SCHOOL OF COMPUTING

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SECI 1013 – DISCRETE STRUCTURE SECTION-08

ASSIGNMENT 2

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1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7
   a. How many numbers are there?
   b. How many numbers are there if the digits are instinct?
   c. How many numbers between 300 to 700 is only odd digits allow?

SOLUTION

a. 6 numbers represented
   Therefore, there are 6 choices for each space in a 3-digit number.

$$6 \times 6 \times 6 = 216$$

b. If digits are instinct = no same numbers are being selected

$$6 \times 5 \times 4 = 120$$

c. Numbers between 300 to 700 + only odd digits
   1st digit place = 3,4,5,6,7
   2nd digit place = 2,3,4,5,6,7
   3rd digit place = 3,5,7

$$5 \times 6 \times 3 = 90$$

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table
   a. Men insist to sit next to each other
   b. The couple insisted to sit next to each other
   c. Men and women sit in alternate seat
   d. Before her friend left, Anita wants to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

   SOLUTION

   a. $(6-1)! \times 5! = 14400$

   b. $(9-1)! \times 2! = 80640$

   c. $(5-1)! \times 5! = 2880$

   d. $12\ people.$

   Task 1: Anita and Her husband are equal to 1 group.

   So, arrangement for Anita and her husband **is $\mathbf{2}$**!

   Task 2: arrangement for her friends (10 people) and 1 group (Anita always stand next to her husband) **is $\mathbf{11}$**!

   $\therefore 11!\ \times 2! = \ 79833600$

3. In a school sport day, five sprinters are competing in a 100-meter race. How many ways are there for the sprinter to finish?
   a. If no ties
   b. Two sprinters tie
   c. Two group of two sprinters tie

SOLUTION

a) If no ties,

   Arranging 5 different positions (5 place),
   Total ways = 5! = 5 x 4 x 3 x 2 x 1 = 120 ways

b) Two sprinters tie,

   Arranging 4 different positions (4 place,1 tie),
   4! = 4 x 3 x 2 x 1 = 24 ways

   choosing which two people will form a tie,
   C (5,2) = 10 ways

   Total ways = 24 x 10 = 240 ways

c) Two group of sprinters tie,

   Arranging 3 different positions (3 place,2 tie),
   3! = 3 x 2 x 1 = 6 ways

   Choosing 1st group tie,
   C (5,2) = 10 ways

   Choosing 2nd group tie,
   C (3,2) = 3 ways

   Total ways = 6 x 10 x 3 = 180 ways

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose?
   a. a dozen croissants?
   b. two dozen croissants with at least two of each kind?
   c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

SOLUTION

a) $n = 6, r = 12$
Means that we have to pick 12 croissants from 6 categories (repetitive allowed)

$$C(6 + 12 - 1,12) = C(17,12)$$

$$= \frac{17!}{12!\,(17 - 12)!}$$

$$= 6188$$

b) 2 dozens = 24 croissants (at least two of each kind)
Select two of each kind = 12 total
Therefore $n = 6, r = 12$

$$C(6 + 12 - 1,12) = C(17,12)$$

$$= \frac{17!}{12!\,(17 - 12)!}$$

$$= 6188$$

c) 24 croissants
At least 5 chocolate and 3 diamond
Remaining = $24 - 5 - 3 = 16$
Therefore $n = 6, r = 16$

$$C(6 + 16 - 1,16) = C(21,16)$$

$$= \frac{21!}{16!\,(21 - 16)!}$$

$$= 20349$$

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.
   a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?
   b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?
   c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

SOLUTION

a. 2 wins for 4 games C (4,2) = $\frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = 6$

   1 win for 3 games C (3,1) = $\frac{3!}{1!(3-1)!} = \frac{3!}{1!(2)!} = 3$

   2 wins and 1 ties/wins: $C\ (4,2) \cdot C(3,1) \cdot 2 = 36$

   1 win and 3 ties/wins : $C(3,1) \cdot C(4,3) \cdot 2^3 = 96$

   $2 \cdot (36 + 96) = 264$

b. $2^{10} = 1024$

   1024-264 = 760

   first round: 760

   second round: 264

   $760\ \times 264 = 200640$

c. first round: 760

   second round: 760

   sudden death: 2+2+2+2+2 = 10

   $760 \times 760\ \times 10 = 5776000$

6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a, b, c, d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical?
(Assume that no answers are left blank.)

SOLUTION

Assuming every student answer every question,

$m = 4^{10} = 1048576$

The minimum students in the examination to guarantee that at least three answer sheets are identical,

$n= km +1$

$n= 2(4^{10}) +1$

$n= 2(1048576) +1$

$n= 2097153$

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

SOLUTION

P(A) = 0.75 (passed in history)
P(A') = 0.25 (failed in history)
P(B) = 0.65 (passed in maths)
P(B') = 0.35 (failed in maths)

$P(A \cap B) = 0.50\ (pass\ both\ history\ and\ maths)$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.75 + 0.65 - 0.50 \\ &= 0.90(students\ who\ passed\ either\ history\ or\ maths) \end{aligned}$$

This means that $P(A' \cup B') = 0.10$ representing students who failed both subjects.

$$\frac{35}{0.10} = 350$$

The total students who sit for exam is 350 students.

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

SOLUTION

S = 780 – 299 = 481
three number for 1 in integer = 0
two number for 1 in integer = {311,411,511,611,711} =5

one number for 1 in integer, for 300 ~ 399 =
{301,310,312,313,314,315,316,317,318,319,321,331,341,351,361,371,381,391} =18

one number for 1 in integer, for 400 ~499
= {401,410,412,413,414,415,416,417,418,419,421,431,441,451,461,471,481,491} =18

one number for 1 in integer, for 500 ~599
= {501,510,512,513,514,515,516,517,518,519,521,531,541,551,561,571,581,591} =18

one number for 1 in integer, for 600 ~699
= {601,610,612,613,614,615,616,617,618,619,621,631,641,651,661,671,681,691} =18

one number for 1 in integer, for 700 ~780
= {701,710,712,713,714,715,716,717,718,719,721,731,741,751,761,771} =16

$$P = \frac{5 + 18 + 18 + 18 + 18 + 16}{481} = 0.1933$$

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.
   a. In how many ways can the cars be parked in the parking lots?
   b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

SOLUTION

a) $\dfrac{P\ (10,6)}{4!\ .\ 2!}$ = 3150 ways

b) $\dfrac{7!}{4!\ .\ 2!}$ = 105

P(A)= $\dfrac{105}{3150}$

P(A)= 0.033

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively

a. Find the probability the trainee receives the message
b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

SOLUTION

a. $P(T) = P(EC)P(TE) + P(LC)P(TL) + P(HC)P(TH)$

$= (0.4 \times 0.6) + (0.1 \times 0.8) + (0.5 \times 1.0)$

$= 0.82$

Where

P(T) = Trainee receives the messages

P(EC) = Coach sent through e-mail

P(TE) = Trainee receives through e-mail

P(LC) = Coach sent through letter

P(TL) = Trainee receives through letter

P(HC) = Coach sent through handphones

P(TH) = Trainee receives through handphones

b. Conditional probability

$$P(E|T) = \frac{P(T \cap E)}{P(T)}$$

$$= \frac{(0.4 \ \times 0.6)}{0.82}$$

$$= 0.29$$

11. In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

SOLUTION

A'- Cars

A –Light truck

B –Fatal Accident

B' - Not a Fatal Accident

Given

P(B|A') = 20/10000 and P(B|A) = 25/100000

P(A) = 0.4

In addition, we know A and A' are complementary events

P(A') =1-P(A)=0.6

Our goal is to compute the conditional probability of a Light truck accident given that it is fatal P(A|B).

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')} = \frac{(0.00025)(0.4)}{(0.00025)(0.4) + P(0.00020)(0.6)} = 45.45\%$$

12. There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

SOLUTION

Total numbers of letters = 9

Total number of boxes = 4

The number of possible ways,
$4^9 = 262144$

Two choices per letter,
$4 \times 3^9 = 78732$ disallowed ways.

to put all letters into one box twice,
$4 \times 1^9 = 4$ ways

The number of allowed assignments of letters to boxes,
$262144 - 78732 + 4 = 183416$ ways