

PSDA Assignment 5

1(a) $H_0: \sigma_1^2 = \sigma_2^2$
 $H_1: \sigma_1^2 > \sigma_2^2$

$$\alpha = 0.05$$

$$\text{numerator, } n_1 - 1 = 10 - 1 \\ = 9$$

$$\text{denominator, } n_2 - 1 = 15 - 1 \\ = 14$$

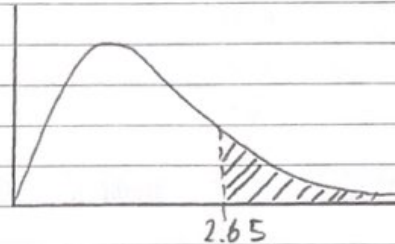
$$\text{critical value: } F_{0.05, 9, 14} = 2.65$$

test statistic:

$$F = \frac{S_1^2}{S_2^2}$$

$$= \frac{66.9^2}{728^2}$$

$$= 0.844$$



Since $F = 0.844 < F_{0.05, 9, 14}$, does not lie in critical region. Hence fail to reject H_0 .

Conclusion:

In 0.05 significant level, there is insufficient evidence to support the claim that the variation of birth weights for the placebo population is less than the variation of the population treated with zinc supplements.

(b) $H_0: \mu_D = 0$
 $H_1: \mu_D > 0$

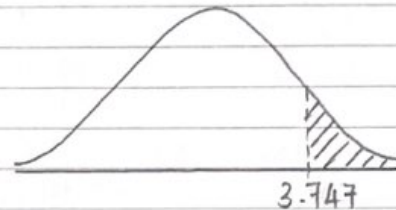
$$\alpha = 0.01$$

$$df = n - 1$$

$$= 5 - 1$$

$$= 4$$

$$\text{critical value: } t_{0.01, 4} = 3.747$$



$$\bar{D} = \frac{274 + 83 + 29 + 236 + 121}{5}$$

$$= 148.6$$

$$S_D = \sqrt{\frac{\sum D^2 - \frac{(\sum D)^2}{n}}{n - 1}}$$

$$= \sqrt{\frac{(75076 + 6889 + 841 + 55696 + 14641) - \frac{(274 + 83 + 29 + 236 + 121)^2}{5}}{5 - 1}}$$

$$= \sqrt{\frac{153143 - \frac{(743)^2}{5}}{4}}$$

$$= 103.360$$

test statistic :

$$t = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}$$

$$= \frac{148.6 - 0}{\frac{103.360}{\sqrt{5}}}$$

$$= 3.215$$

Since $t = 3.215 < t_{0.01, 4}$, does not lie in critical region. Hence fail to reject H_0 .

Conclusion:

In 0.01 significant level, there is insufficient evidence to support the claim that the independent workshop has lower repair costs.

2(a)

Babies having flu	
With medicine	Without medicine
$n_1 = 120$	$n_2 = 280$
$x_1 = 29$	$x_2 = 56$
$\hat{p}_1 = \frac{29}{120} = 0.242$	$\hat{p}_2 = \frac{56}{280} = 0.200$

$$\alpha = 0.01$$

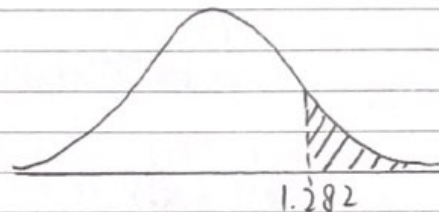
$$H_0 : \hat{p}_1 = \hat{p}_2$$

$$H_1 : \hat{p}_1 > \hat{p}_2$$

$$\alpha = 0.01$$

$$Z_\alpha = 1.282$$

$$\text{critical value : } Z_\alpha = 1.282$$



$$\begin{aligned}\bar{p} &= \frac{x_1 + x_2}{n_1 + n_2} \\ &= \frac{29 + 56}{120 + 280} \\ &= 0.2125\end{aligned}$$

$$\begin{aligned}\bar{q} &= 1 - \bar{p} \\ &= 1 - 0.2125 \\ &= 0.7875\end{aligned}$$

$$\begin{aligned}Z &= \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} \\ &= \frac{(0.242 - 0.200) - 0}{\sqrt{\frac{(0.2125 \times 0.7875)}{120} + \frac{(0.2125 \times 0.7875)}{280}}} \\ &= \frac{0.042}{\sqrt{\frac{0.167}{120} + \frac{0.167}{280}}} \\ &= 0.941\end{aligned}$$

Since $Z = 0.941 < Z_\alpha$, does not lie in critical region. Hence fail to reject H_0

Conclusion:

In 0.01 significant level, there is insufficient evidence to support the company's claim of the effectiveness of the flu medicine.

$$(b) \quad H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

$$\bar{X}_1 = \frac{0.48 + 0.39 + 0.42 + 0.52 + 0.40}{5}$$

$$= 0.442$$

$$\bar{X}_2 = \frac{0.38 + 0.37 + 0.39 + 0.41 + 0.38}{5}$$

$$= 0.386$$

$$S_1^2 = \frac{(0.48 - 0.442)^2 + (0.39 - 0.442)^2 + (0.42 - 0.442)^2 + (0.52 - 0.442)^2 + (0.40 - 0.442)^2}{5-1}$$

$$= 0.00312$$

$$S_2^2 = \frac{(0.38 - 0.386)^2 + (0.37 - 0.386)^2 + (0.39 - 0.386)^2 + (0.41 - 0.386)^2 + (0.38 - 0.386)^2}{5-1}$$

$$= 0.00023$$

$$\alpha = 0.05$$

$$\frac{\alpha}{2} = 0.025$$

$$df = n_1 + n_2 - 2$$

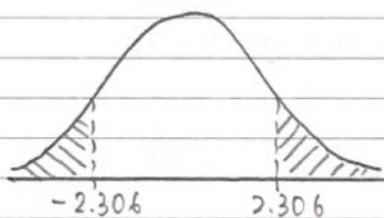
$$= 5 + 5 - 2$$

$$= 8$$

$$t_{0.025, 8} = 2.306$$

$$\text{critical value : } t_{0.025, 8} = 2.306$$

$$-t_{0.025, 8} = -2.306$$



$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(5-1)(0.00312) + (5-1)(0.00023)}{5+5-2}$$

$$= 0.001675$$

$$T = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{0.442 - 0.386 - 0}{(\sqrt{0.001675}) \sqrt{\frac{1}{5} + \frac{1}{5}}}$$

$$= 2.163$$

Since $-t_{0.025, 8} < T = 2.163 < t_{0.025, 8}$, does not lie in critical region. Hence fail to reject H_0 .

In 0.05 significant level, there is insufficient evidence to support the claim that the production line 1 is not producing as consistently as production line 2 in terms of the alcohol content.

(c) $H_0: \mu_0 = 0$
 $H_1: \mu_0 < 0$

											Σ
Before, x_1	204	393	391	265	326	220	423	342	480	464	
After, x_2	223	412	402	285	353	243	443	340	582	490	
$D(x_1 - x_2)$	-19	-19	-11	-20	-27	-23	-20	2	-102	-26	-265
$D^2(x_1 - x_2)^2$	361	361	121	400	729	529	400	4	10404	676	13985

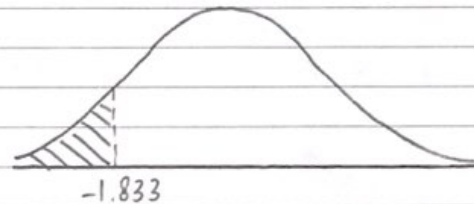
$$\alpha = 0.05$$

$$df = n - 1$$

$$= 10 - 1$$

$$= 9$$

$$\text{critical value: } -t_{0.05, 9} = -1.833$$



$$S_D = \sqrt{\frac{\sum D^2 - \frac{(\sum D)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{13985 - \frac{(-265)^2}{10}}{10-1}}$$

$$= 27.814$$

$$\bar{D} = \frac{\sum D}{n}$$

$$= \frac{-265}{10}$$

$$= -26.5$$

$$t = \frac{\bar{D} - \mu_0}{\frac{S_D}{\sqrt{n}}}$$

$$= \frac{-26.5 - 0}{\frac{27.814}{\sqrt{10}}}$$

$$= -3.013$$

Since $t = -3.013 < -t_{0.05, 9}$, lies in critical region, reject H_0

Conclusion:

In 0.05 significance level, there is sufficient evidence to conclude that training can increase the number of words spelled correctly by the participants.

3(a) $H_0: \mu_1 = \mu_2$
 $H_1: \mu_1 < \mu_2$

$$\bar{X}_1 = \frac{71+75+65+69+73+66+68+71+74+68}{10}$$

$$= 70$$

$$\bar{X}_2 = \frac{72+77+84+78+69+70+77+73+65+75}{10}$$

$$= 74$$

$$S_1^2 = \frac{(71-70)^2 + (75-70)^2 + (65-70)^2 + (69-70)^2 + (73-70)^2 + (66-70)^2 + (68-70)^2 + (71-70)^2 + (74-70)^2 + (68-70)^2}{10-1}$$

$$= \frac{34}{3}$$

$$S_2^2 = \frac{(72-74)^2 + (77-74)^2 + (84-74)^2 + (78-74)^2 + (69-74)^2 + (70-74)^2 + (77-74)^2 + (73-74)^2 + (65-74)^2 + (75-74)^2}{10-1}$$

$$= \frac{262}{9}$$

$$\alpha = 0.05$$

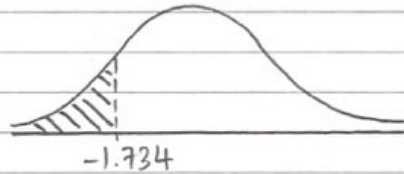
$$df = n_1 + n_2 - 2$$

$$= 10 + 10 - 2$$

$$= 18$$

$$t_{0.05, 18} = 1.734$$

$$\text{critical value: } -t_{0.05, 18} = -1.734$$



$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(10-1)\left(\frac{34}{3}\right)^2 + (10-1)\left(\frac{262}{9}\right)^2}{10+10-2}$$

$$= \frac{39524}{81}$$

$$t = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{70 - 74 - 0}{\left(\sqrt{\frac{39524}{81}}\right) \left(\sqrt{\frac{1}{10} + \frac{1}{10}}\right)}$$

$$= -0.405$$

since $t = -0.405 > -t_{0.05, 18}$, does not lie in critical region. Hence, fail to reject H_0 .

Conclusion:

At 0.05 significance level, there is insufficient evidence to support the claim that Method B is more effective.

(b) $H_0: \mu_0 = 0$
 $H_1: \mu_0 > 0$

Before, x_1	After, x_2	$D(x_1 - x_2)$	$D^2(x_1 - x_2)^2$
95	89	6	36
81	77	4	16
76	77	-1	1
96	94	2	4
82	80	2	4
87	86	1	1
71	72	-1	1
82	82	0	0
77	74	3	9
83	81	2	4
81	78	3	9
65	63	2	4
		$\Sigma D = 23$	$\Sigma D^2 = 89$

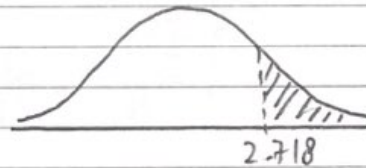
$$\alpha = 0.01$$

$$df = n - 1$$

$$= 12 - 1$$

$$= 11$$

$$\text{critical value: } t_{0.01, 11} = 2.718$$



$$S_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{89 - \frac{(23)^2}{12}}{12-1}}$$

$$= 2.021$$

$$\bar{D} = \frac{\Sigma D}{n}$$

$$= \frac{23}{12}$$

$$t = \frac{\bar{D} - \mu_0}{\frac{S_D}{\sqrt{n}}}$$

$$= \frac{\frac{23}{12} - 0}{\frac{2.021}{\sqrt{12}}}$$

$$= 3.285$$

Since $t = 3.285 > t_{0.01, 11}$, lies in critical region, hence, reject H_0 .

Conclusion:

At 0.01 significance level, there is sufficient evidence to support the claim that the prescribed program of exercise is effective.

4(a)

18-34 age group	35+ age group
$n_1 = 150$	$n_2 = 250$
$\bar{X}_1 = 75.50$	$\bar{X}_2 = 65.20$
$S_1 = 53.20$	$S_2 = 46.10$

$\alpha = 0.05$

Variances unknown

assume variances are not equal

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

Since $n > 30$,

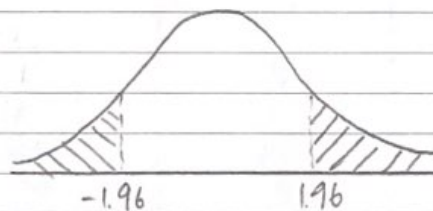
$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$Z_{\frac{\alpha}{2}} = 1.96$

Critical value: $Z_{\frac{\alpha}{2}} = 1.96$

$-Z_{\frac{\alpha}{2}} = -1.96$



$$\begin{aligned}
 Z &= \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \\
 &= \frac{75.50 - 65.20 - 0}{\sqrt{\frac{(53.20)^2}{150} + \frac{(46.10)^2}{250}}} \\
 &= 0.287
 \end{aligned}$$

Since $-Z_{\frac{\alpha}{2}} < Z = 0.287 < Z_{\frac{\alpha}{2}}$, does not lie in critical region. Hence, fail to reject H_0 .

Conclusion:

At 0.05 significant level, there is sufficient evidence to support the claim that there is no difference in the mean spending between these two population.

(b) $H_0: \mu_0 = 3$
 $H_1: \mu_0 > 3$

1 hour later (X_1)	24 hour later (X_2)	$D(X_1 - X_2)$	$D^2(X_1 - X_2)^2$
14	10	4	16
12	4	8	64
18	14	4	16
7	5	2	4
11	8	3	9
9	5	4	16
16	11	5	25
15	12	3	9
13	9	4	16
17	10	7	49
		$\Sigma D = 44$	$\Sigma D^2 = 224$

$$\alpha = 0.05$$

$$df = n - 1$$

$$= 10 - 1$$

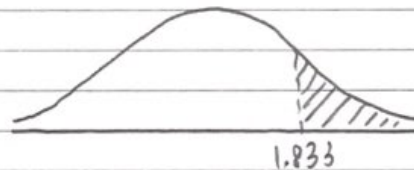
$$= 9$$

$$\text{Critical value: } t_{0.05, 9} = 1.833$$

$$S_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{224 - \frac{44^2}{10}}{10-1}}$$

$$= 1.838$$



$$\bar{D} = \frac{\Sigma D}{n}$$

$$= \frac{44}{10}$$

$$= 4.4$$

$$t = \frac{\bar{D} - \mu_0}{\frac{S_D}{\sqrt{n}}}$$

$$= \frac{4.4 - 3}{\frac{1.838}{\sqrt{10}}}$$

$$= 2.409$$

Since $t = 2.409 > t_{0.05, 9}$, lies in critical region, hence, reject H_0

Conclusion:

At 0.05 significance level, there is sufficient evidence to conclude that the mean number of words recalled after 1 hour exceeds the mean recall after 24 hours by more than 3.