

DISCRETE STRUCTURE (SECI 1013)

TUTORIAL 1

Due date : 24th Nov 2020

1. Let the universal set be the set $\mathbf{R}$ of all real numbers and let $A = \{x \in \mathbf{R} \mid 0 < x \leq 2\}$, $B = \{x \in \mathbf{R} \mid 1 \leq x < 4\}$ and $C = \{x \in \mathbf{R} \mid 3 \leq x < 9\}$. Find each of the following:

- a) $A \cup C$
- b) $(A \cup B)'$
- c) $A' \cup B'$

2. Draw Venn diagrams to describe sets A , B , and C that satisfy the given conditions.

- a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$
- b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$
- c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subseteq B$, $C \not\subseteq B$

3. Given two relations S and T from A to B ,

$$S \cap T = \{(x, y) \in A \times B \mid (x, y) \in S \text{ and } (x, y) \in T\}$$

$$S \cup T = \{(x, y) \in A \times B \mid (x, y) \in S \text{ or } (x, y) \in T\}$$

Let $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$ and defined binary relations S and T from A to B as follows:

$$\text{For all } (x, y) \in A \times B, \quad x S y \leftrightarrow |x| = |y|$$

$$\text{For all } (x, y) \in A \times B, \quad x T y \leftrightarrow x - y \text{ is even}$$

State explicitly which ordered pairs are in $A \times B$, S , T , $S \cap T$, and $S \cup T$.

4. Show that $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.
5. $R_1 = \{(x, y) \mid x + y \leq 6\}$; R_1 is from X to Y ; $R_2 = \{(y, z) \mid y > z\}$; R_2 is from Y to Z ; ordering of X , Y , and Z : 1, 2, 3, 4, 5.

Find:

- a) The matrix A_1 of the relation R_1 (relative to the given orderings)
- b) The matrix A_2 of the relation R_2 (relative to the given orderings)
- c) Is R_1 reflexive, symmetric, transitive, and/or an equivalence relation?
- d) Is R_2 reflexive, antisymmetric, transitive, and/or a partial order relation?

6. Suppose that the matrix of relation R_1 on $\{1, 2, 3\}$ is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R_2 on $\{1, 2, 3\}$ is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

- a) The matrix of relation $R_1 \cup R_2$
- b) The matrix of relation $R_1 \cap R_2$

7. If $f: \mathbf{R} \rightarrow \mathbf{R}$ and $g: \mathbf{R} \rightarrow \mathbf{R}$ are both one-to-one, is $f + g$ also one-to-one? Justify your answer.
8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase. Find a recurrence relation for $c_1, c_2, \dots, c_n$.
9. The Tribonacci sequence (t_n) is defined by the equations,

$$t_1 = t_2 = t_3 = 1, \quad t_n = t_{n-1} + t_{n-2} + t_{n-3} \quad \text{for all } n \geq 4.$$

- a) Find t_7 .
- b) Write a recursive algorithm to compute $t_n, n \geq 1$.