DISCRETE STRUCTURE SECI1013

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ASSIGNMENT 3

LECTURER'S NAME:

DR. NOR AZIZAH ALI

GROUP MEMBERS:

NAME	MATRIC NO
NUR IRDINA ALIAH BINTI ABDUL WAHAB	A20EC0115
YONG ZHI YAN	A20EC0172
MYZA NAZIFA BINTI NAZRY	A20EC0219

Question 1

a) Let A= {1, 2, 3, 4, 5, 6, 7, 8}, B= {2, 5, 9}, and C= {a, b}. Find each of the following:

i. $A-B$

$A-B = \{1, 3, 4, 6, 7, 8\}$

ii. $(A\cap B) \cup C$

$(A\cap B) \cup C = \{2, 5\} \cup \{a, b\}$

$= \{2, 5, a, b\}$

iii. $A\cap B\cap C$

$A\cap B\cap C = \{ \}$

iv. $B\times C$

$B\times C = \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$

v. $P(C)$

$P(C) = 2^2$

$= 4$

b) By referring to the properties of set operations, show that:

$(P\cap ((P'\cup Q)')) \cup (P\cap Q) = P$

$(P\cap ((P'\cup Q)')) \cup (P\cap Q)$	$= (P \cap P'' \cap Q') \cup (P \cap Q)$	De Morgan's Law
	$= (P \cap Q') \cup (P \cap Q)$	Double Complement Law & Idempotent law
	$= P \cap (Q' \cup Q)$	Distributive Law
	$= P \cap U$	Complement Law
	$= P$	Identity Law

c) Construct the truth table for, $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$.

p	q	$\neg p$	$\neg p \vee q$	$q \rightarrow p$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then $(x+2)^2$ is odd"

If $x = 3$, $(x + 2)^2 = 25$, so $(x + 2)^2$ is odd.

Let $P(x)$ = x is an odd integer, $Q(x) = (x + 2)^2$ is an odd integer

$\forall x\, (P(x) \rightarrow Q(x))$

Let a as an odd integer.

$a = 2n + 1$

$(a + 2)^2 = (2n + 1 + 2)^2$

$= (2n + 3)^2$

$= 4n^2 + 12n + 9$

$= 2(n^2 + 6n + 4) + 1$ where $m = n^2 + 6n + 4$, which is an integer

$= 2m + 1$

Hence, $(a + 2)^2$ is an odd integer.

e) Let $P(x, y)$ be the propositional function $x \geq y$. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\exists x\, \exists y\, P(x, y)$

True for only $x \geq 1$ or $0 \leq y \leq 1$

ii. $\forall x\, \forall y\, P(x, y)$

False for $x = 0$ or $y \geq 1$

Question 2

(a) Suppose that the matrix of relation R on {1,2,3}

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ Relative to the ordering 1,2,3

i- Find the domain and the range of R

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

R = {(1,1), (1,2), (2,2), (3,1)

Domain = {1,2,3}
Range = {1,2}

ii- Determine whether the relation is irreflexive and / or anti-symmetric. Justify your answer.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

No, it is not irreflexive because $\exists x \in A$ where (x, x) $\in R$. there is value 1 on the diagonal of the matrix

R = {(1,1), (2,1) (2,2), (1,3)}

It is antisymmetric because $\forall x, y \in A, (x, y) \in R \cap x \neq y \rightarrow (y, x) not\ belong\ to\ R$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

(1,2) $\in R$, (2,1) $not\ belong\ to\ R$

(3,1) $\in R$, (1,3) not belong to R

(b) Let S = {(x, y) | x + y >= 9} is a relation on X = {2,3,4,5} find

i. The element of set S

S= {(4,5), (5,4), (5,5)}

ii- Is S reflexive, symmetric, transitive and / or an equivalence relation? Justify your answer.

$$S = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array}$$

It is not reflexive because $\forall x , \epsilon\, S , (x, x) not\ belong\ to\ R$

(2,2), (3,3), (4,4) do not belong to R

The matrix on its diagonal have value 0

$$S^T = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array}$$

It is symmetric because $\forall x , y\ \epsilon\ S , (x, y) \epsilon\ R , (y, x) \epsilon\ R$

$M_{R=} M_R^T$

$$\begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array} \otimes \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array} = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array}$$

It is not transitive because $M_R\ x\ M_R \neq M_R$

S is not an equivalence relation since it is not reflexive, symmetric and not transitive.

(c) Let X = {1,2,3}, y = {1,2,3,4} and z = {1,2}

i- define a function f :X→Y that is both one to one but not onto

f = {(1,1), (2,2), (3,3)}

ii- Define a function g: X→ Z that is onto but not one to one

g= {(1,1), (2,2), (3,2)}

iii- define a function h: X→ X that is neither one to one nor onto

h= {(1,1), (2,1), (3,1)}

(d) Let m and n be functions from the positive integers to the positive integers defines by the equations:

m(x) = 4x +3, n(x) = 2x-4

i- Find the inverse of m

Let m(x) =y

4x+3 = y

4x = y-3 , X= $\frac{y-3}{4}$, $m^{-1}(x)$= $\frac{x-3}{4}$

ii- find the composition of n. m

n.$m = n(m(x))$

= n (4x+3)

= 2(4x+3) -4

= 8x +6-4

= 8x+2

Question 3

a) Given the recursively defined sequence.

$a_k = a_{k-1} + 2k$, for all integers $k \geq 2$, $a_1 = 1$

i) Find the first three terms.

$a_1 = 1$
$a_2 = 1 + 2(2)$
$= 5$
$a_3 = 5 + 2(3)$
$= 11$

ii) Write the recursive algorithm.

Input = k

Output = a(k)

```
a(k) {
    if (k = 1)
    return 1
    return a(k – 1) + a(2k)
    }
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k–1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for r1, r2, …. r_k.

$r_1 = 7$
$r_2 = 2r_1 = 2 \times 7$
$r_3 = 2r_2 = 2 \times (2 \times 7) = 2^2 \times 7$
.
.
.
$r_k = 2r_{k-1} = 2^{k-1} \times 7$

c) Given the recursive algorithm:

```
Input: n
Output: S (n)
S(n) {
    if (n=1)
        return 5

    return 5*S(n–1)

    }
```

Trace S(4).

S(1) = 5

S(2) = 5 x S(1)

= 5 x 5

= 25

S(3) = 5 x S(2)

= 5 x 25

= 125

S(4) = 5 x S(3)

= 5 x 125

= 625

Question 4

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

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First digit: 9 ways
Second digit: 16 ways
Third digit: 16 ways
Fourth digit: 11 ways

Number of ways = 9 x 16 x 16 x 11
= 25344

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

_ _ _ _ _ _ _

First letter = 1 way	first digit = 9 ways
Second letter = 26 ways	second digit = 10 ways
Third letter = 26 ways	third digit = 1 way
Fourth letter = 26 ways	

Number of ways = 1x 26 x 26 x 26 x 9 x 10 x1

= 1581840 ways

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?

When 1 letter = 8 arrangement

When 2 letters = 1^{st}; 8 arrangements, 2^{nd} = 7 arrangements

= 8 x 7 = 56 arrangement

When 3 letters = 1^{st} = 8 arrangements, 2^{nd} = 7 arrangements, 3^{rd} = 6 arrangements

= 8 x 7 x 6 = 336 arrangements

number of arrangements = 8 + 56 + 336

= 400 arrangements

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

number of ways = $C(7,4).C(\ 6,3)$ = 700 ways

e) How many distinguishable ways can the letters of the word *PROBABILITY* be arranged?

There are 11 letters

P- 1, R -1, O -1, B-2, A-1, I -2, L-1, t-1, Y-1

#Number of ways = $\frac{11!}{2!2!}$ = 9979200 ways.

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

n=6, r =10, n + r =6

C (15, 10) = $\frac{15!}{10!5!}$ = 3003 selections

Question 5

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

k = 3 x 2 = 6
n = 18

s = $\frac{n}{k}$
= $\frac{18}{6}$
= 3

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

Even numbers: 10
Odd numbers: 10

Let n be the number of integers picked in order to be sure getting at least one odd, hence n = 11 as there are only 10 even numbers.

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

Total number of integers: 100
Total number that is divisible by 5: 20
Total number that is not divisible by 5: 100 – 20 = 80

Let n = numbers of integers picked to be sure of getting at least one that is divisible by 5,
hence n = 81 as there are 80 integers not divisible by 5