

PSDA (Assignment 6)

Question 1

(a) $H_0: p_1 = p_2 = p_3 = p_4 = p_5 = p_6$

$\alpha = 0.05, df = k - 1 = 6 - 1 = 5, \chi^2_{5,0.05} = 11.071$

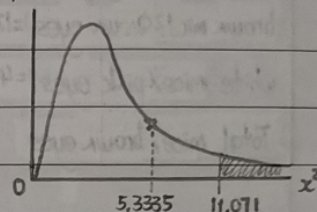
H_1 : At least 1 of the 6 proportions is different from others.

$\therefore$ Since test statistics $\chi^2 = 5.3335 < \text{critical value } \chi^2 = 11.071$, we fail to reject null hypothesis H_0 . There is sufficient evidence to prove that the device is not biased.

$E = \frac{n}{k} = \frac{36}{6} = 6$

Card Number	1	2	3	4	5	6
Observed Frequency	8	5	9	2	7	5
Expected Frequency	6	6	6	6	6	6
$(O-E)^2/E$	0.6667	0.1667	1.5000	2.6667	0.1667	0.1667

$\chi^2 = \sum \frac{(O-E)^2}{E} = 0.6667 + 0.1667 + 1.5 + 2.6667 + 0.1667 + 0.1667 = 5.3335$



(b) i) $H_0: p_{\text{Above}} = 0.159, p_{\text{To+1}} = 0.341, p_{\text{To0}} = 0.341, p_{\text{Below-1}} = 0.159$ ii) Test statistics: $\chi^2 = 12.156$

H_1 : At least one of the proportions is different from the claimed value.

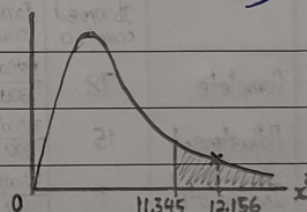
$\alpha = 0.01, df = k - 1 = 4 - 1 = 3, \chi^2_{0.01,3} = 11.345$

$\therefore$ Since test statistics $\chi^2 = 12.156 > \text{critical value } \chi^2 = 11.345$, we reject null hypothesis H_0 . There is sufficient evidence to conclude that the CGPA distribution of the final year students is not normally distributed.

$n = 9 + 60 + 17 + 19 = 105$

Expected frequencies:

Range	Expected Frequency, E
Above 1	$np = 105(0.159) = 16.695$
0 to 1	$np = 105(0.341) = 35.805$
+1 to 1	$np = 105(0.341) = 35.805$
Below -1	$np = 105(0.159) = 16.695$



(c)

	Colour									
Texture	Light			Medium			Dark			Total
	Observed Count, o	Expected Count, e	$[o-e]^2/e$	Observed Count, o	Expected count, e	$[o-e]^2/e$	Observed Count, o	Expected Count, e	$[o-e]^2/e$	
Fine	4	$\frac{32 \times 30}{120} = 8$	$\frac{(4-8)^2}{8} = 2$	20	$\frac{32 \times 66}{120} = 17.6$	$\frac{(20-17.6)^2}{17.6} = 0.3273$	8	$\frac{32 \times 24}{120} = 6.4$	$\frac{(8-6.4)^2}{6.4} = 0.4$	32
Medium	5	$\frac{40 \times 30}{120} = 10$	$\frac{(5-10)^2}{10} = 2.5$	23	$\frac{40 \times 66}{120} = 22$	$\frac{(23-22)^2}{22} = 0.0455$	12	$\frac{40 \times 24}{120} = 8$	$\frac{(12-8)^2}{8} = 2$	40
Coarse	21	$\frac{48 \times 30}{120} = 12$	$\frac{(21-12)^2}{12} = 6.75$	23	$\frac{48 \times 66}{120} = 26.4$	$\frac{(23-26.4)^2}{26.4} = 0.4379$	4	$\frac{48 \times 24}{120} = 9.6$	$\frac{(4-9.6)^2}{9.6} = 3.2667$	48
Total	30	30	$\sum \frac{(o-e)^2}{e} = 11.25$	66	66	$\sum \frac{(o-e)^2}{e} = 0.8107$	24	24	$\sum \frac{(o-e)^2}{e} = 5.6667$	120

Given hypothesis:

H_0 : No relationship between texture and colour

H_1 : Relationship exist between texture and colour

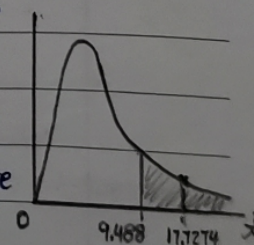
Test statistics, $\chi^2 = \sum \frac{(O-e)^2}{e} = 11.25 + 0.8107 + 5.6667 = 17.7274$

$\alpha = 0.05, df = (3-1)(3-1) = 4, \text{critical value, } \chi^2_{0.05,4} = 9.488$

$\therefore$ Since test statistics $\chi^2 = 17.7274 > \text{critical value}$

$\chi^2 = 9.488$, we reject null hypothesis H_0 .

There is sufficient evidence to conclude that there is association between colour and texture for these waste materials.



Question 2

(a) $H_0: p_{\text{brown, pink}} = 0.56, p_{\text{brown, brown}} = 0.19, p_{\text{white, pink}} = 0.19, p_{\text{white, brown}} = 0.06$

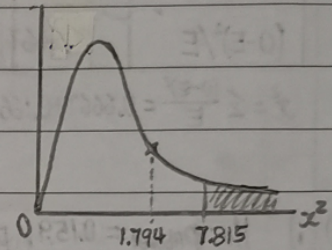
H_1 : At least one of the proportions is different from the claimed value.

Mice	Observed frequency, O	Expected frequency, $E = np$	$(O-E)^2/E$
brown mice, pink eyes	120	$217(0.56) = 121.52$	$\frac{(120-121.52)^2}{121.52} = 0.019$
brown mice, brown eyes	48	$217(0.19) = 41.23$	$\frac{(48-41.23)^2}{41.23} = 1.1116$
white mice, pink eyes	36	$217(0.19) = 41.23$	$\frac{(36-41.23)^2}{41.23} = 0.6634$
white mice, brown eyes	13	$217(0.06) = 13.02$	$\frac{(13-13.02)^2}{13.02} = 3.072 \times 10^{-5}$
$\sum \frac{(O-E)^2}{E} = \chi^2$			1.794

$$n = 120 + 48 + 36 + 13 = 217$$

$$\text{Test statistics, } \chi^2 = \sum \frac{(O-E)^2}{E} = 1.794$$

$$\alpha = 0.05, df = k - 1 = 4 - 1 = 3, \text{ c.v. } \chi_{0.05, 3}^2 = 7.815$$



$\therefore$ Since test statistics, $\chi^2 = 1.794 < \text{critical value } \chi^2 = 7.815$, we fail to reject null hypothesis H_0 . There is sufficient evidence to prove that the data is compatible with the theory prediction of genetic percentage of each mice type.

(b) H_0 : There is no homogeneity among the shops' repair distribution.

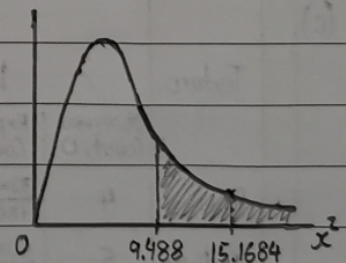
H_1 : Homogeneity exists among the shops' repair distribution.

Repair	Shop 1			Shop 2			Shop 3			Total
	Observed count, O	Expected count, e	$(O-e)^2/e$	Observed count, O	Expected count, e	$(O-e)^2/e$	Observed count, O	Expected count, e	$(O-e)^2/e$	
Complete	78	$\frac{188(100)}{300} = 62.667$	$\frac{(78-62.667)^2}{62.667} = 3.7516$	56	$\frac{180(100)}{300} = 62.667$	$\frac{(56-62.667)^2}{62.667} = 0.7093$	54	$\frac{180(100)}{300} = 62.667$	$\frac{(54-62.667)^2}{62.667} = 1.1981$	188
Adjustment	15	$\frac{76(100)}{300} = 25.333$	$\frac{(15-25.333)^2}{25.333} = 4.2147$	30	$\frac{76(100)}{300} = 25.333$	$\frac{(30-25.333)^2}{25.333} = 0.8598$	31	$\frac{76(100)}{300} = 25.333$	$\frac{(31-25.333)^2}{25.333} = 1.2671$	76
Incomplete	7	$\frac{36(100)}{300} = 12$	$\frac{(7-12)^2}{12} = 2.0833$	14	$\frac{36(100)}{300} = 12$	$\frac{(14-12)^2}{12} = 0.3333$	15	$\frac{36(100)}{300} = 12$	$\frac{(15-12)^2}{12} = 0.75$	36
Total	100	100	$\sum = 10.0496$	100	100	$\sum = 1.9024$	100	100	$\sum = 3.2164$	300

$$\text{Test statistics, } \chi^2 = \sum \frac{(O-e)^2}{e} = 10.0496 + 1.9024 + 3.2164 = 15.1684$$

$$\alpha = 0.05, df = (3-1)(3-1) = 4, \text{ critical value, } \chi_{0.05, 4}^2 = 9.488$$

$\therefore$ Since test statistics, $\chi^2 = 15.1684 > \text{critical value } \chi^2 = 9.488$, we reject null hypothesis H_0 . There is sufficient evidence to conclude that there is homogeneity among the shops' repair distribution.



Question 3

(a) i) $H_0: p_O = 0.44, p_A = 0.45, p_B = 0.08, p_{AB} = 0.03$

H_1 : At least one of the proportions is different from the claimed value.

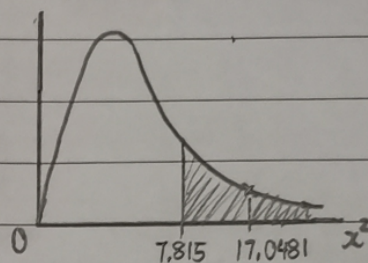
ii) Conclusion: Since test statistics $\chi^2 = 17.0481 >$ critical value $\chi^2 = 7.815$, we reject null hypothesis H_0 . There is sufficient evidence to conclude that the distribution of the blood group is different from expected at 95% confidence interval.

Blood group	Observed frequency, O	Expected frequency, E=np	$(O-E)^2/E$
O	67	$187(0.44) = 82.28$	$\frac{(67-82.28)^2}{82.28} = 2.8376$
A	83	$187(0.45) = 84.15$	$\frac{(83-84.15)^2}{84.15} = 0.0157$
B	29	$187(0.08) = 14.96$	$\frac{(29-14.96)^2}{14.96} = 13.1766$
AB	8	$187(0.03) = 5.61$	$\frac{(8-5.61)^2}{5.61} = 1.0182$
Total	187		$\sum \frac{(O-E)^2}{E} = 17.0481$

Test statistics, $\chi^2 = \sum \frac{(O-E)^2}{E} = 17.0481$

$\alpha = 1 - 0.95 = 0.05$, $df = 4 - 1 = 3$, critical value, $\chi_{0.05, 3}^2 = 7.815$

Evidence: Test statistics $\chi^2 = 17.0481 >$ critical value $\chi^2 = 7.815$



(b) i) H_0 : There is no relationship between students' achievement in Mathematics and Science subjects.

H_1 : Relationship exists between students' achievement in Mathematics and Science subjects.

ii)	Subjects	Achievement (%)								Total
		A		B		C		D		
		Observed count, o	Expected count, e	Observed count, o	Expected count, e	Observed count, o	Expected count, e	Observed count, o	Expected count, e	
	Mathematics	14.7	$\frac{78.4(21.5)}{172.8} = 9.755$	18.3	$\frac{78.4(50.3)}{172.8} = 22.821$	18.4	$\frac{78.4(54.6)}{172.8} = 24.772$	27.0	$\frac{78.4(46.4)}{172.8} = 21.052$	78.4
	Science	6.8	$\frac{94.4(21.5)}{172.8} = 11.745$	32.0	$\frac{94.4(50.3)}{172.8} = 27.479$	36.2	$\frac{94.4(54.6)}{172.8} = 29.828$	19.4	$\frac{94.4(46.4)}{172.8} = 25.348$	94.4
	Total	21.5	21.5	50.3	50.3	54.6	54.6	46.4	46.4	172.8

$\therefore$ Test statistics, $\chi^2 = \sum \frac{(O-E)^2}{E} = 2.5067 + 0.8956 + 1.6390 + 1.6805 + 2.0820$

$+ 0.7438 + 1.3612 + 1.3957$, $\chi^2 = 12.3045$

Subject/Grade	O	e	$(O-E)^2/E$
Mathematics/A	14.7	9.755	$\frac{(14.7-9.755)^2}{9.755} = 2.5067$
Mathematics/B	18.3	22.821	$\frac{(18.3-22.821)^2}{22.821} = 0.8956$
Mathematics/C	18.4	24.772	$\frac{(18.4-24.772)^2}{24.772} = 1.6390$
Mathematics/D	27.0	21.052	$\frac{(27-21.052)^2}{21.052} = 1.6805$
Science/A	6.8	11.745	$\frac{(6.8-11.745)^2}{11.745} = 2.0820$
Science/B	32.0	27.479	$\frac{(32-27.479)^2}{27.479} = 0.7438$
Science/C	36.2	29.828	$\frac{(36.2-29.828)^2}{29.828} = 1.3612$
Science/D	19.4	25.348	$\frac{(19.4-25.348)^2}{25.348} = 1.3957$

$\alpha = 0.05$, $df = (4-1)(2-1) = 3$, critical value $\chi_{0.05, 3}^2 = 7.815$

iii) Since test statistics, $\chi^2 = 12.3045 >$ critical value $\chi^2 = 7.815$, we reject null hypothesis H_0 .

There is sufficient evidence to conclude that there is relationship between achievements of Mathematics and Science subjects.

