UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
FACULTY OF ENGINEERING

ASSIGNMENT 1

SECI 1013-04

DISCRETE STRUCTURE

SESSION: 2020/2021

COURSE: BACHELOR OF COMPUTER SCIENCE (COMPUTER NETWORK AND SECURITY) WITH HONORS

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1. Let the universal set be the set R of all real numbers and let A={x∈R | 0 < x ≤ 2},

B={x∈R | 1 ≤ x < 4} and C={x∈R | 3 ≤ x < 9}. Find each of the following:

A = { 1, 2 }

B = { 1, 2, 3 }

C = { 3,4,5,6,7,8 }

a) A ∪ C

= { x∈R | 1≤ x ≤ 8}

b) (A ∪ B)'

= A' ∩ B'

= { x∈R | (1 ≤ x ≤ 2)' and (1 ≤ x ≤ 3)' }

= { x∈R |(x < 1 or x > 2) and (x < 1 or x > 3)}

= { x∈R | x < 1 or x >3 }

c) A' ∪ B'

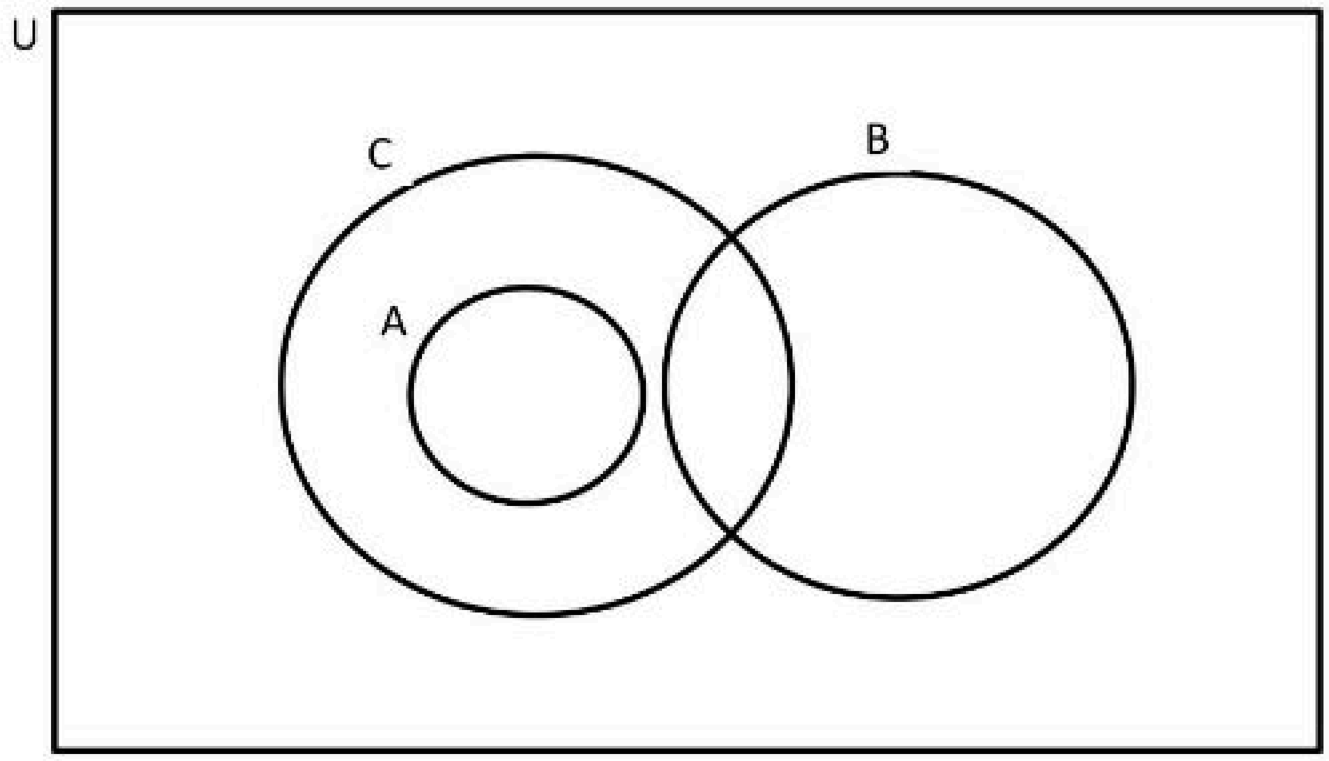
= { x∈R | (1 ≤ x ≤ 2)' or (1 ≤ x ≤ 3)'}

= { x∈R | (x < 1 or x > 2) or (x < 1 or x > 3)}

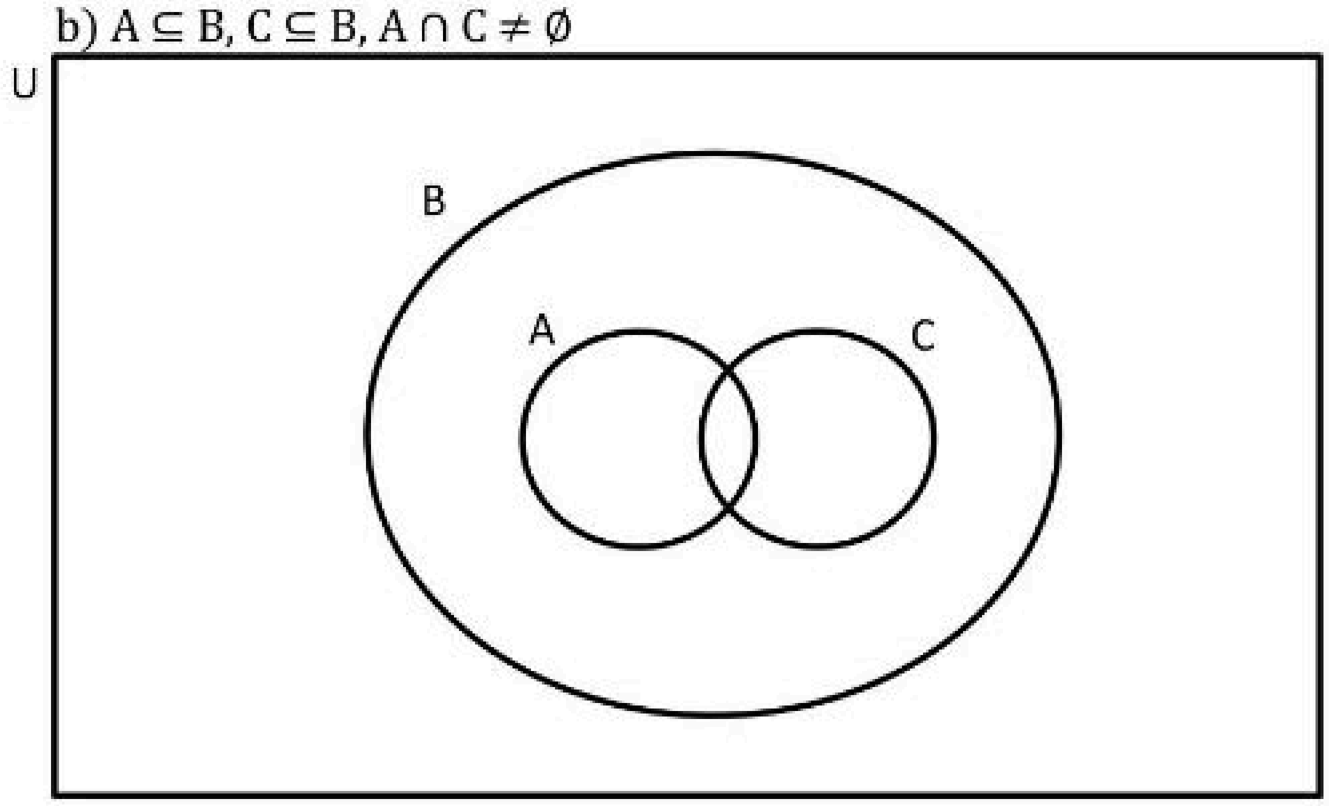
= { x∈R | x < 1 or x > 2}

2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

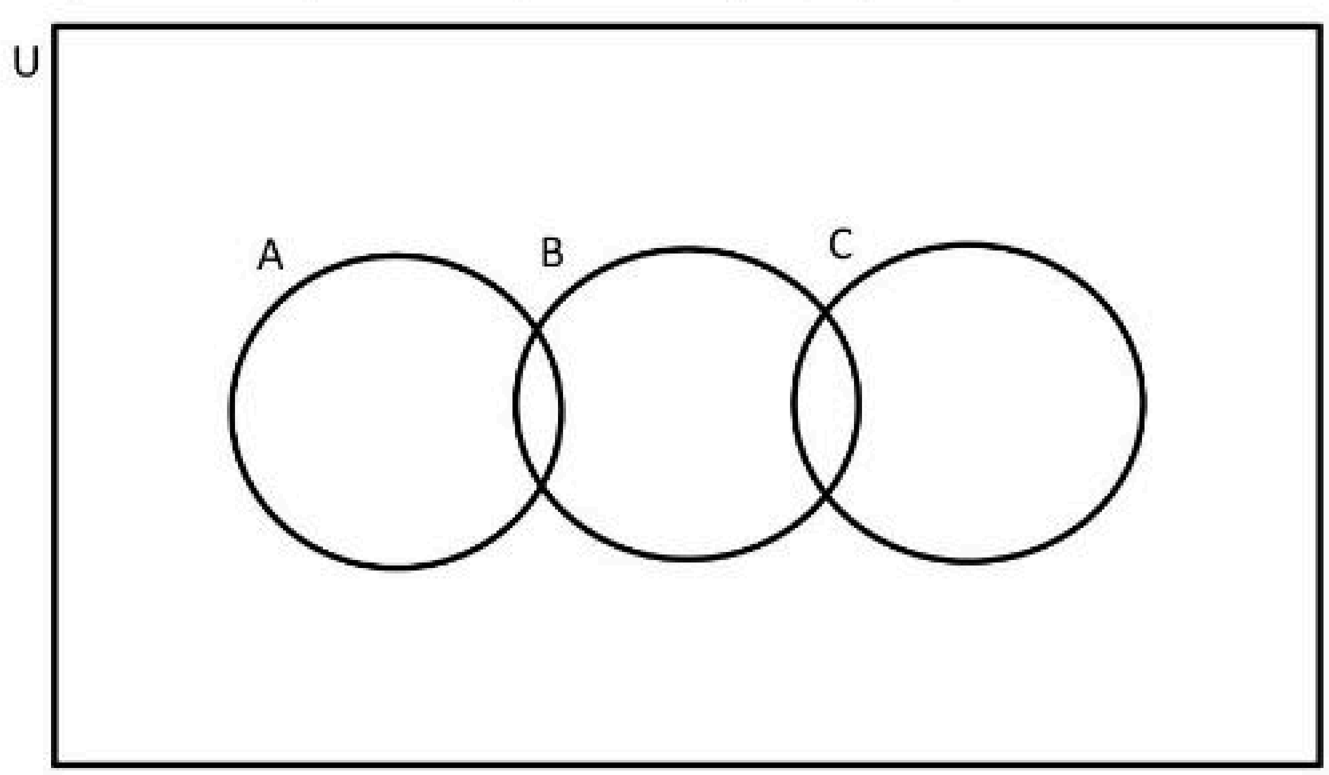
a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$



b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$



c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subset B$, $C \not\subset B$



3. Given two relations S and T from A to B,

$S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$

$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$

Let A={−1, 1, 2, 4} and B={1,2} and defined binary relations S and T from A to B as follows:

For all $(x,y) \in A \times B$, $x\ S\ y \leftrightarrow |x| = |y|$

For all $(x,y) \in A \times B$, $x\ T\ y \leftrightarrow x - y$ is even

State explicitly which ordered pairs are in A×B, S, T, S ∩ T, and S ∪ T.

$A \times B$ = {(−1,1), (−1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)}

S = {(−1,1), (1,1), (2,2)}

T = {(−1,1), (1,1), (2,2), (4,2)}

$S \cap T$ = {(−1,1), (1,1), (2,2)}}

$S \cup T$ = {(−1,1), (1,1), (2,2), (4,2)}

4. Show that $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$ De Morgan's Law

$= \neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q) \vee (p \wedge q)$ De Morgan's Laws

$= (\neg(\neg p) \vee \neg q) \wedge (\neg(\neg p) \vee \neg(\neg q)) \vee (p \wedge q)$ Double Negation Laws

$= ((p \vee \neg q) \wedge (p \vee q)) \vee (p \wedge q)$

$= p \vee (p \wedge q)$ absorption laws

$\equiv p$

Question 5

$R1=\{(x,y)|\ x+y\le 6\}$; $R1$ is from X to Y; $R2=\{(y,z)|\ y>z\}$; $R2$ is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5.

Find

a) The matrix A_1 of the relation R_1 (relative to the given orderings)

$$R_1=\{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)\}$$

$$M_{A_1}=\begin{bmatrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{bmatrix}$$

b) The matrix A_2 of the relation R_2 (relative to the given orderings)

$$R_2=\{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(5,4)\}$$

$$M_{A_2}=\begin{bmatrix}0&0&0&0&0\\1&0&0&0&0\\1&1&0&0&0\\1&1&1&0&0\\1&1&1&1&0\end{bmatrix}$$

c) Is R_1 reflexive, symmetric, transitive, and/or an equivalence relation?

$$M_{A_1}=\begin{bmatrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{bmatrix}$$

R_1 is not reflexive as there is no all 1's on its main diagonal of matrix

$$M_{A_1}=\begin{bmatrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{bmatrix}\qquad M_{A_1}^T=\begin{bmatrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{bmatrix}$$

R_1 is symmetric as $M_{A_1}=M_{A_1}^T$

$$M_{A_1}\otimes M_{A_1}=\begin{bmatrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{bmatrix}\otimes\begin{bmatrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{bmatrix}=\begin{bmatrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{bmatrix}$$

R_1 is not transitive as $M_{A_1}\otimes M_{A_1}\ne M_{A_1}$

So, R_1 is not equivalence relation.

d) Is R_2 reflexive, antisymmetric, transitive, and/or a partial order relation?

$$M_{A_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

R_2 is not a reflexive as there is no all 1's on its main diagonal of matrix.
R_2 is antisymmetric as there are no edges that go in the opposite direction for each edges.It only one directed relation. As $(2,1) \in R_2$, but $(1,2) \notin R_2$.

$$M_{A_2} \otimes M_{A_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

R_2 is not transitive as $M_{A_2} \otimes M_{A_2} \neq M_{A_2}$

6. Suppose that the matrix of relation R_1 on $\{1, 2, 3\}$ is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R_2 on $\{1, 2, 3\}$ is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation $R_1 \cup R_2$

$R_1 = \{(1,1),(2,2),(2,3),(3,1),(3,3)\}$

$R_2 = \{(1,2),(2,2),(3,1),(3,3)\}$

$R_1 \cup R_2 = \{(1,1),(1,2),(2,2),(2,3),(3,1),(3,3)\}$

$$M_{R_1 \cup R_2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Matrix of relation $R_1 \cup R_2$ is reflexive as there are all 1's on its main diagonal of matrix and it is also antisymmetric as there are no edges that go in the opposite direction for each edges. It is only one directed relation as $(3,1) \in (R_1 \cup R_2)$, but $(1,3) \notin (R_1 \cup R_2)$.

b) The matrix of relation $R_1 \cap R_2$

$R_1 \cap R_2 = \{(3,1),(2,2),(3,3)\}$

$$M_{R_1 \cap R_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$R_1 \cap R_2$ is not a reflexive as there have one 'o' on its main diagonal of matrix.

$R_1 \cap R_2$ is antisymmetric , there are only one directed relation.

$(3,1) \in (R_1 \cap R_2)$, but $(1,3) \notin (R_1 \cap R_2)$.

$R_1 \cap R_2$ is transitive. The product of Boolean show that this matrix is transitive.

$$M_{R_1 \cap R_2} \otimes M_{R_1 \cap R_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{R_1 \cap R_2} \otimes M_{R_1 \cap R_2} = M_{R_1 \cap R_2}$$

7. If f :**R**→ **R** and g:**R**→ **R** are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

f is one-to-one as $f(x_1) = f(x_2)$

g is one-to-one as $g(x_1) = g(x_2)$

For f+g ,

$$f(x_1) + g(x_1) = f(x_2) + g(x_2)$$
$$x_1 + x_1 = x_2 + x_2$$
$$2x_1 = 2x_2$$
$$x_1 = x_2$$

So, f + g is one-to-one function as $f(x_1)+g(x_1)=f(x_2)+g(x_2)$

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase. Find a recurrence relation for $c_1, c_2, \ldots, c_n$.

When **n = 1**, there is only 1 way to climb this staircase which is taking one stair at a time.

So, when **n = 1,** $\mathbf{C_1 = 1}$

When **n = 2**, means can climb the entire staircase taking two at a time.

So, when **n = 2,** $\mathbf{C_2 = 2}$

When **n ≥ 3**, means we can climb the entire staircase by taking a combination of one or two stair increments. So, there is 2 ways to climb the staircase but depends on the last step taken is either a single stair or two stairs together. If the last step is a single stair, the number of ways is $\mathbf{C_{n-1}}$, whereas if the last step is two stairs together, the ways to climb are $\mathbf{C_{n-2}}$. Therefore, $\mathbf{C_n = C_{n-1} + C_{n-2}}$.

9. The Tribonacci sequence (tn) is defined by the equations,

$t_0 = 0$, $t_1 = t_2 = 1$, $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$.

a) Find t_7.

$t_0 = 0$

$t_1 = 1$

$t_2 = 1$

$t_3 = t_2 + t_1 + t_0 = 1 + 1 + 0 = 2$

$t_4 = t_3 + t_2 + t_1 = 2 + 1 + 1 = 4$

$t_5 = t_4 + t_3 + t_2 = 4 + 2 + 1 = 7$

$t_6 = t_5 + t_4 + t_3 = 7 + 4 + 2 = 13$

$t_7 = t_6 + t_5 + t_4 = 13 + 7 + 4 = 24$

b) Write a recursive algorithm to compute t_n, $n \geq 0$.

```
Input = n ,   n ≥ 0
output = t_n
t_n
{ if (n =0)
  return 0;
  else if ((n =1) ||(n=2))
  return 1;
  else
  return (t_{n-1}+ t_{n-2} + t_{n-3});
}
```