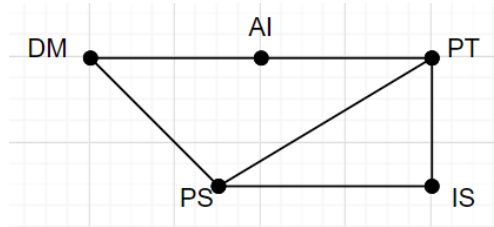
Adjacency matrix:

$$a_F = \begin{matrix} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

- (b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot:-

- i. DM and IS
- ii. DM and PT
- iii. AI and PS
- iv. IS and AI

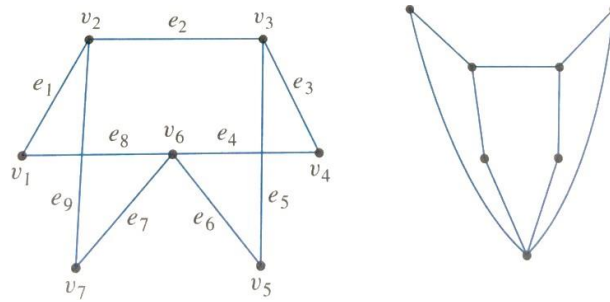
SOLUTION:



Adjacency matrix:

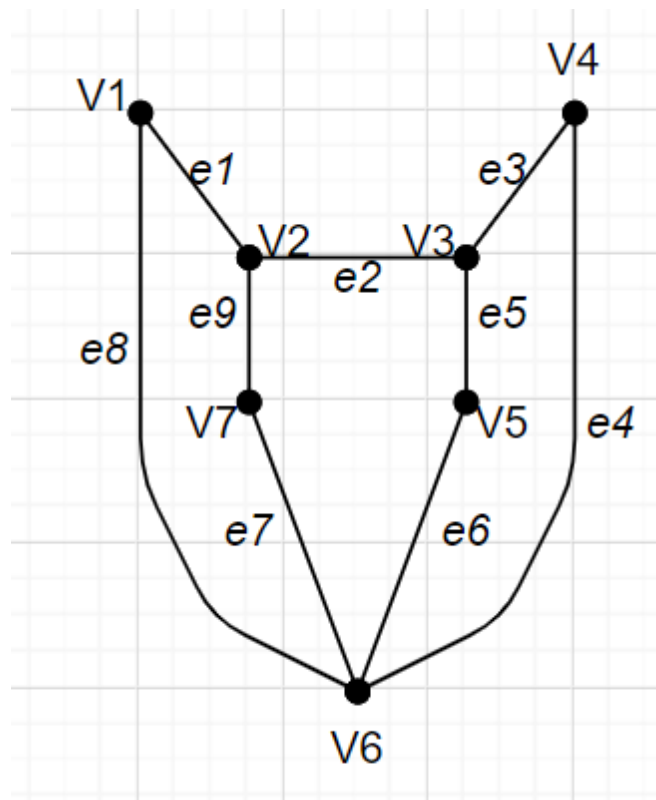
$$a_F = \begin{matrix} & \begin{matrix} DM & IS & PT & AI & PS \end{matrix} \\ \begin{matrix} DM \\ IS \\ PT \\ AI \\ PS \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

3. Show that the two drawings represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.

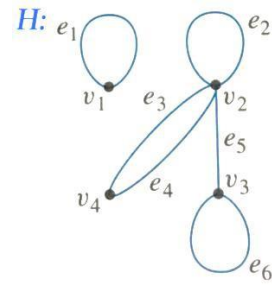
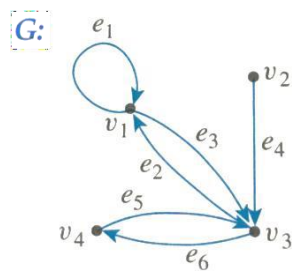


SOLUTION:

Firstly, we need to detect the edges connected to the respective vertices. Make sure the number of edges connected is equal on both graphs.



4. Find the adjacency and incidence matrices for the following graphs.



SOLUTION:

Adjacency matrix:

$$A_G = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{matrix}$$

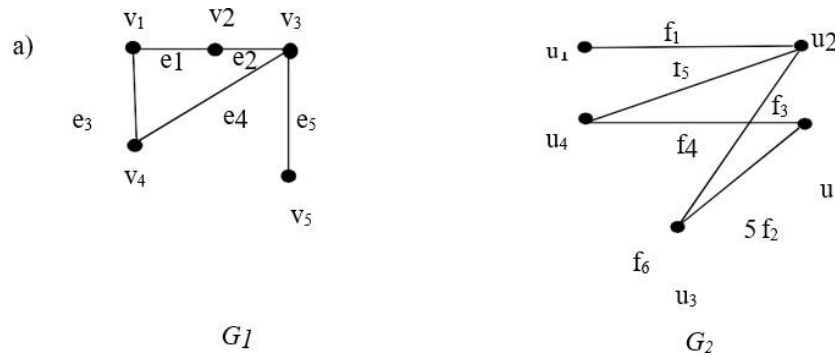
$$A_H = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix} \end{matrix}$$

Incidence matrix:

$$I_G = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{pmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \end{matrix}$$

$$I_H = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

5. Determine whether the following graphs are isomorphic.



SOLUTION:

a) - G_1 and G_2 have the same number of vertices and the same number of edges.

G_1 :

Vertex	V1	V2	V3	V4	V5
Degree	2	2	3	2	1

G_2 :

Vertex	U1	U2	U3	U4	U5
Degree	1	3	2	2	2

G_1 : 1 vertex has 1 degree
 3 vertices has 2 degree
 1 vertex has 3 degree

G_2 : 1 vertex has 1 degree
 3 vertices has 2 degree
 1 vertex has 1 degree

Define $f : G_1 \Rightarrow G_2$, where

$G_1 = \{v_1, v_2, v_3, v_4, v_5\} : G_2 = \{u_1, u_2, u_3, u_4, u_5\}$

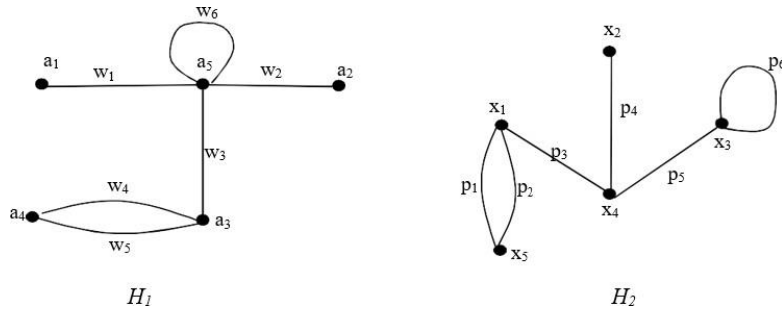
Then,

$f(v_1) = u_5; f(v_2) = u_4; f(v_3) = u_2; f(v_4) = u_3; f(v_5) = u_1;$

$$\begin{matrix}
 & v_1 & v_2 & v_3 & v_4 & v_5 \\
 \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{matrix} & \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix} & A_{G_1} = A_{G_2} & \begin{matrix} u_5 & u_4 & u_2 & u_3 & u_1 \\ \begin{matrix} u_5 \\ u_4 \\ u_2 \\ u_3 \\ u_1 \end{matrix} & \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}
 \end{matrix}$$

Thus, G_1 and G_2 is isomorphic

b)



SOLUTION:

b) – H_1 and H_2 have the same number of vertices and the same number of edges

H_1 :

Vertex	A1	A2	A3	A4	A5
Degree	1	1	3	2	5

H_2 :

Vertex	X1	X2	X3	X4	X5
Degree	3	1	3	3	2

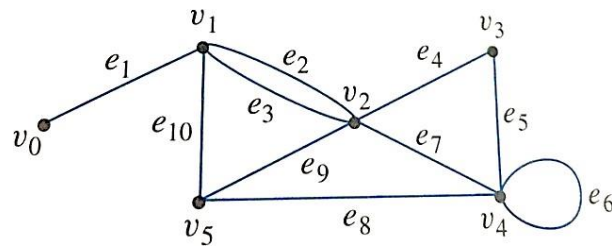
H_1 : 2 vertices has 1 degree
 1 vertex has 2 degree
 1 vertex has 3 degree
 1 vertex has 5 degree

H_2 : 1 vertex has 1 degree
 1 vertex has 2 degree
 3 vertices has 3 degree

H_1 and H_2 not having a same degree for corresponding vertices

Thus, H_1 and H_2 is not isomorphic

6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.

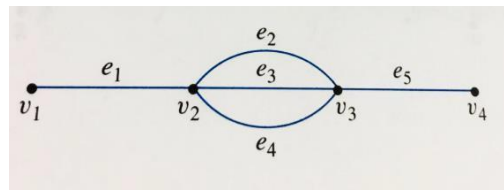


- a) $v_0 e_1 v_1 e_{10} v_5 e_9 v_2 e_2 v_1$
b) $v_4 e_7 v_2 e_9 v_5 e_{10} v_1 e_3 v_2 e_9 v_5$
c) v_2
d) $v_5 e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4 e_8 v_5$
e) $v_2 e_4 v_3 e_5 v_4 e_8 v_5 e_9 v_2 e_7 v_4 e_5 v_3 e_4 v_2$
f) $v_3 e_5 v_4 e_8 v_5 e_{10} v_1 e_3 v_2$

SOLUTION:

- a) trail
b) walk
c) closed walk
d) closed walk
e) closed walk
f) path

7. Consider the following graph.



- a) How many paths are there from v_1 to v_4 ?
- b) How many trails are there from v_1 to v_4 ?
- c) How many walks are there from v_1 to v_4 ?

SOLUTION:

- a) $(v_1, e_1, v_2, e_2, v_3, e_5, v_4)$
 $(v_1, e_1, v_2, e_3, v_3, e_5, v_4)$
 $(v_1, e_1, v_2, e_4, v_3, e_5, v_4)$

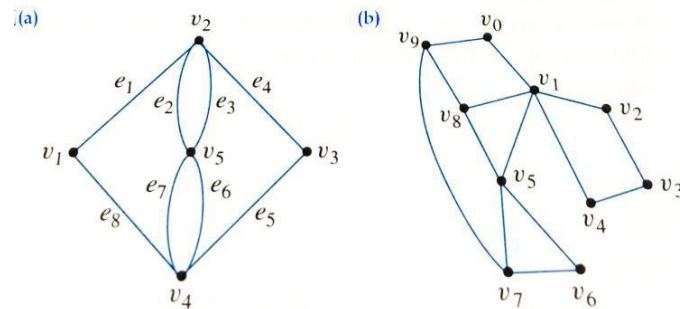
3 paths

- b) $3! + 3 = 9$

- c) As the walk may have repeated edges, a walk from v_1 to v_4 may contain an arbitrarily large number of edges joining a pair of vertices along the way.

Thus, the number of walks from v_1 to v_4 is infinite.

8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



SOLUTION:

a)

Vertex	V1	V2	V3	V4	V5
Degree	2	4	2	4	4

a is Euler circuit as its has connected graph and every vertex has even degree
(v2, e4, v5, e6, v4, e7, v5, e2, v2, e1, v1, e8, v4, e5, v3, e4, v2)

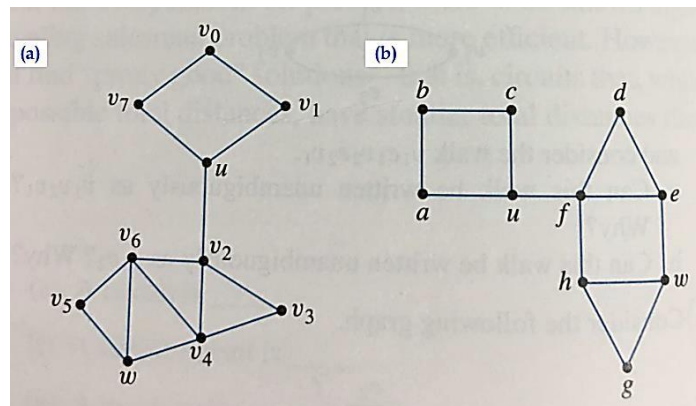
Have Euler Circuit because if it starts at v2 and ends at v2 which at the same vertex, uses every vertex of a at least once and uses every edge of a exactly once.

b)

Vertex	V0	V1	V2	V3	V4	V5	V6	V7	V8	V9
Degree	2	4	2	2	2	4	2	3	3	3

b is not Euler Circuit as not all vertex has even degree which is v7, v8 and v9 has odd degree

9. For each of graph in (a) – (b), determine whether there is an Euler path from u to w . If there is, find such a path.



SOLUTION:

a)

Vertex	u	V0	V1	V2	V3	V4	V5	V6	V7	w
Degree	3	2	2	4	2	4	2	4	2	3

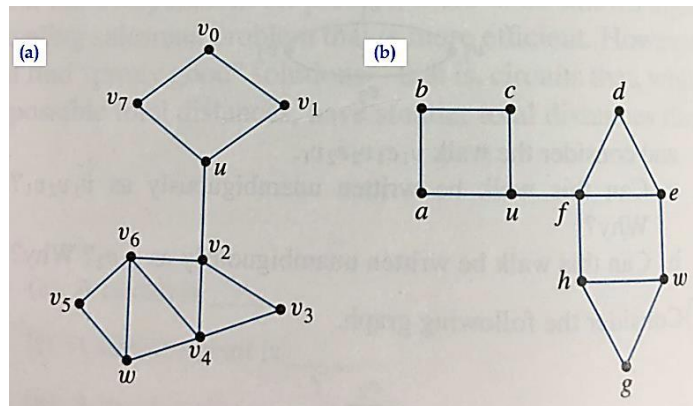
a is Euler path as u and w are the only vertices having odd degree

b)

Vertex	u	a	b	c	d	e	f	G	h	w
Degree	3	2	2	2	2	4	4	2	3	3

b is not Euler path as not only u and w have the odd degree but h also have odd degree

10. For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.



SOLUTION:

- a) Not having Hamiltonian Circuit as it cannot have hamilton circuit because there is one vertex that must have at least go through twice to go through all the vertex.

For example, (v5, v6, v2, u, v7, v0, v1, u, v2, v3, v4, w, v5)

- b) Not having Hamiltonian Circuit as it cannot have hamilton circuit because there is one vertex that must have at least go through twice to go through all the vertex.

For example, (a, b, c, u, f, d, e, w, g, h, f, u, a)

11. How many leaves does a full 3-ary tree with 100 vertices have?

SOLUTION:

let $m = 3, n = 100, l = ?$

since it is a full m -ary tree, we can use the following formula:

$$n = \frac{ml - 1}{m - 1}$$

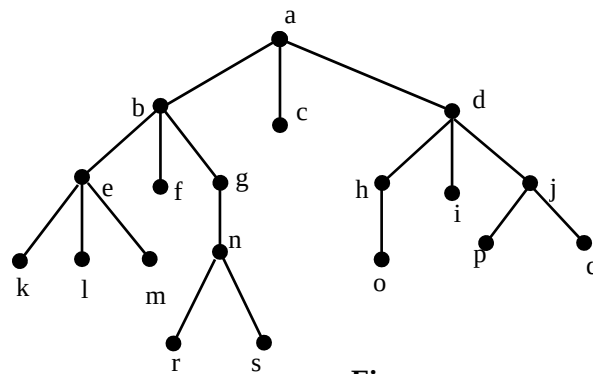
$$100 = \frac{3(l) - 1}{3 - 1}$$

$$200 = 3(l) - 1$$

$$3(l) = 201$$

$$l = 67 \text{ leaves}$$

12. Find the following vertex/vertices in the rooted tree illustrated below.

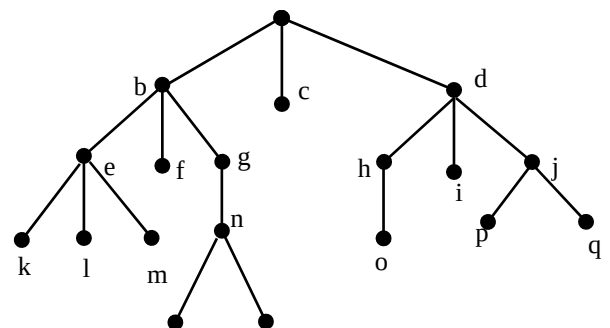


Figure

- a) Root
- b) Internal vertices
- c) Leaves
- d) Children of n
- e) Parent of e
- f) Siblings of k
- g) Proper ancestors of q
- h) Proper descendants of b

SOLUTION:

- a) Root = a
- b) Internal vertices = b, e, g, n, d, h, j
- c) Leaves = $k, l, m, r, s, c, o, i, p, q$
- d) Children of n = r, s
- e) Parent of e = b
- f) Siblings of k = l, m
- g) Proper ancestors of q = j, d, a
- h) Proper descendants of b = $e, f, g, k, l, m, n, r, s$



13. In which order are the vertices of ordered rooted tree in **Figure 1** is visited using *preorder*, *inorder* and *postorder*.

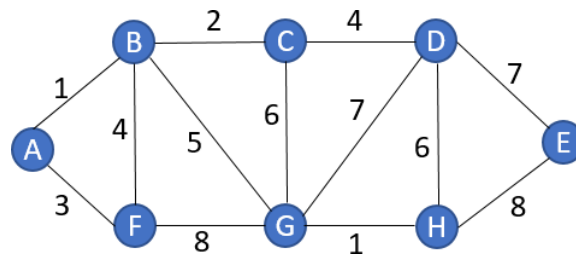
SOLUTION:

Preorder: a,b,e,k,l,m,f,g,n,r,s,c,d,h,o,i,j,p,q

Inorder: k,e,l,m,b,f,r,n,s,g,a,c,o,h,d,i,p,q,j

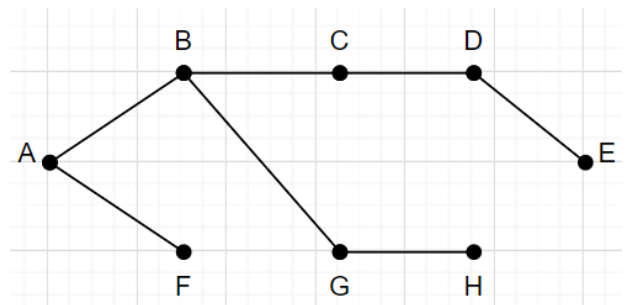
Postorder: k,l,m,e,f,r,s,n,g,b,c,o,h,i,p,q,j,d,a

14. Find the minimum spanning tree for the following graph using Kruskal's algorithm.



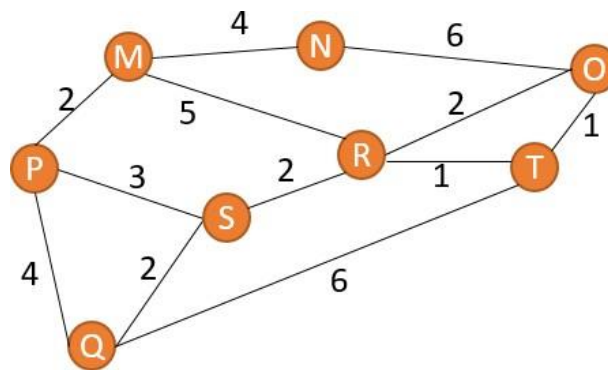
SOLUTION:

AB	1
GH	1
BC	2
AF	3
CD	4
FB	4
BG	5
GC	6
DH	6
GD	7
DE	7
HE	8
FG	8



$$\text{Total} = 1+3+5+1+2+4+7 \\ = 23$$

15. Use Dijkstra's algorithm to find the shortest path from **M** to **T** for the following graph.



SOLUTION:

Iteration	S	N	L(M)	L(P)	L(N)	L(R)	L(S)	L(O)	L(Q)	L(T)
0	{}	{M,P,N,R,S,O,Q,T}	0	∞	∞	∞	∞	∞	∞	∞
1	{ M }	{P,N,R,S,O,Q,T}	0	2	4	5	∞	∞	∞	∞
2	{M,P}	{N,R,S,O,Q,T}	0	2	4	5	5	∞	6	∞
3	{M,P,N}	{R,S,O,Q,T}	0	2	4	5	5	10	6	∞
4	{M,P,N, R }	{S,O,Q,T}	0	2	4	5	5	7	6	6
5	{M,P,N,R,S}	{O,Q,T}	0	2	4	5	5	7	6	6
6	{M,P,N,R,S,Q}	{O,T}	0	2	4	5	5	7	6	6
7	{M,P,N,R,S,Q, T }	{O}	0	2	4	5	5	7	6	6

Shortest path $M \rightarrow R \rightarrow T$

Shortest length = 6