DISCRETE STRUCTURE

SECI 1013 (04)

ASSIGNMENT 1

(Group Assignment)

Student Name	:	TAY JIA YI
Matric No	:	A20EC0158
Student Name	:	NUR SYAKIRAH BINTI MOHD SHUKRI
Matric No	:	A19EM0384
Student Name	:	TASIF AHMED
Matric No	:	A18CS4055
Lecturer's Name	:	DR. HASWADI BIN HASAN

DISCRETE STRUCTURE (SECI 1013)

TUTORIAL 3

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QUESTION 1 **[25 marks]**

a) Let A={1, 2, 3, 4, 5, 6, 7, 8}, B={2, 5, 9}, and C={a, b}. Find each of the following:
   i. $A–B$ (9 marks)
   ii. $(A \cap B) \cup C$
   iii. $A \cap B \cap C$
   iv. $B \times C$
   v. $P(C)$

a) $A = \{1,2,3,4,5,6,7,8\}$ $C = \{a, b\}$
$B = \{2,5,9\}$

i. $A - B$
$\{1,2,3,4,5,6,7,8\} - \{2,5,9\}$
$= \{1,3,4,6,7,8\}$

$A - B = \{1,3,4,6,7,8\}$ #

ii. $(A \cap B) \cup C$
$A \cap B = \{1,2,3,4,5,6,7,8\} \cap \{2,5,9\}$
$= \{2,5\}$

$(A \cap B) \cup C = \{2,5\} \cup \{a,b\}$ $(A \cap B) \cup C = \{a,b,2,5\}$ #
$= \{a,b,2,5\}$

iii. $A \cap B \cap C$
$A \cap B \cap C = \{1,2,3,4,5,6,7,8\} \cap \{2,5,9\} \cap \{a,b\}$
$= \emptyset$
$A \cap B \cap C = \emptyset$

iv $B \times C$
$B \times C = \{2,5,9\} \times \{a,b\}$
$= \{(2,a),(5,a),(9,a),(2,b),(5,b),(9,b)\}$

$B \times C = \{(2,a),(5,a),(9,a),(2,b),(5,b),(9,b)\}$ #

v. $P(C)$
$P(C) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$ #

b) By referring to the properties of set operations, show that: (4 marks)

$(P\cap ((P'\cup Q)')) \cup (P\cap Q) = P$

b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$ — Double complement law $(P')' = P$ ✓

$= (P \cap (P \cap Q')) \cup (P \cap Q)$ → De Morgan's law $(P' \cup Q)' = P \cap Q'$

$= ((P \cap P) \cap Q') \cup (P \cap Q)$ → Associative law $P \cap (P \cap Q') = (P \cap P) \cap Q'$

$= (P \cap Q') \cup (P \cap Q)$ → Idempotent law $P \cap P = P$

$= P \cap (P \cup Q')$ → Absorption law $P \cap (P \cup Q') = P$

$= P$ #

c) Construct the truth table for, **A** = $(\neg p \vee q) \leftrightarrow (q \rightarrow p)$. (4 marks)

c) $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$

p	q	$\neg p$	$(\neg p \vee q)$	$(q \rightarrow p)$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then $(x+2)^2$ is odd" (4 marks)

d) Proof

"For all integer x, if x is odd, then $(x+2)^2$ is odd"

x = odd
$= 1, 3, 5, 7$

$(x+2)^2$ odd?

$x = 1$
$= (1+2)^2$
$= 9$ ← odd ✓

$x = 3$
$= (3+2)^2$
$= 25$ ← odd

$x = 5$
$= (5+2)^2$
$= 49$ ← odd

∴ proven that if x is odd, $(x+2)^2$ also odd #

e) Let $P(x,y)$ be the propositional function $x \geq y$. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\exists x \, \exists y \, P(x, y)$ (4 marks)

ii. $\forall x \, \forall y \, P(x, y)$

e) $x \geq y$, all positive integers.

i $\exists x \exists y \; P(x,y)$

let $x = 2$ $y = 1$

$P(x,y) = 2 \geq 1$ True

ii $\forall x \forall y \; P(x,y)$

let $x = 5$ $y = 9$

$P(x,y) = 5 \geq 9$ False.

QUESTION 2 **[25 marks]**

a) Suppose that the matrix of relation R on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ relative to the ordering 1, 2, 3. (7 marks)

i. Find the domain and the range of R.

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

2a) $R = \begin{array}{cc} \begin{array}{ccc} 1 & 2 & 3 \end{array} & \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$

i) $R = \{(1,1), (1,2), (2,2), (3,1)\}$

domain $= \{1, 2, 3\}$

range $= \{1, 2\}$

ii) Matrix R is not irreflexive, as $(1,1), (2,2), \in R$, the main diagonal are not all 0's.

Matrix R is antisymmetric, as $(1,2) \in R$, $(2,1) \notin R$

$(1,3) \notin R$, $(3,1) \in R$.

b) Let $S=\{(x,y) \mid x+y \geq 9\}$ is a relation on $X=\{2, 3, 4, 5\}$. Find: (6 marks)

i. The elements of the set S.

ii. Is S reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.

bi) $S = \{(4,5), (5,4), (5,5)\}$

ii) $S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Matrix S is not reflexive, as $(2,2), (3,3), (4,4) \notin S$, it has value 0 and 1 on main diagonal.

Matrix S is symmetric, as $M_s = M_s^T$

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = M_s^T$$

Matrix S is not transitive, as $M_s \otimes M_s \neq M_s$.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

M_s transitive $\neq M_s$

$\therefore$ Therefore, S is not reflective, not transitive, but it is symmetric. S is not equivalence equation.

c) Let $X=\{1, 2, 3\}$, $Y=\{1, 2, 3, 4\}$, and $Z=\{1, 2\}$. (6 marks)

i. Define a function $f: X \to Y$ that is one-to-one but not onto.

ii. Define a function $g: X \to Z$ that is onto but not one-to-one.

iii. Define a function $h: X \to X$ that is neither one-to-one nor onto.

ci) $f: X \to Y$ is one-to-one, as each element Y has at most one arrow or pair, and it is not onto, as each element Y does not has at least one arrow or pair, for example, there are three elements in X and four elements in Y.

ii) $g: X \to Z$ is onto, as each element Z has at least one arrow or pair from element X, and it is not one-to-one, as each element Z does not have at most one arrow or pair, as the number of elements X are greater than number of elements Z.

iii) $h: X \to X$ is not one-to-one, as each element X has more than one arrow or pair, it is not onto, as each element X does not has at least one arrow or pair with element X, itself.

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

$m(x) = 4x+3$, $\quad n(x) = 2x-4$ (6 marks)

i. Find the inverse of m.

ii. Find the compositions of n o m.

di)
$$m(x) = 4x + 3$$
$$y = 4x + 3$$
$$x = \frac{y-3}{4}$$
$$m^{-1}(y) = x$$
$$m^{-1}(y) = \frac{y-3}{4}$$
$$m^{-1}(x) = \frac{x-3}{4}$$

ii)
$$n \circ m = (n \circ m)(x)$$
$$= n(m(x))$$
$$= 2(4x+3) - 4$$
$$= 8x + 6 - 4$$
$$= 8x + 2$$
$$= 2(4x+1)$$

QUESTION 3 **[15 marks]**

a) Given the recursively defined sequence.

$$a_k = a_{k-1} + 2k, \text{ for all integers } k \geq 2, \quad a_1 = 1$$

i) Find the first three terms. (2 marks)

ii) Write the recursive algorithm. (5 marks)

a) $a_k = a_{k-1} + 2k \quad , k \geq 2 \quad a_1 = 1$

i) first three terms.

$a_1 = 1$

$a_2 = 1 + 2(2) = 5$

$a_3 = 5 + 2(3) = 11$

ii) recursive algorithm

```
input : k
output : a(k)
a(k)
{
   a(1) = 1
   if (k >= 2)
      a(k) = a(k-1) + 2(k)
      return a(k)
}
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k–1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for r_1, r_2, …. r_k. (4 marks)

b)

$r_1 = 7$

$r_k = 2r_{k-1}$, $k \geq 2$

Let $P_k = r_{k+1}$

$P_0 = r_1 = 7$

$P_k = 2P_{k-1}$, $k \geq 1$

$r_k = 2r_{k-1} = r_1 = Zr_{k-1}$

$2r_{k-1} = Zr_{k-1}$

$2 = Z$

$P_0 = 7$

$P_k = r\ 2^k$

$= 7 \times 2^k$

For $r_k = P_{k-1}$

$r_k = P_{k-1}$

$= 7 \times 2^{k-1}$

$r_1 = 7$

$r_2 = 7 \times 2^{2-1} = 14$

$r_3 = 7 \times 2^{3-1} = 28$

c) Given the recursive algorithm:

Input: *n*

Output: *S* (*n*)

S(*n*) {

if (*n*=1)

return 5

return 5**S*(*n*–1)

}

Trace *S*(4). (4 marks)

c) S(4)

Input : n

Output S(n)

S(n) {

if (n = 1)

return 5

return 5*S(n-1)

}

S(1) = 5

S(2) = 5(5) = 25

S(3) = 5(25) = 125

S(4) = ?

- not valid first condition

- valid for second condition

S(4) = 5*S(3)

= 5 (125)

= 625

S(4) = 625 #

QUESTION 4 **[25 marks]**

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

(4 marks)

(a) Let;

MNOP is the desired 4 digits number.

Here,

M,N,O,P are four digits from (0 to F)

1st digit M : can contain 9 most possible values.

they are from (3 to B)

2nd digit N : } Both can hold 16 possible
3rd digit O : } values where no condition mentioned.

4th digit P : can take 11 most possible values and they are from 5 to F.

So; Total 4 digits numbers = 9X16X16X11
= 25344

[Ans]

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

(4 marks)

(b) Need to fix up 4 letters & 3 digits where A will be the 1st letter & end digit must be 0.

Total Alphabet numbers: 26

Total digits: (0-9) =10

Here,

$1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1$

$= 1757600$

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?

(5 marks)

(c). There are 8 letters in COMPUTER word. There is no repitation of any digits in this word.

1st single words = 8

2 letter words $= P(8,2) = 8 \times 7 = 56$

3 letter word $= P(8,3)$

$= 8 \times 7 \times 6$

$= 336$

∴ Total numbers of possibility = 8+56+336

$= 400$

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

(4 marks)

(d)

Number of ways to select 4 women

$$c(7,4) = \frac{7!}{4!(7-4)!} = 35$$

Number of ways to select 3 men

$$c(6,3) = \frac{6!}{3!(6-3)!} = 20$$

Number of ways = Number of ways to select 3 men X Number of ways to select 4 women.

$= 35 \times 20$

$= 700$

e) How many distinguishable ways can the letters of the word *PROBABILITY* be arranged?

(4 marks)

(e). Total letters in PROBABILITY = 11

I & B ~~occurred~~ repeat 2 times.

without repetition case we get

11 − 4 = 7 letters that used only

once, = P R O A L T Y

So, in this term distinguishable ways = 11!

So,

Total possible ways $= \frac{11!}{2! \times 2!}$

$= 9979200$

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

(4 marks)

(f). 1st Kind of pastries = P1

2nd Kind of pastries = P2

3rd Kind of pastries = P3

— — — — — — — — —

6th Kind of pastries = P6.

The number of selections are non-negative integer solutions to the equation = P1 + P2 + P3 + P4 + P5 + P6 = 10

We know,

number of solutions to $P_1 + P2 + P3 \ldots + PK = n$.

Here,

$n = 6,\ r = 10$

$C(6+10-1,\ 10) = C(15,\ 10)$

$= \dfrac{15!}{10!\ 5!}$

$= 3003.$

QUESTION 5 **[10 marks]**

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

(4 marks)

(a) Given that,

18 persons have (i) Ali, Bahar and Carlie as first name

(ii). Daud & Elyas last name

Need to show, at least 3 peoples first name and last name are same.

Here,

We can see question is related to specific person with specific names.

By using pigeonhole principle:

n pigeons are placed into r pigeon holes with ; then there exists a pigeonhole with atleast n/r pigeons.

Continue..

Let,

n = Number of persons have any combination = 18

r = Number of combinations of first name & last names combinations

$= 2 \times 3$

$= 6$ (Multiplicity Principle)

So; 6 different (first + second) name combinations.

Now, applying pigeonhole principle.

we get: $\frac{n}{r}$

$= \frac{18}{6}$

$= 3$ people, with same first & second name.

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

(3 marks)

(b) Number of odd & even integers from 1 to 20 is same.

So;

The possibility of picking an odd integer is for every pick = $1/2$

Then even if we pick 10 integer randomly then there is a chance that all of them could be even also.

Therefore we have picked 11 numbers to be sure that atleast 1 of them will be odd.

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

(3 marks)

(c) There are 80 integers from 1 to 100
which are non-divided by 5
So;
We must pick 81 integers to make it
sure that at least 1 of them is
divided by 5.