SCHOOL OF COMPUTING

SEMESTER I 2020/2021

SECI 1013 – DISCRETE STRUCTURE SECTION-08

ASSIGNMENT 1

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DISCRETE STRUCTURE (SECI 1013)

2020/2021 – SEMESTER 1

ASSIGNMENT# 1

1. Let the universal set be the set **R** of all real numbers and let $A=\{x\in\mathbf{R} \mid 0 < x \leq 2\}$, $B=\{x\in\mathbf{R} \mid 1 \leq x < 4\}$ and $C=\{x\in\mathbf{R} \mid 3 \leq x < 9\}$. Find each of the following:

   a) $A \cup C$
   b) $(A \cup B)'$
   c) $A' \cup B'$

   Answer:

   A = { 1, 2}
   B = { 1,2,3}
   C = { 3,4,5,6,7,8}

   a) $\mathrm{A} \cup \mathrm{C} = \{1,2,3,4,5,6,7,8\}$

   b) $(\mathrm{A} \cup \mathrm{B})' = \{4,5,6,7,8\}$

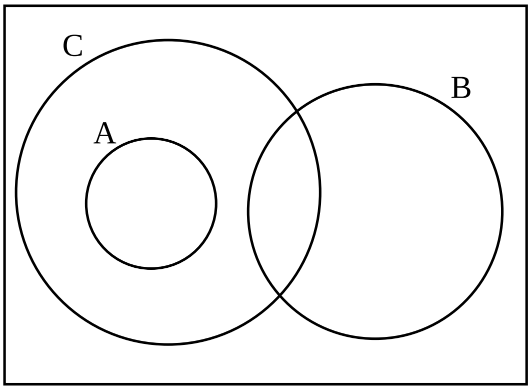
   c) $\mathrm{A}' = \{3,4,5,6,7,8\}$
   $\mathrm{B}' = \{4,5,6,7,8\}$
   $\mathrm{A}' \cup \mathrm{B}' = \{3,4,5,6,7,8\}$

2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

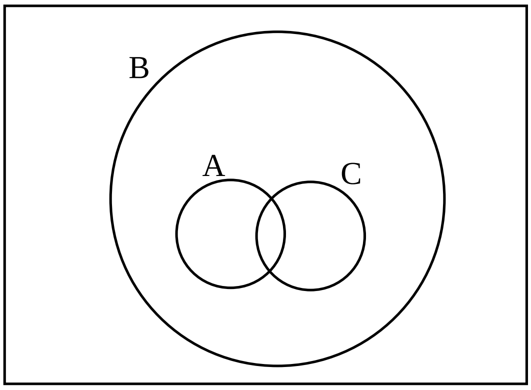
a) $A \cap B = \varnothing,\ A \subseteq C, C \cap B \neq \varnothing$
b) $A \subseteq B,\ C \subseteq B,\ A \cap C \neq \varnothing$
c) $A \cap B \neq \varnothing, B \cap C \neq \varnothing, A \cap C = \varnothing,\ A \not\subset B, C \not\subset B$

Answer:

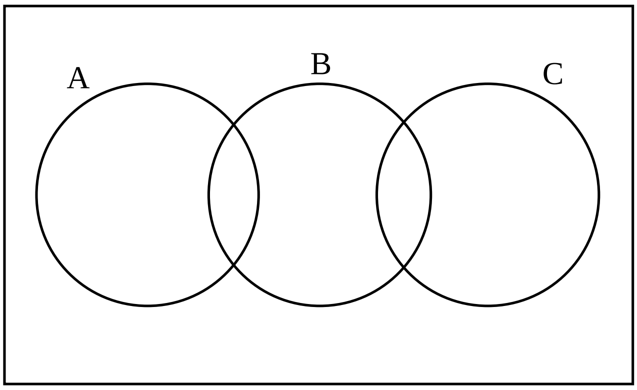
a) $A \cap B = \emptyset, A \subseteq C, C \cap B \neq \emptyset$



b) $A \subseteq B, C \subseteq B, A \cap C \neq \emptyset$



c) $A \cap B \neq \emptyset, B \cap C \neq \emptyset, A \cap C = \emptyset, A \not\subset B, C \not\subset B$



3. Given two relations S and T from A to B, $S \cap T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$

   $S \cup T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$

   Let $A=\{-1, 1, 2, 4\}$ and $B=\{1,2\}$ and defined binary relations S and T from A to B as follows:

   For all $(x,y) \in A\times B$, $\quad x\,S\,y \leftrightarrow |x| = |y|$

   For all $(x,y) \in A\times B$, $\quad x\,T\,y \leftrightarrow x - y$ is even

   State explicitly which ordered pairs are in $A\times B$, S, T, $S \cap T$, and $S \cup T$.

   Answer:

   Let $A = \{-1,1,2,4\}$ and $B = \{1,2\}$,

   Given the condition for both sets S and T are:

   For all $(x,y) \in A \times B$, $\quad x\,S\,y \leftrightarrow |x| = |y|$

   For all $(x,y) \in A \times B$, $\quad x\,T\,y \leftrightarrow x - y \; is\ even$

   Therefore,

   $A \times B = \{ (-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2) \}$

   $S = \{ (-1,1), (1,1), (2,2) \}$

   $T = \{ (-1,1), (1,1), (2,2), (4,2) \}$

   $S \cup T = \{ (-1,1), (1,1), (2,2), (4,2) \}$

   $S \cap T = \{ (-1,1), (1,1), (2,2) \}$

4. Show that $\neg ((\neg p\wedge q) \vee (\neg p\wedge\neg q)) \vee (p\wedge q) \equiv p$. State carefully which of the laws are used at each stage.

   Answer:

   $$\neg\big((\neg p \wedge q) \vee (\neg p \wedge \neg q)\big) \vee (p \wedge q) \equiv p$$

   $from\ LHS,$

   $\neg\big((\neg p \wedge q) \vee (\neg p \wedge \neg q)\big) \vee (p \wedge q)$

   $= \big(\neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q)\big) \vee (p \wedge q)$ De Morgan's Law

   $= \big((p \vee \neg q) \wedge (p \vee q)\big) \vee (p \wedge q)$ De Morgan's Law, Double Negation Law

   $= p \vee (\neg q \wedge q) \vee (p \wedge q)$ Distributive Law

   $= (p \vee \emptyset) \vee (p \wedge q)$ Compliment Law

   $= p \vee (p \wedge q)$ Absorption Law

   $= p$ (shown)

5. R_1={(x,y)| x+y ≤6}; R_1 is from X to Y; R_2={(y,z)| y>z}; R_2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5.

Find:

a) The matrix A_1 of the relation R_1 (relative to the given orderings)
b) The matrix A_2 of the relation R_2 (relative to the given orderings)
c) Is R_1 reflexive, symmetric, transitive, and/or an equivalence relation?
d) Is R_2 reflexive, antisymmetric, transitive, and/or a partial order relation?

Answer:

a) $R_1 = \left\{ \begin{matrix} (1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4), \\ (2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1) \end{matrix} \right\}.$

$$A_1 = \begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

b) $R_2 = \{ (5,1),(5,2),(5,3),(5,4),(4,1),(4,2),(4,3),(3,1),(3,2),(3,1),(2,1)\}$

$$A_1 = \begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

c) R_1 is not reflexive.
R_1 is symmetric.
R_1 is transitive.
R_1 is not equivalence relation.

d) R_2 is irreflexive.
R_2 is not antisymmetric
R_2 is not transitive.
R_2 is not partial order relation.

6. Suppose that the matrix of relation R_1 on {1, 2, 3} is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R_2 on {1, 2, 3} is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation $R_1 \cup R_2$
b) The matrix of relation $R_1 \cap R_2$

Answers:

$\boldsymbol{R_1 = \{(1,1),(2,2),(2,3),(3,1),(3,3)\}}$
$\boldsymbol{R_2 = \{(1,2),(2,2),(3,1),(3,3)\}}$

a. $R_1 \cup R_2 = \{(1,1),(1,2),(2,2),(2,3),(3,1),(3,3)\}$

$$R_1 \cup R_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

b. $R_1 \cap R_2 = \{(2,2),(3,1),(3,3)\}$

$$R_1 \cap R_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

7. If f :**R**→ **R** and g:**R**→ **R** are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

   Answer:

   $f(R) = R$
   $g(R) = R$

   $f(R) + g(R) = R + R = 2R$

   Let,
   $(f + g)(R_1) = (f + g)(R_2)$
   $2R_1 = 2R_2$
   $R_1 = R_2$

   ∴ this shows that $f + g$ is one-to-one.

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase. Find a recurrence relation for c_1, c_2, …., c_n.

   Answer:

   n = numbers of stairs
   c_n = number of different ways to climb the staircase

   When n = 1 only 1 stair can be taken to climb it
   $c_1 = 1$

   When n = 2 either take 2 stairs at once or take the stairs one by one
   $c_2 = 2$

   When n ≥3 we need to use combination of 1 stair and 2 stairs

   If we take 1 step: Cn-1 more ways to climb the stairs
   If we take 2 step: Cn-2 more ways to climb the stairs

   $c_n = c_{n-1} + c_{n-2}$ when $n \geq 3$

9. The Tribonacci sequence (t_n) is defined by the equations,
 $t_0 = 0$, $t_1 = t_2 = 1$, $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$.
   a) Find t_7.
   b) Write a recursive algorithm to compute t_n, $n \geq 3$.

Answer:

a) $t_3 = t_2 + t_1 + t_0 = 1 + 1 + 0 = 2$
   $t_4 = t_3 + t_2 + t_1 = 2 + 1 + 1 = 4$
   $t_5 = t_4 + t_3 + t_2 = 4 + 2 + 1 = 7$
   $t_6 = t_5 + t_4 + t_3 = 7 + 4 + 2 = 13$
   $t_7 = t_6 + t_5 + t_4 = 13 + 7 + 4 = 24$
   $t_7 = 24$

b)
```
Input n integer positive
Output t(n)
t(n)
{
   if ( n=0 or n=1 or n=2)
   return 1
   return t(n-1) + t(n-2) + t(n-3)
}
```