UNIVERSITI TEKNOLOGI MALAYSIA

ASSIGNMENT 1

COURSE NAME: DISCRETE STRUCTURE

COURSE CODE: SECI 1013

SECTION: 03

LECTURER'S NAME: Dr. Nor Azizah Ali

GROUP NUMBER: 7

GROUP MEMBERS:

Name	**Matric No.**
LUQMAN ARIFF BIN NOOR AZHAR	A20EC0202
TERENCE A/L LOORTHANATHAN	A20EC0165
MADINA SURAYA BINTI ZHARIN	A20EC0203

Question 1

Let the universal set be the set **R** of all real numbers and let $A = \{x \in \mathbf{R} \mid 0 < x \leq 2\}$, $B = \{x \in \mathbf{R} \mid 1 \leq x < 4\}$ and $C = \{x \in \mathbf{R} \mid 3 \leq x < 9\}$. Find each of the following:

a) $A \cup C$
$= \{x \in \boldsymbol{R} \mid (0 < x \leq 2) \text{ or } (3 \leq x < 9)\}$

b) $(A \cup B)'$
$= \{x \in \boldsymbol{R} \mid \neg((0 < x \leq 2) \text{ or } (1 \leq x < 4))\}$

$= \{x \in \boldsymbol{R} \mid x < 0 \text{ or } x > 4\}$

c) $A' \cup B'$

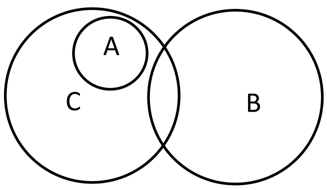
$= \{x \in \boldsymbol{R} \mid (0 < x \leq 2)' \text{ or } (1 \leq x < 4)'\}$

$= \{x \in \boldsymbol{R} \mid x < 1 \text{ or } x > 2\}$

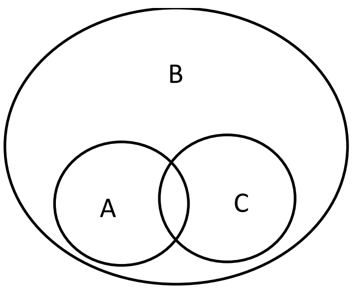
Question 2

Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions

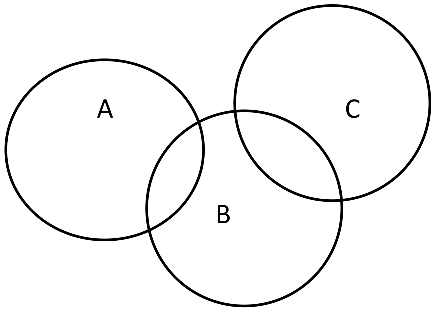
a) $A \cap B = \emptyset,\ A \subseteq C,\ C \cap B \neq \emptyset$



b) $A \subseteq B,\ C \subseteq B,\ A \cap C \neq \emptyset$



c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subset B$, $C \not\subset B$



Question 3

Given two relations S and T from A to B, $S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$

$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$

Let $A = \{-1, 1, 2, 4\}$ and $B = \{1,2\}$ and defined binary relations S and T from A to B as follows:

For all $(x,y) \in A \times B$, $\quad x\, S\, y \leftrightarrow |x| = |y|$

For all $(x,y) \in A \times B$, $\quad x\, T\, y \leftrightarrow x - y$ is even

State explicitly which ordered pairs are in $A \times B$, S, T, $S \cap T$, and $S \cup T$.

Solution:

S = { (-1,1) , (1,1) , (2,2) }

T = { (4,2) , (2,1) , (1,1) , (2,2) }

S ∩ T = { (1,1) , (2,2) }

S ∪ T = { (-1,1) , (1,1) , (2,2) , (4,2) , (2,1) }

A × B = { (-1,1) , (-1,2) , (1,1) , (1,2) , (2,2) , (2,1) , (4,1) , (4,2) }

Question 4

Show that $\neg\,((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

Solution:

Right side:

$= \neg\,((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$ [Distributive Law]

$= \neg\,(\neg p \wedge (q \vee \neg q)) \vee (p \wedge q)$ [Negation Law]

$= \neg\,(\neg p \wedge (T)) \vee (p \wedge q)$ [De Morgan's Law]

$= (p \vee F) \vee (p \wedge q)$ [Identity Law]

$= p \vee (p \wedge q)$ [Absorption Law]

$= p$

Question 5

$R_1 = \{(x, y) \mid x + y \leq 6\}$; R_1 is from X to Y; $R_2 = \{(y, z) \mid y > z\}$; R_2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5. Find:

a) The matrix A_1 of the relation R_1 (relative to the given orderings)

$$A_1 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}}$$

b) The matrix A_2 of the relation R_2 (relative to the given orderings)

$$A_2 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}}$$

c) Is R_1 reflexive, symmetric, transitive, and/or an equivalence relation?

- R_1 is not reflexive as the diagonal of matrix not all 1 or $\nabla x \in R, (x, x) \notin R_1$.
- R_1 is not transitive as $A_1 \otimes A_1 \neq A_1$
- R_1 is not an equivalence relation as it is only symmetric but not reflexive and not transitive.
- R_1 is a symmetric relation as $\nabla x, y \in R, (x, y) \in R_1 \rightarrow (y, x) \in R_1$.

d) Is R_2 reflexive, antisymmetric, transitive, and/or a partial order relation?

- R_2 is not reflexive as the diagonal of matrix not all 1 or $\nabla x \in R, (x, x) \notin R_2$.
- R_2 is not transitive as $A_2 \otimes A_2 \neq A_2$
- R_2 is not partial order relation as it is only antisymmetric but not reflexive and not transitive.
- R_2 is an antisymmetric relation as $\nabla x, y \in R, (x, y) \in R_2 \rightarrow (y, x) \notin R_2 \rightarrow x \neq y$.

Question 6

Suppose that the matrix of relation $R1$ on $\{1, 2, 3\}$ is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation $R2$ on $\{1, 2, 3\}$ is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation $R1 \cup R2$

Solution:

$= \boldsymbol{R1} \cup \boldsymbol{R2}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \cup \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b) The matrix of relation $R1 \cap R2$

Solution:

$= \boldsymbol{R1} \cap \boldsymbol{R2}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \cap \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Question 7

If f:**R**→ **R** and g:**R**→ **R** are both one-to-one, is f+ g also one-to-one? Justify your answer.

Solution

Assume $f(x) = 4x - 1$ and $g(x) = x + 3$

.$(f + g)(x) = (4x - 1) + (x + 3)$

$= 5x - 2$

Let $(f + g)(x_1) = (f + g)(x_2)$, then

$5x_1 - 2 = 5x_2 - 2$

Then, $5x_1 = 5x_2$

Therefore $x_1 = x_2$ proves $(f + g)(x)$ is a one-to-one function

Question 8

With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer n≥1, if the staircase consists of n stairs, let cn be the number of different ways to climb the staircase. Find a recurrence relation for c1, c2, …., cn.

Solution:

$n \ = \ stairs \quad (n \geq 1)$

$c_n \ = \ number\ of\ different\ ways\ to\ climb\ the\ staircase$

If n = 1, $c_1 = 1$ = [(1)]

If n = 2, $c_2 = 2$ = [(1,1), (2)]

If n = 3, $c_3 = 3$ = [(1,1,1), (1,2), (2,1)]

If n = 4, $c_4 = 5$ = [(1,1,1,1), (1,1,2), (1,2,1), (2,1,1), (2,2)]

If n = 5, $c_5 = 8$ = [(1,1,1,1,1), (1,1,1,2), (1,1,2,1), (1,2,1,1), (2,1,1,1), (1,2,2), (2,1,2), (2,2,1)]

If n = 6, $c_6 =?$

Notice that every $\boldsymbol{c_n}$ is fixed for every n $[where\ (\boldsymbol{n} \geq \mathbf{1})]$, and every first step taken it can only be

taken in two forms either 1 step or 2 steps. If, one step is taken as a first step then $\boldsymbol{n} = n - \mathbf{1}$ staircases are left to climb and if two steps are taken as first steps then $\boldsymbol{n} = n - \mathbf{2}$ staircases are left to climb. Therefore the $\boldsymbol{c_n}$ also will be relatively reduced. For example, $n = 4$ if one step is taken as the first step there will be 3 staircases left to climb and if 2 steps are taken as first there will be 2 staircases left to climb. So, the summation of ways to climb the 3 staircases if one step is taken as first step and the ways to climb the 2 staircases if two steps are taken as first is the number of ways to climb $n = 4$ staircases.

So, in conclusion we can see that $\boldsymbol{c_n}$ depends on the summation of its previous two $\boldsymbol{c_n}$. Therefore, the recurrence relation which is a Fibonacci sequence is:

$$\boldsymbol{c_n} = \boldsymbol{c_{n-1}} + \boldsymbol{c_{n-2}}, for\ n > 2 \qquad [Where, c_1 = 1, c_2 = 2]$$

$$Therefore, \qquad c_6 = c_5 + c_4$$

$$c_6 = 8 + 5 = 13\ ways$$

Question 9

The Tribonacci sequence (t_n) is defined by the equations, $t_0 = 0$, $t_1 = t_2 = 1$, $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$.

a) Find t_7.

$t_7 = t_{7-1} + t_{7-2} + t_{7-3}$

$= t_6 + t_5 + t_4$

$= 13 + 7 + 4$

$= 24$

b) Write a recursive algorithm to compute t_n, $n \geq 3$.

```
f(n)
{
        if (n = 1) or (n= 2)
                return 1
        return f (n – 1) + f (n – 2) + f (n – 3)
}
```