DISCRETE STRUCTURE (SECI1013)

TUTORIAL 1

GROUP 4: -

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1. Let the universal set be the set **R** of all real numbers and let A= {x∈**R** | 0 < x ≤ 2}, B= {x∈**R** | 1 ≤ x < 4} and C= {x∈**R** | 3 ≤ x < 9}. Find each of the following:

a) A ∪ C

b) (A ∪ B)′

c) A′ ∪ B′

1. A ∪ C

= {x ∈ R | 0 < x ≤ 2 or 3 ≤ x < 9}

= {x ∈ R | 0 < x < 9}

 **A ∪ C = {x ∈ R | 0 < x < 9}**

1. (A ∪ B) ′

(A ∪ B) = {x ∈ R | 0 < x ≤ 2 or 1 ≤ x < 4} = {x ∈ R | 0 < x < 4}

(A ∪ B) ′ = {x ∈ R | ¬ (0 < x < 4)} = {x ∈ R | x ≤ 0 or x ≥ 4}

**(A ∪ B) ′ = {x ∈ R | x ≤ 0 or x ≥ 4}**

1. A′ ∪ B′

A′ = {x ∈ R | ¬ (0 < x ≤ 2)}

B′ = {x ∈ R | ¬ (1 ≤ x < 4)}

 A′ ∪ B′ = {x ∈ R |¬ (0 < x ≤ 2) or ¬ (1 ≤ x < 4)} = {x ∈ R | x < 1 or x > 2}

 **A′ ∪ B′ = {x ∈ R | x < 1 or x > 2}**

2. Draw Venn diagrams to describe sets *A*, *B*, and *C* that satisfy the given conditions.

a) *A* ∩ *B* = ∅, *A* ⊆ *C*, *C* ∩ *B* ≠ ∅

 

 b) *A* ⊆ *B*, C ⊆ *B*, A ∩ *C* ≠ ∅

 c) *A* ∩ *B* ≠ ∅, *B* ∩ *C* ≠ ∅, *A* ∩ *C* = ∅, *A* ⊄ *B,* *C* ⊄ *B*

 

 3.Given two relations *S* and *T* from *A* to *B*,

*S* ∩ *T* = {(*x,y*) ∈*A*×*B* | (*x,y*) ∈ *S* and (*x,y*) ∈ T}

*S* ∪ *T* = {(*x,y*) ∈*A*×*B* | (*x,y*) ∈ *S* or (*x,y*) ∈ T}

Let *A*= {*−*1, 1, 2, 4} and *B*= {1,2} and defined binary relations *S* and *T* from *A* to *B* as follows:

For all (*x,y*) ∈*A*×*B*, *x S y* ↔ |*x*| = |*y*|

For all (*x,y*) ∈*A*×*B*, *x T y* ↔ *x− y* is even

State explicitly which ordered pairs are in *A*×*B*, *S*, *T*, *S* ∩ *T*, and *S* ∪ *T*.

 *A*= {*−*1, 1, 2, 4}

 *B*= {1,2}

**A x B** = {(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)}

 **S** = {(-1,1), (1,1), (2,2)}

  **T** = {(-1,1), (1,1), (2,2), (4,2)}

 **S ∩ T** *=* {(-1,1), (1,1), (2,2)}

 **S ∪ T =** {(-1,1), (1,1), (2,2), (4,2)}

4. Show that $¬ \left(\left(¬ p ∧q \right) ∨ \left(¬ p ∧¬ q \right)\right)∨ \left(p ∧q\right)≡ p . $ State carefully which of the laws are used at each stage.

From Left Hand side (LHS): -

$¬ \left( \left( ¬ p ∧q \right) ∨ \left(¬ p ∧¬ q \right)\right) ∨ \left( p ∧q\right)≡ p $ (LHS)

$= ( ¬ \left(¬ p ∧q\right) ∧¬\left(¬ p ∧ ¬ q\right)) ∨\left(p ∧q\right)$ (De Morgan ‘s laws)

$= \left(¬ ¬ p ∨ ¬ q\right) ∧ \left(¬ ¬ p ∨ ¬ ¬ q\right)∨(p ∧q ) $ (De Morgan ‘s laws)

$= \left( p ∨ ¬ q \right) ∧ \left( p ∨q \right) ∨( p ∧q) $ (Double negation law)

$= p ∨ \left( ¬ q ∧q \right)∨( p ∧q )$ (Distributive laws)

$= p ∨ ∅ ∨( p ∧q )$ (Complement law)

$= p ∨( p ∧q )$ (Identity law)

$= p$ (proven) (Absorption law)

5. $R\_{1}=\left\{ \left(x ,y\right)\right| x+y\leq 6 \} ; R\_{1} $ is from $X $to $Y$ ; $R\_{2 }=\left\{\left(y ,z\right)\right| y >z\} ; R\_{2}$ is from Y to Z ; ordering of X<Y and Z : 1, 2 ,3 ,4 ,5.

a) The matrix $A\_{1}$of the relation $R\_{1}$ (relative to the given ordering)

$$A\_{1}:-$$

 $ \{\left(1,1\right),\left(1 ,2\right),\left(1,3\right),\left(1,4\right),\left(1,5\right),\left(2,1\right),\left(2,2\right),\left(2,3\right),\left(2,4\right),\left(3,1\right),\left(3,2\right),\left(3,3\right),\left(4,1\right),\left(4,2\right),\left(5,1\right)\}$

$$A\_{1}= \left[\begin{matrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{matrix}\right]$$

b) The matrix $A\_{2}$of the relation $R\_{2}$ (relative to the given ordering)

$$A\_{2}=\{\left(2,1\right),\left(3,1\right),\left(3,2\right),\left(4,1\right),\left(4,2\right),\left(4,3\right),\left(5,1\right),\left(5,2\right),\left(5,3\right),(5,4)\}$$

$$A\_{2}= \left[\begin{matrix}0&0&0&0&0\\1&0&0&0&0\\1&1&0&0&0\\1&1&1&0&0\\1&1&1&1&0\end{matrix}\right]$$

1. **i**s $R\_{1}$ reflexive, symmetric, transitive and /or equivalence relation?

- Reflexive: it’s not reflexive since the main diagonal for $R\_{1}$is not all have value 1.

- Symmetric: its symmetric since $\left(x,y\right)\in R\_{1} and \left(y,x\right) \in R\_{1}$

- Transitive: it’s not transitive since: -

- It’s not equivalence relation because it’s not reflexive and not transitive

$\left[\begin{matrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{matrix}\right]$ $⨂$ $\left[\begin{matrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{matrix}\right]$ = $\left[\begin{matrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{matrix}\right]$

$$M\_{R} \ne M\_{R}$$

1. is $R\_{2}$ reflexive, Antisymmetric, transitive and /or a partial order relation?

- Reflexive: it’s not reflexive since the main diagonal for $R\_{2}$ is not all have value 1.

- Antisymmetric: it’s not antisymmetric since $2\ne 1 and ( 1,2) $not belong to $R\_{2}$

- Transitive: it’s not transitive since: -

- it’s not partial order relation since $R\_{2}$ not reflexive and not transitive

$\left[\begin{matrix}0&0&0&0&0\\1&0&0&0&0\\1&1&0&0&0\\1&1&1&0&0\\1&1&1&1&0\end{matrix}\right]$ $⨂$ $\left[\begin{matrix}0&0&0&0&0\\1&0&0&0&0\\1&1&0&0&0\\1&1&1&0&0\\1&1&1&1&0\end{matrix}\right]$ = $\left[\begin{matrix}0&0&0&0&0\\1&0&0&0&0\\1&0&0&0&0\\1&1&0&0&0\\1&1&1&0&0\end{matrix}\right]$

$$M\_{R} \ne M\_{R}$$

6. a) The matrix of relation $R\_{1}∪ R\_{2}$

$$R\_{1}=\{\left(1,1\right),\left(2,2\right),\left(2,3\right),\left(3,1\right),\left(3,3\right)\}$$

$$R\_{2}=\{\left(1,2\right),\left(2,2\right),\left(3,1\right),\left(3,3\right)\}$$

$$R\_{1}∪ R\_{2}=\{\left(1,1\right),\left(1,2\right),\left(2,2\right),\left(2,3\right),\left(3,1\right),\left(3,3\right)\}$$

$$\left[\begin{matrix}1&1&0\\0&1&1\\1&0&1\end{matrix}\right]$$

. b) The matrix of relation $R\_{1}∩ R\_{2}$

$$R\_{1}=\{\left(1,1\right),\left(2,2\right),\left(2,3\right),\left(3,1\right),\left(3,3\right)\}$$

$$R\_{2}=\{\left(1,2\right),\left(2,2\right),\left(3,1\right),\left(3,3\right)\}$$

$$R\_{1}∩ R\_{2}=\{\left(2,2\right),\left(3,1\right),\left(3,3\right)\}$$

$$\left[\begin{matrix}0&0&0\\0&1&0\\1&0&1\end{matrix}\right]$$

7. Let $f:R\rightarrow R$ and $g:R\rightarrow R$ are functions and the function $(f+g):R\rightarrow R$ is defined by the formula $\left(f+g\right)\left(x\right):f\left(x\right)+g(x)$ for all real number x.

Let us assume that $f:R\rightarrow R$ and $g:R\rightarrow R$ are both one-to-one, then $(f+g)$ is not one-to-one.

For example:

Let us assume that $f\left(x\right)=x$ and $g\left(x\right)=-x$ for all real number x.

Then f and g are both one-to-one.

Because for all real number $x\_{1}$ and $x\_{2}$, if $f(x\_{1})=f(x\_{2})$ then $x\_{1}=x\_{2}$.

And, if $g(x\_{1})=g(x\_{2})$, then $-x\_{1}=-x\_{2}\rightarrow x\_{1}=x\_{2}$.

$∴$For $\left(f+g\right)\left(x\right)=f\left(x\right)+g\left(x\right)=x=(-x)$ for all real number x.

Thus, $\left(f+g\right)\left(1\right)=\left(f+g\right)\left(2\right)=0$ but 1≠2.

$∴\left(f+g\right)$ is not one-to-one.

8. Given: Climb staircase with 1 stair at a time or 2 stairs at a time or any combination of those two.

Let $C\_{n}$=number of different wats to climb a staircase with n stairs.

When $n=1$, the staircase contains only 1 stair thus only climb once, thus there is only one way that is climbing using 1-stair step.

$$C\_{1}=1$$

When $n=2$, the staircase contains only 2 stairs thus only climb twice making it to have two ways to climb the stairs either we climb twice with 1-stair step or climb once with 2-stair step.

$$C\_{2}=2$$

When $n\geq 3$, the staircase contains more than 2 stairs and thus we will need to use a combination of 1-stair and 2-stair steps.

If the last move will be a 1-stair step, then there were $C\_{n-1}$ ways to arrive at the previous stair which was a staircase with n-1 stairs.

If the last move will be a 2-stair step, then there were $C\_{n-2}$ ways to arrive at the previous stair which was a staircase with n-2 stairs.

$∴$We can conclude that $C\_{n}=C\_{n-1}+C\_{n-2}$

$$∴C\_{1}=1$$

$$C\_{2}=2$$

$C\_{n}=C\_{n-1}+C\_{n-2}$ when n≥3

9. (a) $t\_{7}=?$

 Given $t\_{0}=0$, $t\_{1}= t\_{2}=1$, $t\_{n}= t\_{n-1}+ t\_{n-2}+ t\_{n-3}$.

 $t\_{7}= t\_{6}+ t\_{5}+ t\_{4}$

 $=(t\_{5}+ t\_{4}+ t\_{3})+ t\_{5}+ t\_{4}$

 $=2t\_{5}+ 2t\_{4}+ t\_{3}$

 $=2\left( t\_{4}+ t\_{3}+ t\_{2}\right)+ 2t\_{4}+t\_{3}$

 $=4t\_{4}+ t\_{3}+ t\_{2}$

 $=4(t\_{3}+ t\_{2}+ t\_{1})+ 3t\_{3}+ 2t\_{2}$

 $=7t\_{3}+ 6t\_{2}+ 4t\_{1}$

 $=7\left(1+1+0\right)+6\left(1\right)+4(1)$

 $=7\left(2\right)+6\left(1\right)+4(1)$

 $=14+6+4$

 $=24$

$$∴t\_{7}=24$$

9. (b) Recursive algorithm to compute $t\_{n}, n\geq 0$.

 Tribonacci $(n :n \geq 0)$

 if $n=0$ return 0

 else if $n=1$ return 1

 else if $n=2$ return 1

 Else return $tribonacci\left(n-1\right)+tribonacci\left(n-2\right)+tribonacci(n-3)$

 $∴Output is tribonacci\left(n\right)$

#include <iostream>

using namespace std;

int main ()

{

int printTribRec (int n)

{

 if (n == 0)

 return 0;

 if (n == 1 || n == 2)

 return 1;

 else

 return printTribRec (n - 1) + printTribRec (n - 2) + printTribRec(n - 3);

}

void printTrib (int n)

{

 for (int i = 1; i < n; i++)

 cout << printTribRec(i) << " ";

}

}