

Assignment 5 (PSDA)

1) (a)

Zinc Supplement Group	Placebo Group
$n_1 = 10$	$n_2 = 15$
$\bar{x}_1 = 3214$	$\bar{x}_2 = 3088$
$s_1 = 669$	$s_2 = 728$

$$H_0: \sigma_1 = \sigma_2, H_1: \sigma_1 > \sigma_2, \alpha = 0.05$$

$$\text{Test statistic, } F = \frac{s_1^2}{s_2^2} = \frac{(669)^2}{(728)^2} = \frac{447561}{529984} = 0.84$$

$$\begin{aligned} \text{Degree of freedom} \Rightarrow \text{Numerator} &= n_1 - 1 = 10 - 1 = 9 \\ \text{Denominator} &= n_2 - 1 = 15 - 1 = 14 \end{aligned}$$

$$F\text{-table} \Rightarrow F_{0.05, 9, 14} = 2.65$$

* Since $F = 0.84 < F_{0.05, 9, 14} = 2.65$, fail to reject H_0 . There is no sufficient evidence that the variation of birth weights for placebo population is less than the variation for the population treated with zinc supplements.

(b)

x_1	797	571	904	1147	418	
x_2	523	488	875	911	297	
$D(x_1 - x_2)$	274	83	29	236	121	$\Sigma D = 743$
$D^2(x_1 - x_2)^2$	75076	6889	841	55696	14641	$\Sigma D^2 = 153143$

$H_0: \mu_D = 0$, $H_1: \mu_D > 0$ (claimed), $n=5$, $\alpha=0.01$, $df=4$

Critical value $\Rightarrow t_{0.01, 4} = 3.747$

$$\bar{D} = \frac{\Sigma D}{n} = \frac{743}{5} = 148.6$$

$$\text{Standard deviation, } s_D = \sqrt{\frac{\Sigma D^2 - (\Sigma D)^2}{n}} = \sqrt{\frac{153143 - (743)^2}{5}} = \sqrt{\frac{153143 - 551949}{5}} = \sqrt{\frac{153143 - 551949}{5-1}}$$

$$s_D = 103.36$$

$$\text{Test value, } t = \frac{\bar{D} - \mu_0}{s_D / \sqrt{n}} = \frac{148.6 - 0}{103.36 / \sqrt{5}} = 3.22$$

* Since $3.22 < t_{0.01, 4} = 3.747$ at $\alpha = 0.01$, fail to reject H_0 .
There is insufficient evidence to support the claim that the independent workshop has lower repair costs.

2) (a)

	Meds	No Meds	
Flu	$n_1 = 120$	$n_2 = 280$	$p_1 = 29/120 = 0.2417$
Recovered	$x_1 = 29$	$x_2 = 56$	$p_2 = 56/280 = 0.2$

$$H_0: p_1 = p_2, H_1: p_1 > p_2, \alpha = 0.01, q_1 = 0.7583, q_2 = 0.8$$

$$\text{Test statistic, } z = \frac{p_1 - p_2}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} = \frac{0.2417 - 0.2}{\sqrt{\frac{(0.2417)(0.7583)}{120} + \frac{(0.2)(0.8)}{280}}}$$

$$z = 0.91$$

$$\begin{aligned} \text{Right-tailed test } \Rightarrow P(z > 0.91) &= 1 - P(z < 0.91) \\ &= 1 - 0.8186 \\ &= 0.1814 \end{aligned}$$

* Since $p\text{-value} = 0.1814 > \alpha = 0.01$, fail to reject H_0 . There is no sufficient evidence to support the company's claim of the effectiveness of the flu medicine.

(b)

Line 1	Line 2
$n_1 = 5$	$n_2 = 5$
$\bar{x}_1 = 0.442$	$\bar{x}_2 = 0.386$
$s_1 = 0.05$	$s_2 = 0.01$

$$H_0: \mu_1 - \mu_2 = 0, H_1: \mu_1 \neq \mu_2, \alpha = 0.05$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(5 - 1)(0.05)^2 + (5 - 1)(0.01)^2}{5 + 5 - 2}$$

$$s_p^2 = 0.0013$$

$$s_p = 0.036$$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{s_p \sqrt{1/n_1 + 1/n_2}} = \frac{0.442 - 0.386 - 0}{(0.036)(\sqrt{1/5 + 1/5})}$$

$$t_0 = 2.46$$

$$\text{Two-tailed test} \Rightarrow \alpha = 0.05 \div 2 = 0.025$$

$$t_0 > t_{0.025, 4} = 2.776$$

$$t_0 < t_{0.025, 4} = -2.776$$

* Since $-2.776 < t_0 = 2.46 < 2.776$ at $\alpha = 0.05$, fail to reject H_0 . There is no sufficient evidence to conclude that production line 1 is not producing as consistently as production line 2 in terms of the alcohol content.

(c)

x_1	204	393	391	265	326	220	423	342	480	464	
x_2	223	412	402	285	353	243	443	340	582	490	
$D(x_1 - x_2)$	-19	-19	-11	-20	-27	-23	-20	2	-102	-26	$\sum D = -265$
$D^2(x_1 - x_2)^2$	361	361	121	400	729	529	400	4	10404	676	$\sum D^2 = 13985$

$$H_0: \mu_0 = 0, H_1: \mu_0 < 0, n = 10, \alpha = 0.05, df = 9, t_{0.05, 9} = -1.833$$

$$\bar{D} = \frac{\sum D}{n} = \frac{-265}{10} = -26.5$$

Standard deviation, $s_D = \sqrt{\frac{\sum D^2 - \frac{(\sum D)^2}{n}}{n-1}} = \sqrt{\frac{13985 - \frac{(-265)^2}{10}}{10-1}}$

$$s_D = 27.82$$

$$\text{Test value, } t = \frac{\bar{D} - \mu_D}{s_D / \sqrt{n}} = \frac{(-26.5) - 0}{27.82 / \sqrt{10}}$$

$$t = -3.012$$

* Since $t = -3.012 < t_{0.05, 9} = -1.833$, reject H_0 . There is sufficient evidence to conclude that training can increase the number of words spelled correctly by the participants.

3. (a) $H_0: \mu_1 = \mu_2$, $H_1: \mu_1 \neq \mu_2$, $n = 10$, $\alpha = 0.05$, $df = 18$

$$\bar{x}_1 = 70, \bar{x}_2 = 74, s_1^2 = 11.33, s_2^2 = 29.11, n_1 = 10, n_2 = 10$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(10 - 1)(11.33) + (10 - 1)(29.11)}{10 + 10 - 2}$$

$$s_p^2 = 20.22$$

$$s_p = \sqrt{20.22}$$

$$s_p = 4.497$$

$$\text{Test statistic, } t_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{s_p \sqrt{1/n_1 + 1/n_2}} = \frac{70 - 74 - 0}{(4.497)(\sqrt{1/10 + 1/10})}$$

$$t_0 = -1.989$$

$$\text{Critical value} \Rightarrow t_{0.05, 18} = -1.734$$

* Since $t_0 = -1.989 < t_{0.05, 18} = -1.734$, reject H_0 . There is sufficient evidence to conclude that method B is more effective.

(b)

x_1	95	81	76	96	82	87	71	82	77	83	81	65	
x_2	89	77	77	94	80	86	72	82	74	81	78	63	
$D(x_1 - x_2)$	6	4	-1	2	2	1	-1	0	3	2	3	2	$\Sigma D = 23$
$D^2(x_1 - x_2)^2$	36	16	1	4	4	1	1	0	9	4	9	4	$\Sigma D^2 = 89$

$$H_0: \mu_0 = 0, H_1: \mu_0 < 0, n = 12, \alpha = 0.01, df = 11$$

$$\bar{D} = \frac{\Sigma D}{n} = \frac{23}{12} = 1.92$$

$$\text{Standard deviation, } s_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{n}}{n-1}} = \sqrt{\frac{89 - \frac{(23)^2}{12}}{12-1}}$$

$$s_D = 2.02$$

$$\text{Test value, } t = \frac{\bar{D} - \mu_0}{s_D / \sqrt{n}} = \frac{1.92 - 0}{2.02 / \sqrt{12}} = 3.29$$

$$\text{Critical value } \Rightarrow t_{0.01, 11} = 2.718$$

* Since $t = 3.29 > t_{0.01, 11} = 2.718$, fail to reject H_0 . There is no sufficient evidence to conclude that the prescribed program of exercise is effective.

4. (a) $H_0: \mu_1 = \mu_2$, $H_1: \mu_1 \neq \mu_2$, $\alpha = 0.05$

$n_1 = 150$, $n_2 = 250$, $\bar{x}_1 = 75.50$, $\bar{x}_2 = 65.20$, $s_1 = 53.20$, $s_2 = 46.10$

$$T_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{75.50 - 65.20 - 0}{\sqrt{\frac{(53.20)^2}{150} + \frac{(46.10)^2}{250}}} = 1.969$$

$$V = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}} = \frac{\left(\frac{(53.2)^2}{150} + \frac{(46.1)^2}{250} \right)^2}{\frac{((53.2)^2/150)^2}{150 - 1} + \frac{((46.1)^2/250)^2}{250 - 1}}$$

$V = 279.6$

$V \approx 280$

Critical value $\Rightarrow t_{0.05, 280} = 1.645$

Reject H_0 if $t_0 > 1.645$ or $t_0 < -1.645$

* Since $T_0 = 1.969 > t_{0.05, 280} = 1.645$, fail to reject H_0 . There is sufficient evidence to conclude that there is no difference in the mean spending between the two populations.

(b)	x_1	14	12	18	7	11	9	16	15	13	17	
	x_2	10	4	14	5	8	5	11	12	9	10	
	$D(x_1 - x_2)$	4	8	4	2	3	4	5	3	4	7	$\sum D = 44$
	$D^2(x_1 - x_2)^2$	16	64	16	4	9	16	25	9	16	49	$\sum D^2 = 224$

$$H_0: \mu_0 = 0, H_1: \mu_0 < 0 \text{ (claimed)}, \alpha = 0.05, df = 9, n = 10$$

$$\bar{D} = \frac{\sum D}{n} = \frac{44}{10} = 4.4$$

$$\text{Standard deviation } s_D = \sqrt{\frac{\sum D^2 - \frac{(\sum D)^2}{n}}{n-1}} = \sqrt{\frac{224 - \frac{(44)^2}{10}}{10-1}}$$

$$s_D = 1.84$$

$$\text{Test value, } t = \frac{\bar{D} - \mu_0}{s_D / \sqrt{n}} = \frac{4.4 - 0}{1.84 / \sqrt{10}} = 7.57$$

$$\text{Critical value } \Rightarrow t_{0.05, 9} = 1.833$$

* Since $t = 7.57 > t_{0.05, 9} = 1.833$ at $\alpha = 0.05$, fail to reject H_0 . There is no sufficient evidence to conclude that the mean number of words recalled after 1 hour exceeds the mean recall after 24 hours by more than 3.