QUESTION 1 **[15 Marks]**

a) Draw a Venn diagram illustrating set $(A - B) \cap (C - B)$

Give that $A \cap B \cap C \neq \{\}$ (2 marks)

b) There are 20 people in your neighbourhood own pets. Five people own cats, rabbit and hamster. Three of them own only hamsters, five own only rabbit and another three own cat. How many total pets in your neighbourhood. (4 marks)

c) Let P and Q are set, prove that $((P \cup Q)' \cap Q')' = P \cup Q$ (4 marks)

d) Prove the following theorem using indirect proof method.

For all integers, if $a^2 - 3b$ is even then a is even and b is even (5 marks)

QUESTION 2 **[20 Marks]**

a) Write the following statement using *p*, *q*, *r* and logical connective

p : I go to the beach

q : it is a sunny summer day

r : it is Sunday

i) I go to the beach whenever it is Sunday and sunny summer day (2 marks)

ii) If it is not either Sunday or sunny summer day, then I do not go to the beach (2 marks)

iii) If I do not go to the beach, then it is not either Sunday or sunny summer day (2 marks)

iv) Which of these (i), (ii) and (iii) are equivalence statement using truth table. (2 marks)

b) Write the negation of $\forall x(x^2 + 2x - 3 = 0)$ and determine the resulting proposition is TRUE or FALSE with the domain of discourse is integer. (5 marks)

c) Express the following statement using predicates, quantifier and logical connective with the domain of discourse consist of all students at your school (7 marks)

i) There is a student at your school who can speak Russian but does not know C++

ii) Every student at your school either can speak Russian or knows C++

iii) No student at your school can speak Russian or knows C++

QUESTION 3 **[22 Marks]**

a) Let $A = \{1,3,5\}$. Define R on A by xRy if $3x + y$ is a multiple of 6.

i) Find the element of R. (3 marks)

ii) Draw the corresponding digraph. (2 marks)

iii) Determine the domain and range R (2 marks)

iv) Determine whether the relation R is irreflexive? (2 marks)

b) Consider the relation R on $B = \{a, b, c, d\}$ given by the matrix

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i) List in-degrees and out-degrees of all vertices. (4 marks)

ii) Detemine whether the relation R is reflexive, symmetric, antisymmetric and/ or transitive? Justify your answers. (4 marks)

iii) Is R a partial order? Why or why not? (2 marks)

c) Let $C = \{1, 2, 3, 4\}$. Find a relation R on C that has exactly 3 ordered pair members and is both symmetric and antisymmetric. (3 marks)

QUESTION 4 **[13 Marks]**

a) For each of the following mappings indicate what type of function they are (if any). Use the following key:

1. Not a function
2. A function which is onto but not one-to-one
3. A function which is one-to-one but not onto
4. A function which is bijection

i) The mapping f from $\mathbb{Z}$ *to* $\mathbb{Z}$ defined by $f(n) = -2(n)$ (2 marks)

ii) The mapping f from $\mathbb{Z}$ *to* $\mathbb{Z}$ defined by $f(n) = |n|$ (2 marks)

b) Find the inverse of $f(x) = \dfrac{x-5}{2x+1}$ (5 marks)

c) Functions f, g and h are given by

$f(x) = 2x + 3$ and $g(x) = -x^2 + 1$, $h(x)= 1/x$

Find the composite function defined by $h\circ(g\circ f)$ (4 marks)

QUESTION 5 **[15 Marks]**

a) Suppose that the number of bacteria in a colony triples every hour.

i) Set up a recurrence relation for the number of bacteria after n hours have elapsed. (2 marks)

ii) If 10 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours? (3 marks)

b) Find a recurrence relation for the balance B_k owed at the end of k months on a loan of RM9000 at rate of 8% if a payment of RM150 is made each month.

[Hint: Express B_k in terms of B_{k-1}; the monthly interest is $(0.08/12)\,B_{k-1}$]

(5 marks)

c) Write a recursive algorithm for computing n^2 where n is a nonnegative integer using the fact that $(n+1)^2 = n^2 + 2n + 1$. (5 marks)

QUESTION 6 **[10 Marks]**

(i) Ricky is going for a winter holiday and thinking of dressing warm for the winter. He will be layering three shirts over each other, and two pairs of socks. If he has twenty shirts to choose from, along with twelve different kinds of socks, how many ways can he layer up? (3 marks)

(ii) There are 4 red cups, 5 blue cups and 3 yellow cups. In how many ways can you arrange these so that each yellow cup is in between two blue cups? (4 marks)

(iii) A company has 7 software engineers and 5 civil engineers. In how many ways can they be seated in a row so that no two of the civil engineers will sit together? (3 marks)

QUESTION 7 **[15 Marks]**

(i) How many bit strings of length 8 either start with a 1 bit or end with the two bits 00? (4 marks)

(ii) There are 12 intermediate stations between two places A and B. Find the number of ways in which a train can be made to stop at 4 of these intermediate stations so that no two stopping stations are consecutive? (2 marks)

(iii) A cricket club in University of ABC is organizing a cricket tournament. Each team should plays one match with every other team.

(a) Find the number of matches in this tournament if there are 7 teams registered for the tournament. (2 marks)

(b) If the tournament should have 45 matches, find the number of teams that need to register in this tournament. (3 marks)

(iv) In a small village, there are 25 families, of which 18 families have at most 2 children. In a rural development programme 16 families are to be chosen for assistance, of which at least 14 families must have at most 2 children. In how many ways can the choice be made? (4 marks)