SCHOOL OF COMPUTING

SEMESTER I 2020/2021

SECI 1013 – DISCRETE STRUCTURE SECTION-08

ASSIGNMENT 3

NAMA	NO MATRIK
KHAIRUL IZZAT BIN HASHIM	**A20EC0058**
AIMAN NA'IM BIN ARIFFIN	**A20EC0008**
BRENDAN DYLAN GAMPA ANAK JOSEPH DUSIT	**A20EC0021**

LECTURER: DR NUR AZIZAH ALI

DATE OF SUBMISSION: 7th January 2021

QUESTION 1 **[25 marks]**

a) Let $A=\{1, 2, 3, 4, 5, 6, 7, 8\}$, $B=\{2, 5, 9\}$, and $C=\{a, b\}$. Find each of the following: (9 marks)

i. $A–B$
ii. $(A\cap B)\cup C$
iii. $A\cap B\cap C$
iv. $B\times C$
v. $P(C)$

ANSWER:

i. $A\text{-}B = \{1,3,4,6,7,8\}$
ii. $(A\cap B) = \{2,5\}$
$(A\cap B)\cup C = \{2,5,a,b\}$
iii. $A\cap B\cap C = \emptyset$
iv. $B\times C = \{(2,a),(2,b),(5,a),(5,b),(9,a),(9,b)\}$
v. $P(C) = \{ \emptyset, \{a\}, \{b\}, \{a,b\}\}$

b) By referring to the properties of set operations, show that: (4 marks)

$(P\cap ((P'\cup Q)')) \cup (P\cap Q) = P$

ANSWER:

$(P\cap ((P'\cup Q)')) \cup (P\cap Q) = P$

$\big(P \cap (P'' \cap Q')\big) \cup (P \cap Q)$ - De Morgan Law

$\big(P \cap (P \cap Q')\big) \cup (P \cap Q)$ - Double complement Law

$((P \cap P) \cap Q') \cup (P \cap Q)$ - Associative Law

$(P \cap Q') \cup (P \cap Q)$ - Idempotent Law

$P \cap (Q' \cup Q)$ - Distributive Law

$P \cap S$ - Universal Set

$P = P$

c) Construct the truth table for, **A** = $(\neg p \vee q) \leftrightarrow (q \rightarrow p)$. (4 marks)

ANSWER:

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$q \rightarrow p$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
0	0	1	1	1	1	1
0	1	1	0	1	0	0
1	0	0	1	0	1	0
1	1	0	0	1	1	1

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then $(x+2)^2$ is odd" (4 marks)

ANSWER:

$$\forall x\ (P(x) \rightarrow Q((x+2)^2))$$

$P(x)$ is odd integer.
$Q((x+2)^2)$ is odd integer

Let,
$a = 2k + 1$
$a + 2 = 2k + 1 + 2$
$(a+2)^2 = (2k+3)^2$
$(a+2)^2 = 4k^2 + 12k + 9$
$(a+2)^2 = 2\left(2k^2 + 6k + \frac{9}{2}\right)$
Let, $t = 2k^2 + 6k + \frac{9}{2}$
$(a+2)^2 = 2t$
Thus, it is even. So,

$P(x)$ is odd integer.
$Q((x+2)^2)$ is not odd integer.

e) Let $P(x,y)$ be the propositional function $x \geq y$. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\exists x\, \exists y\; P(x, y)$ (4 marks)

ii. $\forall x\, \forall y\; P(x, y)$

ANSWER:

i. $\exists x\, \exists y\; P\,(x, y)$

assume,
x = 1
y = 2

1 ≥ 2 - not true

Assume,
x = 2
y=2
2 ≥ 2 – true

Assume,
X= 3
Y=2
3 ≥ 2 – true

It shows that only some of x is more than equal to y.
Thus, the statement is TRUE.

ii. $\forall x\, \forall y\; P\,(x, y)$

Assume
x= 1
y=2
1< 2 - not true
Its shows that not all x is more than all y.

Thus, the statement is FALSE.

QUESTION 2 **[25 marks]**

a) Suppose that the matrix of relation R on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ relative to the ordering 1, 2, 3. (7 marks)

i. Find the domain and the range of *R*.

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

ANSWER:

R = {(1,1),(1,3),(2,1),(2,2)}

a) Domain = {1,2}
Range = {1,2,3}

b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Since (1,1) and (2,2) ∈ R and only (3,3) ∈ $\cancel{R}$, thus R is not irreflexive.

Since (1,2) ∈ R and (2,1) ∈ R, thus R is not antisymmetric.

b) Let $S=\{(x,y) \mid x+y \geq 9\}$ is a relation on $X=\{2, 3, 4, 5\}$. Find: (6 marks)

i. The elements of the set *S*.

ii. Is *S* reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.

ANSWER:

i. S = { {4,5},{5,4},{5,5}}
ii. Not reflexive = The diagonal matrix of set S is not all 1's.
Symmetric = Since there's $(4{,}5) \in S$, $(5{,}4) \in S$ also exist.
Not transitive = The matrix product of set S does not produce the same value.

Thus, it is not equivalence relation since it is not reflexive and transitive.

c) Let $X=\{1, 2, 3\}$, $Y=\{1, 2, 3, 4\}$, and $Z=\{1, 2\}$. (6 marks)

i. Define a function $f: X \rightarrow Y$ that is one-to-one but not onto.

ii. Define a function $g: X \rightarrow Z$ that is onto but not one-to-one.

iii. Define a function $h: X \rightarrow X$ that is neither one-to-one nor onto.

ANSWER:

i. $f: X \rightarrow Y = \{(1,1), (2,2), (3,3)\}$
ii. $f: X \rightarrow Z = \{(1,1), (2,1), (3,2)\}$
iii. $f: X \rightarrow X = \{(1,3), (2,2), (3,1)\}$

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

$m(x) = 4x+3$, $n(x) = 2x-4$ (6 marks)

i. Find the inverse of m.

ANSWER:

$m(x) = 4x + 3$
$let, y = 4x + 3$

$y - 3 = 4x$

$$\frac{y-3}{4} = x$$

$$m^{-1}(x) = \frac{x-3}{4}$$

ii. Find the compositions of n o m.

ANSWER:

$n\big(m(x)\big) = 2(4x + 3) - 4$
$n\big(m(x)\big) = 8x + 6 - 4$
$n\big(m(x)\big) = 8x + 2$

QUESTION 3 **[15 marks]**

a.) Given the recursively defined sequence.

$$a_k = a_{k-1} + 2k, \text{ for all integers } k \geq 2, \quad a_1 = 1$$

i) Find the first three terms. (2 marks)

ii) Write the recursive algorithm. (5 marks)

ANSWER:

i. First 3 terms

$$\begin{aligned} a_2 &= a_1 + 2k \\ &= 1 + 2(2) \\ &= 5 \end{aligned}$$

$$\begin{aligned} a_3 &= a_2 + 2k \\ &= 5 + 2(3) \\ &= 11 \end{aligned}$$

$\therefore$ 1,5,11

ii.
```
input = k
output = a(k)

a(k)
{
   if (k==2)
      return 5
   return a(k-1) + 2k
}
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k–1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for r_1, r_2, …. r_k. (4 marks)

ANSWER:

$$r_k = 2r_{k-1} \ , k \geq 2 \ , r_{k1} = 7$$

c) Given the recursive algorithm:

Input: *n*

Output: *S* (*n*)

S(*n*) {

if (*n*=1)

return 5

return 5**S*(*n*–1)

}

Trace *S*(4). (4 marks)

ANSWER:

S(4)

n = 4
because n ≠ 1
return 5*S(3)

S(4) = 625
return 5*125 = 625

S(3)

n = 3
because n ≠ 1
return 5*S(2)

S(3) = 125
return 5*25 = 125

S(2)

n = 2
because n ≠ 1
return 5*S(1)

S(2) = 25
return 5*5 = 25

S(1)

n = 1
because n = 1
return 5

S(1) = 5
return 5

S(4) = 625

QUESTION 4 **[25 marks]**

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

ANSWER:

Each of the 4 digits long will be divided into 4 tasks.

1^{st} task 2^{nd} task 3^{rd} task 4^{th} task

1^{st} task for first digit
0 1 2 3 4 5 6 7 8 9 A B C D E F
=9 ways

2^{nd} task for second digit
0 1 2 3 4 5 6 7 8 9 A B C D E F
=16 ways

3^{rd} task for third digit
0 1 2 3 4 5 6 7 8 9 A B C D E F
=16 ways

4^{th} task for fourth digit
0 1 2 3 4 5 6 7 8 9 A B C D E F
=11 ways

$9 \times 16 \times 16 \times 11$ = 25344 ways

(4 marks)

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

(4 marks)

ANSWER:

$$\begin{aligned} Letters &= 1 \times 26 \times 26 \times 26 \\ &= 17576 \end{aligned}$$

$$\begin{aligned} Numbers &= 9 \times 10 \times 1 \\ &= 90 \end{aligned}$$

$$Total = 17576 \times 90 = 1581840 \ license\ plates$$

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)? (5 marks)

ANSWER:

1 letter = 8 ways
2 letter = 8 x 7 = 56 ways
3 letter = 8 x 7 x 6 = 336 ways

Total ways= 8 + 56 + 336 = 400 ways

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

(4 marks)

ANSWER:

$$Girls = C(7,4) = \frac{7!}{4!\,(7-4)!} = 35$$

$$Boys\ = C(6,3) = \frac{6!}{3!\,(6-3)!} = 20$$

$$Total\ = 35 \times 20 = 700\ groups\ of\ seven$$

e) How many distinguishable ways can the letters of the word *PROBABILITY* be arranged? (4 marks)

ANSWER:

$$\frac{11!}{(2!)(2!)} = \frac{39\ 916\ 800}{4} = 9\ 979\ 200$$

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

(4 marks)

ANSWER

6 kinds of pastry. Pick 10.
n = 6, r = 10

$$C(n+r-1,r) = \frac{(n+r-1)!}{r!\,(n-1)!}$$

$$C(6+10-1,10) = \frac{(6+10-1)!}{10!\,(6-1)!}$$

$$C(15,10) = \frac{(15)!}{10!\,(5)!}$$

$C(15,10) = 3003$ ways

QUESTION 5 **[10 marks]**

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names. (4 marks)

ANSWER:

Pigeon (n) = 18
Pigeonholes (k) = Combination of first and last name
= 3 x 2 = 6

$$\left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{18}{6} \right\rceil = \lceil 3 \rceil$$

At least 3 people with the same first and last name

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

(3 marks)

ANSWER:

2,4,6,8,10,12,14,16,18,20 – 10 even integers
1,3,5,7,9,11,13,15,17,19 – 10 odd integers

Therefore, in total, we need to pick at least 10+1 =11 integers from 1 through 20 to be sure of getting at least one that is odd.

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

(3 marks)

ANSWER:

$let\ N = numbers\ divisible\ by\ 5$

$N = \{5,10,15,20,25,30,35,40,45,50,55,60,65,70,75,80,85,90,95,100\}$

$|N| = 20$

Since there are 20 integers that are divisible by 5, thus 100-20 = 80 is not divisible by 5. If there is at least one integer selected is divisible by 5, then at least 81 integers must be selected.

Answer = 81 integers