DISCRETE STRUCTURE

SECI 1013 (04)

ASSIGNMENT 2

(Group Assignment)

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Tutorial #2

SECI1013 – Discrete Structure

Chapter 3

DUE DATE : 27 DECEMBER, 2020

1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7
   a. How many numbers are there?
   b. How many numbers are there if the digits are distinct?
   c. How many numbers between 300 to 700 is only odd digits allow?

Ans:

a. 6P1 x 6P1 x 6P1 = 216

b. 6P3 = 120

c.
For digit starts from 3 and ends with 3, 5 & 7 = 1P1 x 6P1 x 3P1
For digit starts from 4 and ends with 3, 5 & 7 = 1P1 x 6P1 x 3P1
For digit starts from 5 and ends with 3, 5 & 7 = 1P1 x 6P1 x 3P1
For digit starts from 6 and ends with 3, 5 & 7 = 1P1 x 6P1 x 3P1

In conclusion,
Num = 1P1 x 6P1 x 3P1
= 18

Total num = 18 x 4
= 72

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table
   a. Men insist to sit next to each other
   b. The couple insisted to sit next to each other
   c. Men and women sit in alternate seat
   d. Before her friend left, Anita want to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

Ans:

a. Number of ways = (7 - 1)! x 4!

= 17280

b. Number of ways = (9 - 1)! x 2!

= 80640

c. Number of ways = (4 - 1)! x 4! x 2!

= 288

d. Number of ways = 11! X 2!

= 79833600

3. In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish
   a. If no ties
   b. Two sprinters tie
   c. Two group of two sprinters tie

Ans;

a. Number of ways = 5!
= 120

b. Number of ways = 5C2 x 4!
= 240

c. Number of ways = 5C2 x 3C2 x 3!
= 180

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose
   a. a dozen croissants?
   b. two dozen croissants with at least two of each kind?
   c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

Ans:

a. Dozen = 12

n = 6, r = 12

C(n+r-1, r)

= C(17, 12)

$$= \frac{17!}{12!\,(17-12)!}$$

= 6188

b. Two dozen = 12 x 2 = 24

Since at least 2 of each kind, 6 croissants x 2 = 12 croissants

For another 12,

n = 6, r = 12

C(n+r-1, r)

= C(17, 12)

$$= \frac{17!}{12!\,(17-12)!}$$

= 6188

c. Two dozen = 12 x 2 = 24

Since at least 5 of chocolate & almond, 24 - 8 = 16 croissants

n = 6, r = 16

C(n+r-1, r)

= C(21, 16)

$$= \frac{21!}{16!\,(21-16)!}$$

= 20349

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.
   a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?
   b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?
   c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Ans:

a. 2 win in 4 match, $C(4, 2) = \frac{4!}{2! \times 2!}$

1 win in 3 match, $C(3, 1) = 3$

For only 1 team win the round,

2 win & 1 tie/win = $2 \times [C(4, 2) \times C(3, 1)]$

$= 36$

1 win & 3 tie/win = $2^3 \times [C(4, 3) \times C(3, 1)]$

$= 96$

Num of scenarios = $2 \times (36 + 96)$

$= 264$

b. Prob of 10 penalty kick = 2^{10}

$= 1024$

Prob of game settled in first 10 kicks = 264, but game ends at second round

1024 – 264 = 760

For 1st unsettled round = 760

For 2nd unsettled round = 264

In conclusion, num = 760×264

$= 200640$

c. For 1st unsettled game = 760 scenarios

For 2nd unsettled game = 760 scenarios

Sudden death = 2 + 2 +2 + 2+ 2

= 10 scenarios

Num = 760 x 760 x 10

= 5776000

6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical?
(Assume that no answers are left blank.)

Ans:

N = total num of students (pigeon)
k = total num of possible sheets (pigeonhole)

$k = 4^{10}$
$\quad = 1048576$

Pigeonhole principle formula: $\left(\left\lceil \frac{N}{k} \right\rceil - 1\right)$

$$\left\lceil \frac{N-1}{1048576} \right\rceil + 1 = 3$$

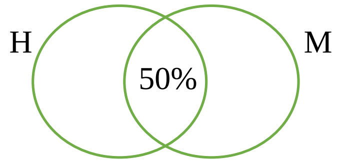
N – 1 = 2097152

N = 2097153

Minimum num of students = 2097153

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Ans:



H – History

M – Mathematics

% of student failed = 100% - (75% + 65% - 50%)

= 100% - 90%

= 10%

Number of candidates = 35 / 10% x 100%

= 350 candidates

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Ans:

The total number of possible outcomes= 780-299 = 481

So, we have 481 number of possible outcomes

Now, we must find 1 in 3 digits, 1 in 2 digits, 1 in 1 digit

1 in 3 digit:
301,310,311,312,313,314,315,316,317,318,319,321,331,341,351,361,371,381,391

1 in 4 digit:

401, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 431, 441, 451, 461, 471, 481, 491

1 in 5 digit:

501, 510, 511, 512, 513, 515, 515, 516, 517, 518, 519, 521, 531, 551, 551, 561, 571, 581, 591

1 in 6 digit:

601, 610,611, 612, 613, 616, 616, 616, 617, 618, 619, 621, 631, 661, 661, 661, 671, 681, 691

1 in 7 digit:

701, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 721, 731, 741, 751, 761, 771

For 1 in 3 digit until 1 in 6 digit, each has 19 outcomes, so,
19 x 4 = 76 outcomes

For 1 in 7 digit, there are only 17 outcomes

So,
Total num of successful outcomes = 76 + 17 = 93

Prob = 93/481
= 0.1933

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.

   a. In how many ways can the cars be parked in the parking lots?
   b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

Ans:

a. Number of ways = $\dfrac{10P6}{2! \times 4!}$

$= 3150$

b. Number of ways = $\dfrac{7!}{2! \times 4!}$

$= 105$

Prob = $\dfrac{\text{Num of ways car parked with empty lots next to each other}}{\text{Total num of ways car parked in parking lots}}$

$= \dfrac{105}{3150}$

$= \dfrac{1}{30}$

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4, 0.1 and 0.5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6, 0.8 and 1 respectively.

a. Find the probability the trainee receives the message.
b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email.

Ans:

a. Prob email = 0.4 x 0.6 = 0.24
Prob letter = 0.1 x 0.8 = 0.08
Prob handphone = 0.5 x 1 = 0.5

Prob trainee reveive message = 0.24 + 0.08 + 0.5
= 0.82

b. A = receive via email
B = trainee receive message

$$P(A|B) = \frac{P(A \text{ n } B)}{P(B)}$$

$$= \frac{0.4 \times 0.6}{0.82}$$

= 0.2927

11. In a recent News, it was reported that light trucks, which include SUV's , pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

Ans:

From question we are getting in total 4 events:

A = Light Truck

A' = Cars

B = Fatal Accident

B' = Non-fatal Accident

From the question we get:

$P\left(\frac{B}{A\prime}\right) = (20 \div 100000)$ $\qquad$ $P\left(\frac{B}{A}\right) = (25 \div 100000)$

P(A') = 1 - P(A) $\qquad$ P(A) = 0.4

= 1 – 0.4

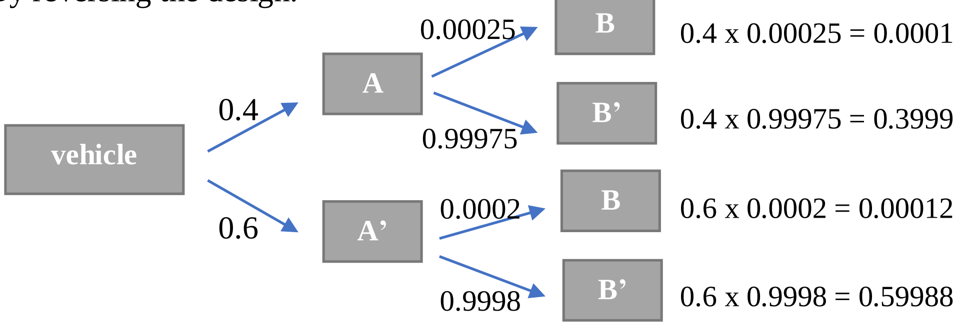
= 0.6

Considering $P\left(\frac{A}{B}\right)$ conditional probability of light truck involved accident given that it is fatal,

Using theory:

$$P\left(\frac{A}{B}\right) = \frac{P\left(\frac{B}{A}\right).\ P(A)}{P\left(\frac{B}{A}\right).\ P(A) + P\left(\frac{B}{A`}\right).\ P(A`)}$$

By reversing the design:



$$P\left(\frac{B}{A}\right) = \frac{(0.00025)\times(0.4)}{(0.00025)\times(0.4) + (0.00020)\times(0.6)}$$

= 0.4545

12. There are 9 letters having different colours (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

Ans:

Num of ways (without restrictions) $= 4^9$
$= 262144$

Num of ways put all letters into each shape twice $= 4 \text{ x } 1^9$
$= 4$

Num of disallowed ways $= 4 \text{ x } 2^9$
$= 2048$

In conclusion,
Num of ways = 262144 – 2048 + 4
= 260100