

PSDA - Assignment 6

1.(a) $k=6$, $n=36$, $\alpha=0.05$

$$H_0: p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = \frac{1}{6}$$

 H_1 : At least one of the proportion is different from others

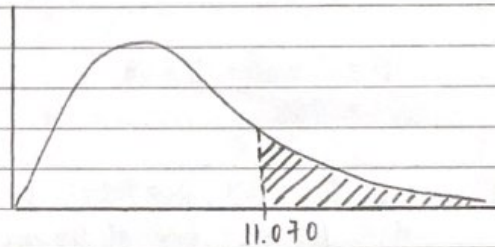
$$\alpha = 0.05$$

$$df = k-1$$

$$= 6-1$$

$$= 5$$

$$\chi^2_{0.05,5} = 11.070$$

Critical value: $\chi^2_{0.05,5} = 11.070$ 

Card Number	O	$E = \frac{n}{k}$ or $n \times p$	$(O-E)^2 / E$
1	8	6	$\frac{2}{3}$
2	5	6	$\frac{1}{6}$
3	9	6	$\frac{3}{2}$
4	2	6	$\frac{4}{3}$
5	7	6	$\frac{1}{6}$
6	5	6	$\frac{1}{6}$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{2}{3} + \frac{1}{6} + \frac{3}{2} + \frac{4}{3} + \frac{1}{6} + \frac{1}{6}$$

$$= 5.333$$

since $\chi^2 = 5.333 < \chi^2_{0.05,5}$, does not lie in critical region. Fail to reject H_0

Conclusion:

At 0.05 significant level, there is sufficient evidence to prove that the device is not biased.

(b)(i)

Range	P	O	E = n × P
Above 1	0.159	9	17
0 to 1	0.341	60	35
-1 to 0	0.341	17	36
Below -1	0.159	19	17

$$N = 9 + 60 + 17 + 19$$

$$= 105$$

(ii)

$$H_0: p_1 = 0.159, p_2 = 0.341, p_3 = 0.341, p_4 = 0.159$$

H_1 : At least one of the proportions is different from the claimed value

$$\alpha = 0.01$$

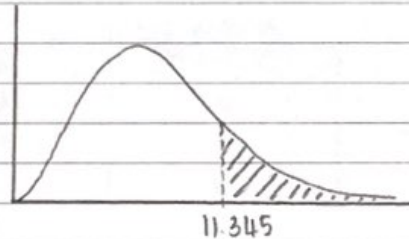
$$df = k - 1$$

$$= 4 - 1$$

$$= 3$$

$$\chi^2_{0.01, 3} = 11.345$$

$$\text{critical value: } \chi^2_{0.01, 3} = 11.345$$



$$\text{test statistic: } \chi^2 = 12.156$$

Since $\chi^2 = 12.156 > \chi^2_{0.01, 3}$, lies in critical region, reject H_0

Conclusion:

At 0.01 significance level, there is sufficient evidence to conclude that the CGPA distribution of the final year students is normally distributed

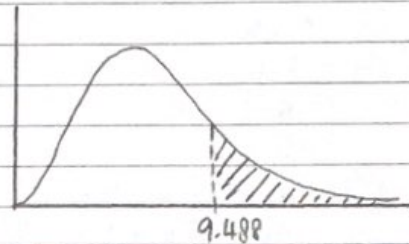
- (c) H_0 : No relationship between texture and colour
 H_1 : Relationship exist between texture and colour

$$\alpha = 0.05$$

$$\begin{aligned} df &= (r-1)(c-1) \\ &= (3-1)(3-1) \\ &= 4 \end{aligned}$$

$$\chi^2_{0.05,4} = 9.488$$

$$\text{critical value: } \chi^2_{0.05,4} = 9.488$$



Texture	Colour			Total
	Light	Medium	Dark	
Fine	4	20	8	32
Medium	5	23	12	40
Coarse	21	23	4	48
Total	30	66	24	120

Cell, i,j	O_{ij}	E_{ij}	$(O_{ij} - E_{ij})^2 / E_{ij}$
1,1	4	$\frac{32 \times 30}{120} = 8$	2.0000
1,2	20	$\frac{32 \times 66}{120} = 17.6$	0.3273
1,3	8	$\frac{32 \times 24}{120} = 6.4$	0.4000
2,1	5	$\frac{40 \times 30}{120} = 10$	2.5000
2,2	23	$\frac{40 \times 66}{120} = 22$	0.0455
2,3	12	$\frac{40 \times 24}{120} = 8$	2.0000
3,1	21	$\frac{48 \times 30}{120} = 12$	6.7500
3,2	23	$\frac{48 \times 66}{120} = 26.4$	0.4379
3,3	4	$\frac{48 \times 24}{120} = 9.6$	3.2667

$$\begin{aligned} \chi^2 &= 2.0000 + 0.3273 + 0.4000 + 2.5000 + 0.0455 + 2.0000 + 6.7500 + 0.4379 + 3.2667 \\ &= 17.727 \end{aligned}$$

since $\chi^2 = 17.727 > \chi^2_{0.01,4}$, lies in critical region, reject H_0

Conclusion:

At 0.05 significance level, there is sufficient evidence to conclude that there is relationship exist between texture and colour.

- 2(a) $H_0: p_1 = 0.56, p_2 = 0.19, p_3 = 0.19, p_4 = 0.06$
 $H_1: \text{At least one of the proportions is different from the claimed value}$

$$\alpha = 0.05$$

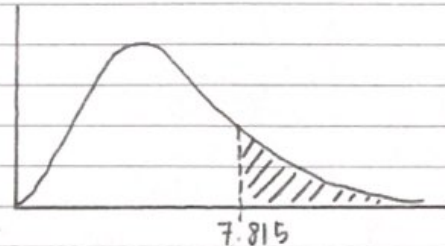
$$df = k - 1$$

$$= 4 - 1$$

$$= 3$$

$$\chi^2_{0.05, 3} = 7.815$$

$$\text{Critical value: } \chi^2_{0.05, 3} = 7.815$$



Mice	p	O	$E = n \times p$	$(O - E)^2 / E$
brown mice with pink eyes	0.56	120	121.52	0.0190
brown mice with brown eyes	0.19	48	41.23	1.1116
white mice with pink eyes	0.19	36	41.23	0.6634
white mice with brown eyes	0.06	13	13.02	0.0003

$$n = 120 + 48 + 36 + 13$$

$$= 217$$

$$\chi^2 = 0.0190 + 1.1116 + 0.6634 + 0.0003$$

$$= 1.794$$

Since $\chi^2 = 1.794 < \chi^2_{0.05, 3}$, does not lie in critical region, fail to reject H_0

Conclusion :

At 0.05 significance level, there is sufficient evidence to conclude that the theory that predicts the types of mice are obtained with the genetic percentage of 56%, 19%, 19% and 6% respectively.

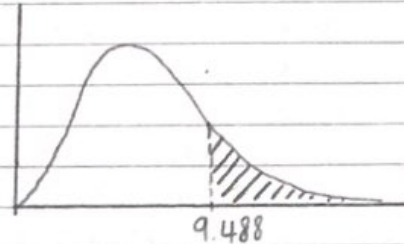
- (b) H_0 : There is no homogeneity among the shops' repair distribution
 H_1 : There is homogeneity among the shops' repair distribution

$$\alpha = 0.05$$

$$df = (r-1)(c-1) \\ = (3-1)(3-1) \\ = 4$$

$$\chi^2_{0.05,4} = 9.488$$

$$\text{Critical value: } \chi^2_{0.05,4} = 9.488$$



Repair	Shop1	Shop2	Shop3	Total
Complete	78	56	54	188
Adjustment	15	30	31	76
Incomplete	7	14	15	36
Total	100	100	100	300

Cell, ij	O_{ij}	E_{ij}	$(O_{ij} - E_{ij})^2 / E_{ij}$
1,1	78	$\frac{188 \times 100}{300} = \frac{188}{3}$	3.7518
1,2	56	$\frac{188 \times 100}{300} = \frac{188}{3}$	0.7092
1,3	54	$\frac{188 \times 100}{300} = \frac{188}{3}$	1.1986
2,1	15	$\frac{76 \times 100}{300} = \frac{76}{3}$	4.2149
2,2	30	$\frac{76 \times 100}{300} = \frac{76}{3}$	0.8596
2,3	31	$\frac{76 \times 100}{300} = \frac{76}{3}$	1.2675
3,1	7	$\frac{36 \times 100}{300} = 12$	2.0833
3,2	14	$\frac{36 \times 100}{300} = 12$	0.3333
3,3	15	$\frac{36 \times 100}{300} = 12$	0.7500

$$\chi^2 = 3.7518 + 0.7092 + 1.1986 + 4.2149 + 0.8596 + 1.2675 + 2.0833 + 0.3333 + 0.7500 \\ = 15.168$$

Since $\chi^2 = 15.168 > \chi^2_{0.05,4}$, lies in critical region, reject H_0 .

Conclusion:

At 0.05 significance level, there is sufficient evidence to conclude that there is homogeneity among the shops' repair distribution.

$$3(a)(i) \quad H_0 : p_1 = 0.44, p_2 = 0.45, p_3 = 0.08, p_4 = 0.03$$

H_1 : At least one of the proportions is different from the claimed value

$$1 - \alpha = 0.95$$

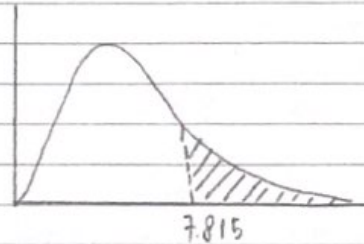
$$\alpha = 0.05$$

$$df = k - 1$$

$$= 4 - 1$$

$$= 3$$

$$\text{critical value} : \chi^2_{0.05, 3} = 7.815$$



Blood group	p	O	E = n * p	(O-E) ² /E
O	0.44	67	82.28	2.8376
A	0.45	83	84.15	0.0157
B	0.08	29	14.96	13.1766
AB	0.03	8	5.61	1.0182

$$\chi^2 = 2.8376 + 0.0157 + 13.1766 + 1.0182$$

$$= 17.0481$$

since $\chi^2 = 17.0481 > \chi^2_{0.05, 3}$, lies in critical region, hence reject H_0

- (ii) At 0.05 significance level, there is sufficient evidence to conclude that there is at least one of the proportions is different from the claimed value.

(b)(i) H_0 : There is no relationship between student's achievement in Mathematics and science subjects.

H_1 : There is relationship between student's achievement in Mathematics and science subjects.

(ii)

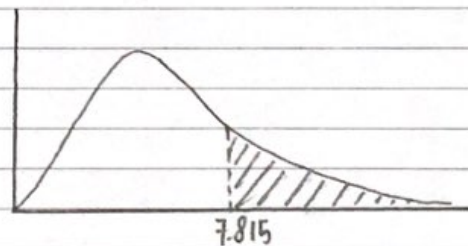
Subjects / Grade	Achievement (%)				Total
	A	B	C	D	
Mathematics	14.7	18.3	18.4	27.0	78.4
Science	6.8	32.0	36.2	19.4	94.4
Total	21.5	50.3	54.6	46.4	172.8

Cell, ij	O_{ij}	E_{ij}	$(O_{ij} - E_{ij})^2 / E_{ij}$
1,1	14.7	$\frac{78.4 \times 21.5}{172.8} = 9.7546$	2.5072
1,2	18.3	$\frac{78.4 \times 50.3}{172.8} = 22.8213$	0.8957
1,3	18.4	$\frac{78.4 \times 54.6}{172.8} = 24.7772$	1.6391
1,4	27.0	$\frac{78.4 \times 46.4}{172.8} = 21.0519$	1.6806
2,1	6.8	$\frac{94.4 \times 21.5}{172.8} = 11.7454$	2.0823
2,2	32.0	$\frac{94.4 \times 50.3}{172.8} = 27.4787$	0.7439
2,3	36.2	$\frac{94.4 \times 54.6}{172.8} = 29.8278$	1.3613
2,4	19.4	$\frac{94.4 \times 46.4}{172.8} = 25.3481$	1.3958

$$\chi^2 = 2.5072 + 0.8957 + 1.6391 + 1.6806 + 2.0823 + 0.7439 + 1.3613 + 1.3958$$

$$= 12.306$$

(iii) $\alpha = 0.05$
 $df = (r-1)(c-1)$
 $= (2-1)(4-1)$
 $= 3$
 $\chi^2_{0.05,3} = 7.815$



Since $\chi^2 = 12.306 > \chi^2_{0.05,3}$, lies in critical region, hence reject H_0 .

Conclusion:

At 0.05 significance level, there is sufficient evidence to conclude that there is relationship between student's achievement in Mathematics and Science subjects.