

Tutorial 2 (Chapter 3)

1. Consider 3-digit number whose digit are either 2, 3, 4, 5, 6 or 7.
- How many numbers are there?
 - How many numbers are there if the digits are distinct?
 - How many numbers between 300 to 700 is only odd digits allow?

2, 3, 4, 5, 6, 7 — 6 num

- a)
- | | | | |
|---|---|---|--|
| — | — | — | each digit can
be fill with 6 numbers |
| 6 | 6 | 6 | |
| ↓ | ↓ | ↓ | |
| 2 | 2 | 2 | |
| 3 | 3 | 3 | |
| 4 | 4 | 4 | |
| 5 | 5 | 5 | |
| 6 | 6 | 6 | |
| 7 | 7 | 7 | |

$$6 \times 6 \times 6 = 216$$

- b)
- | | | | |
|---|---|---|----------------------------------|
| — | — | — | each digit cannot be
repeated |
| 6 | 5 | 4 | |

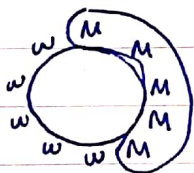
$$6 \times 5 \times 4 = 120$$

- c)
- | | | |
|---|---|---|
| — | — | — |
| 3 | 2 | 3 |
| 4 | 3 | 5 |
| 5 | 4 | 7 |
| 6 | 5 | |
| | 6 | |
| | 7 | |

$$4 \times 6 \times 3 = 72$$

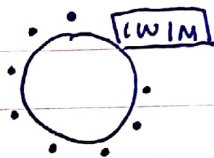
2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table
- Men insist to sit next to each other
 - The couple insisted to sit next to each other
 - Men and women sit in alternate seat
 - Before her friend left, Anita want to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other.

- a. 5 men and 5 women

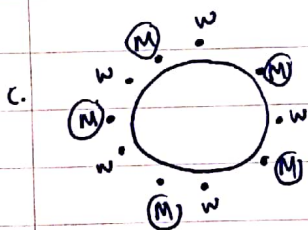


$$\begin{aligned}
 & (6-1)! \times 5! \\
 & = 5! \times 5! \\
 & = 14400 \text{ ways}
 \end{aligned}$$

- b. couple sit next to each other



$$\begin{aligned}
 & (9-1)! \times 2! \\
 & = 8! \times 2! \\
 & = 80640 \text{ ways}
 \end{aligned}$$



$$\begin{aligned}
 & (5-1)! \times 5! \\
 & = 4! \times 5! \\
 & = 2880 \text{ ways}
 \end{aligned}$$

- d. Anita + Husband 6 6 6 6 6 6 6 6 6 6

$$11! \times 2! = 79833600 \text{ ways}$$

Subject

3. In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for sprinter to finish

- If no ties
- Two sprinters tie
- Two group of two sprinters tie

a) $\begin{array}{ccccc} _ & _ & _ & _ & _ \\ 5 & 4 & 3 & 2 & 1 \end{array} = 5! = 120 \text{ ways}$

b) $\begin{array}{cccc} _ & _ & _ & _ \\ 2 & & & \end{array} \quad \begin{array}{l} n=5, r=2 \\ {}^nC_r \\ {}^5C_2 = 10 \\ 10 \times 4! \\ = 240 \text{ ways} \end{array}$

c) $\begin{array}{ccc} _ & _ & _ \\ 2 & 2 & \end{array} \quad \begin{array}{l} {}^5C_2 \times {}^3C_2 \times 3! \\ = 180 \text{ ways} \end{array}$

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a. a dozen croissants?

b. two dozen croissants with at least two of each kind?

c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

a. $n = 6, r = 12$ (dozen)

$$C(n+r-1, r) = \binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$

$$C(6+12-1, 12) = \binom{6+12-1}{12} = \frac{(6+12-1)!}{12!(6-1)!}$$

$$= \frac{17!}{12! \times 5!}$$

$$= 6188 \text{ ways}$$

b. $n = 6, r = 12 \times 2 = 24$ (2 dozen)

$$r = 24 - (6 \times 2) = 12$$

*at least two of each kind

$$C(n+r-1, r) = \binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$

$$C(6+12-1, 12) = \binom{6+12-1}{12} = \frac{(6+12-1)!}{12!(6-1)!}$$

$$= \frac{17!}{12! \times 5!}$$

$$= 6188 \text{ ways}$$

Plain = 2

Cherry = 2

Chocolate = 2

Almond = 2

Apple = 2

Broccoli = 2

c) $n = 6$, $r = 2 \times 12 = 24$ (2 dozens)
 $24 - (5 + 3) =$ at least 5 chocolate
 $r = 16$ at least 3 almond

$$C(6+16-1, 16) = \frac{(6+16-1)!}{16! (6-1)!} = \frac{21!}{16! \times 5!}$$
$$= 20349 \text{ ways}$$